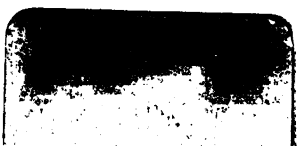

# HANDBOOK

OF

# STREET-RAILROAD LOCATION.

BY

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# PREFACE.

THIS book is designed to present to the student and engineer that part of the subject of street-railroad construction which is to be performed by the civil, rather than by the electrical engineer. It therefore includes, besides the trigonometrical problems pertaining to railroad curves, the general principles governing the choice of site, grades and curves. It is assumed that every one using the book will be familiar with the practice of land surveying and with the manipulation of surveying instruments, and no space is given to these subjects.

The arrangement is believed to be that which gives the best result in the classroom, while the form and size are convenient for use in the field. Every principle is illustrated by the solution of a numerical example, and at the end of each article several problems are proposed to test the proficiency of the student.

Especial attention is called to the presentation of the subject of compound curves and the use of them as transition curves. The table prepared gives the deflection angles from every chord to every point on the transition curve which is located by rationally increasing radii. It is explained how this table may be applied to main curves of radii between forty and twenty-five hundred feet. A similar table adapted to steam railroads is given by Mr. Godwin in his "Railroad Engineer's Field-book."

It is hoped that this book may be instrumental in obtaining for street-railroad construction the same degree of precision and permanence that is found in the best examples of American steam-railway practice.

JOHN P. BROOKS.

LEXINGTON, KY., Sept. 24, 1897.

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# HANDBOOK

OF

# STREET RAILROAD LOCATION.

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## CHAPTER I.

### RAILROAD CURVES.

#### ART. 1. TRIGONOMETRICAL RELATIONS.

Railroad curves are, in general, arcs of circles, and other forms which are sometimes employed will be considered else-

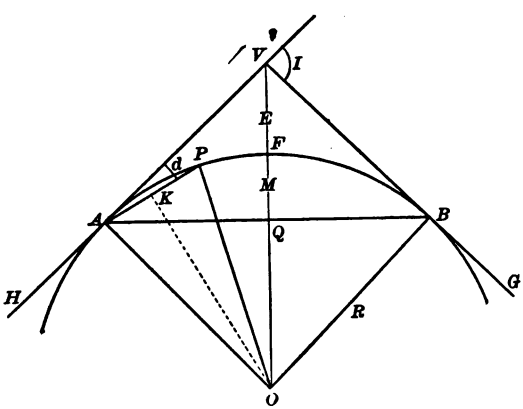


FIG. 1.

where. In Fig. 1, $AFB$ is a curve joining two straight lines or tangents $HV$ and $GV$, tangent at $A$ and $B$ respectively.

The initials of the words Point of Curve and Point of Tangency, P.C. and P.T., indicate the beginning and end of the curve. The lines $AV$ and $BV$ are called Tangent Distances, $T$. The distance $AB$ is called the Long Chord to distinguish it from any other chord, $C$, as $AP$. On the line $VO$, joining the vertex and the center, $VF$ is the External Distance, $E$, and $FQ$ the Middle Ordinate, $M$. The exterior angle at $V$ between the tangents is the Intersection Angle, $I$, which equals $AOB$, and the angle between any chord and a tangent at the end of the chord is a Deflection Angle, $d$.

In Fig. 1, $AO$ is perpendicular to $AV$, hence

$$AV = AO \tan AOV,$$

$$\text{or} \qquad T = R \tan \frac{I}{2},$$

$$\text{and} \qquad R = T \cot \frac{I}{2}.$$

Let $OK$ be drawn perpendicular to $AP$, then $VAP = \frac{1}{2}AOP = AOK$, and

$$\sin d = \frac{C}{2R}.$$

From Fig. 1,

$$AB = 2T \cos \tfrac{1}{2}I;$$

$$AB = 2R \sin \tfrac{1}{2}I;$$

$$\text{also} \qquad AB = 2M \cot \tfrac{1}{2}I.$$

The relation between the length of an arc and its subtended angle is the same as that of a semi-circumference to 180°; hence if $L$ be the length of $AFB$, $L : AOB = \pi R : 180°$, whence $L = IR\frac{\pi}{180}$, or

$$L = .0174533IR,$$

in which $I$ is the angle $AOB$ expressed in degrees and fraction of a degree.

In Fig. 1, $QF = OF - OQ$, or $R - R \cos \frac{I}{2}$; then

$$M = R\left(1 - \cos \frac{I}{2}\right) = R \operatorname{vers} \frac{I}{2};$$

$$M = E \cos \frac{I}{2}.$$

In the triangle $VFB$ the angle $VBF = \frac{I}{4}$ and $BVF = \frac{1}{2}(180 - I)$; so

$$VF : VB = \sin VBF : \sin VFB,$$

or $$E = T \tan \frac{I}{4};$$

also $$E = R\left(\frac{1}{\cos \frac{I}{2}} - 1\right)$$

$$= R \operatorname{exsec} \frac{I}{2}.$$

Many other equations may be formed by solving any of the above for a different factor; for example, let it be required to determine the proper radius for a curve that shall pass 11 feet from the vertex of its tangents, which make an intersection angle 65° 82'. From the next to the last equation,

$$R = \frac{E \cos \frac{I}{2}}{1 - \cos \frac{I}{2}}$$

$$R = \frac{22 \times .84088}{.15912} = 58.1 \text{ feet}$$

With a table of exsecants at hand the labor of computation is somewhat lessened by using the next formula to that selected.

Prob. 1. In the above example compute the tangent distance, long chord, middle ordinate, and the deflection angle from the tangent to a 10-foot chord.

Prob. 2. Given: $E = 4\ 64$ feet, $M = 4\ 02$ feet. Compute $R$ and $I$.

## ART. 2. LOCATING CURVES.

(*a*) **By Offsets from a Tangent.**—In Fig. 2 the distance along

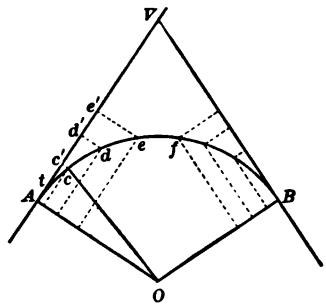


FIG. 2.

the tangent from $A$ to the foot of an offset, as $c'$, and the length of the offset, $c'c$, are given by

$$Ac' = t = R \sin cOA,$$

and $$cc' = f = 1 - R \cos cOA;$$

or $$f = R \operatorname{vers} cOA.$$

Let $C$ be the length of any chord, as $Ae$, then $f = \frac{C^2}{2R}$.

If the angles $cod$, $doe$, $eof$, . . . be equal, the arcs subtending them will, of course, be equal, which is a better arrangement than to make $Ac'$, $c'd'$, . . . equal. In the field, after $c$ is fixed, $Ad'$ is measured and $d$ is located at the intersection of arcs of radii $dd'$ and $dc$, with less chance of error than by erecting a perpendicular at $d'$. For example, let it be required to set stakes 10 feet apart on a curve of 50 feet radius and $I = 88° 24'$.

$$\sin \frac{1}{2} AOC = \frac{5}{R} = .100, \quad \text{and} \quad AOC = 11° 28'.$$

$$t_1 = 50 \times \sin 11° 28' = 9\ 94 \text{ feet};$$

$$t_2 = 50 \times \sin 2 \,.\, cOA = 19.48 \text{ feet};$$

$$t_3 = 50 \times \sin 3 \,.\, cOA = 28.25;$$

$$t_4 = 50 \times \sin 4 \,.\, cOA = 35.89.$$

The corresponding offsets are 1.0, 3.95, 8.74, and 15.18 feet The number of arcs in the curve is found from 88° 24′÷11° 28′ to be 7 +. The points $A$ and $B$ are located by measuring from $V$ the tangent distance $T = R \tan \frac{1}{2}I$, and four arcs of 10 feet chords may be laid off from $A$ and three from $B$. The remaining chord, $c = 2R \sin \frac{1}{2}(88° 24' - 7 \times 11° 28') = 7.09$ feet, whose measurement affords a test upon the accuracy of the work.

Prob. 3. In the above example compare the length of the curve with that of the eight chords.

(*b*) By Offsets from a Chord.—In Fig. 3 it is required to

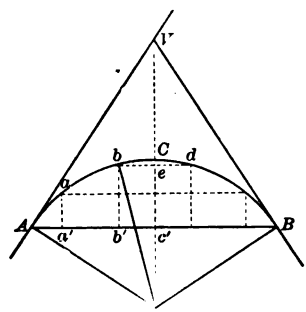


FIG. 3.

locate the curve $ACB$ by offsets from the chord $AB$. The distance $Cc'$ is first determined by $M = R\left(1 - \cos \frac{I}{2}\right)$. Then any offset, as $bb'$, is $c'C - Ce$.

$$bb' = c'C - R(1 - \cos COb),$$

or $$bb' = c'C - R \text{ vers } COb,$$

and $$cb' = R \sin COb.$$

$A$ and $B$ are fixed by measuring the tangent distances from $V$.

For example: to locate points ten feet apart on a curve when the long chord and radius are given as 58.4 feet and 45 f

feet respectively, $A$ and $B$ being fixed. Sin $\frac{1}{2}I = \frac{58\ 4}{90.0}$ and $I = 80°\ 56'$. $M = R(1 - \cos 40°\ 28') = 10.76$ feet. The angle subtended by a chord of ten feet will be 12° 46′, the distances from the middle of the chord to the offsets from $c'A$ and $c'B$ will be 9.95, 19.40, and 27.89, and the offsets 9.65, 6.36, and 1.08 feet respectively. The small chords at the ends of the curve are found to be $C' = 2R \sin \frac{1}{2}(80°.56' - 6 \times 12°\ 46') = 1.70$ feet. If it be desired to have the equal chords begin at $A$, the first arc $COb$ will be 2° 10′ and the second 12° 46′; the first arc on the right of the middle will be 10° 36′, and the last one 4° 20′. Since the points on the curve are no longer symmetrical with respect to $Cc'$, the offsets and distances along the chord must be computed for each one.

On very flat arcs the middle ordinate is $M = \frac{C^2}{8R}$ without material error.

(*c*) By Intersection of Two Chords.—If, in Fig. 4, the chord $Aa$ be assumed, the chord $aB$ can be computed by

$$aB = 2R \sin \tfrac{1}{2}(I - AOa).$$

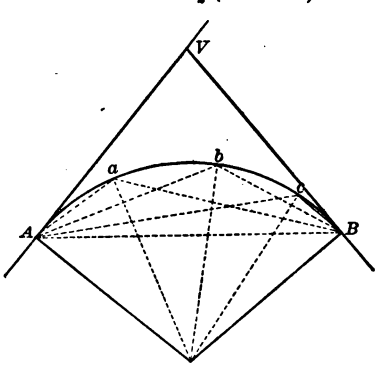


FIG 4.

$Ab$ subtends twice the arc $AOa$ and $bOB = I - 2AOa$. The chords from each point to $A$ and to $B$ may thus be computed and any point, as $b$, located by intersecting a small arc of radius

$Ab$ by one of radius $Bb$. This is done by first determining within a foot or so where the point will fall and driving temporary stakes, one inside and one outside the curve, about two feet apart, into which nails are driven on an arc of radius $Ab$, and a string is stretched between them. The divergence of this chord from its arc will be inappreciable, and the point on the string and the arc of radius $bB$ will be directly over $b$. This is not a good method of locating curves unless each of the chords is less than the length of the tape; it may, however, be applied to longer curves by first locating the middle of the curve by measuring the external distance from $V$ toward $O$ and locating the intermediate points in the manner described above. For example, let $I = 96° 42'$ and $R = 60$ feet. $E$ will be 30.28 feet, and the chord of $\frac{I}{2}$ equals 49.14 feet, so the middle point of the curve may be found at the intersection of arcs of these radii from $V$ and $A$ respectively as centers. The points 10 feet from $A$ and from $B$ will be 39.84 feet from the middle, those 10 feet from the last will be 30.27 feet from the middle, and so on.

(*d*) By Offsets from a Chord produced.—The angle $mab$, Fig. 5, between a chord and the previous chord produced is the

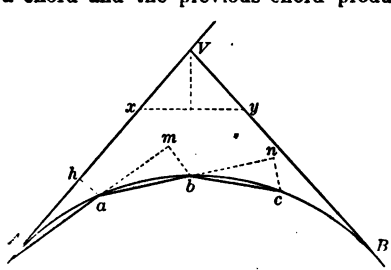


FIG. 5.

same as that at the center subtended by the chord, and the angle between the tangent and the first chord $VAa$ is one half this amount, or, if $bam = d$,

$$\sin \tfrac{1}{2}d = \frac{C}{2R};$$

$$mb = C \sin d,$$
$$ma = C \cos d.$$

The length of the last chord, $c'$, ending at $B$ will subtend an angle at the center of $I - nd$, when $n$ is the number of chords laid off from $A$.

$$C' = 2R \sin \tfrac{1}{2}(I - nd).$$

This is an effective method of setting the stakes on a curve without the aid of an instrument. The intersection angle, if unknown, may be approximately determined by laying off $Vx = Vy$ on $VA$ and $VB$ and measuring $xy$; then

$$\cos \frac{I}{2} = \frac{xy}{2\,Vx}.$$

The points $A$, $B$ and $a$ are fixed as described before. The ring at the end of the tape is hooked over the nail in stake $A$, and $C + am$ is measured on $Aa$ produced, locating $m$. The ring is then hooked at $a$, and the mark $C + mb$ held at $m$; the division on the tape indicating a chord distance from $a$ will be over $b$ when both sections of the tape are drawn taut.

For instance, let $Vx = 50$ feet, $xy = 72.02$ feet, and $R = 45$ feet, to set stakes 10 feet apart on the curve. Cos $\frac{1}{2}I = .7202$, $I = 92° 08'$. $T = 46.71$ feet. Sin $\frac{1}{2}d = .1111$, $d = 12° 46'$. The last chord will subtend an angle of $92\ 08 - 7\ (12° 46')$ or $2° 46'$ and be equal to 2.17 feet. The chord produced will be $am = 10 \cos 12° 46' = 9.75$ feet, and $mb = 10 \sin 12° 46' = 2.21$ feet.

(*e*) By Chords and Deflection Angles.—If the chords in Fig. 6 be all equal, they will subtend angles at $A$, each equal to $VAa$ or $d$, when $2d$ is the angle at the center of the curve subtended by the same chord. The angle at any point on the curve, as $b$, between a chord from $A$ produced, and the chord $bd$ from $b$ to any other point on the curve, is the same as the angle at $A$ between the tangent and $Ad$;

or $\quad gbd = VAd,$
also $\quad ecd = VAc$ or $aAd.$

With the instrument at $A$ the first chord $Aa$ is measured

making an angle $d$ with $AV$. The angle $VAg$ is next deflected from the tangent, and with the end of the tape at $a$, the mark indicating the length of chord is moved through a small arc till it comes into the line of sight, when the point $b$ is located. If the transitman be unable to read the graduations on the tape, a pencil may be held vertically on the proper division, and he will order it moved to the right or left till it is bisected by the line of sight.

After several points have been located from $A$ in the manner described above, the chords between them become nearer and nearer perpendicular to those from $A$, and hence successive points are fixed with less precision than those nearer $A$. In Fig. 6, if the end of the tape be at $d$, the pencil, held on the

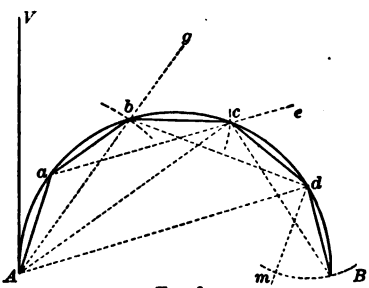


FIG 6.

proper division in swinging through the arc of radius $dB$ will move with reference to $AB$ nearly as the sine of $AdB$ varies. If this angle be near 90°, a small movement along the small arc could not be detected by the transitman, and in an extreme case $B$ might even be located at $m$. The proper method is to move the instrument to a point on the curve whose deflection angle from the tangent is about 45° and to locate the remaining points from there.

Before beginning the work of running the curve the deflection angle for each chord and the total deflection from the tangent to each point are computed and tabulated as shown below. When the instrument is moved from $A$ to some station as $b$, the back-sight is taken to any of the points already fixed, preferably a distant one, with the vernier set at the deflecti

from the tangent to the chord from $A$ to that point, so that the vernier will always read zero when the telescope is pointed to $A$. The line of sight is turned through 180°, either in azimuth or by revolving the telescope on its axis, and the veriner readings for all the following points are given in the column of total deflections in the following field-notes for the curve in Fig. 6:

| Station. | Distance. | Chord Deflection. | Total Deflection. |
|---|---|---|---|
| $A$ | 0 | 0° 00′ | 0° 00′ |
| $a$ | $C$ | $d$ | $d$ |
| $b$ | $C$ | $d$ | $2d$ |
| $c$ | $C$ | $d$ | $3d$ |
| $d$ | $C$ | $d$ | $4d$ |
| $B$ | $C'$ | $d'$ | $\frac{1}{2}I$ |

If the instrument be at $c$ and a back-sight is taken to $A$, the vernier reading is zero; if to $a$ or $b$, readings taken from the notes are $d$ or $2d$. The telescope is turned through 180° and the notes indicate that $4d$ is the proper setting for locating the point $d$. The last deflection is $\frac{1}{2}I - 4d$, and the measured and computed lengths of $dB$ should agree.

As an example let it be required to make a table of field-notes for a curve whose radius is 50 feet and intersection angle 106° 18′. Then $T = 66.72$ feet. The deflection for a chord of 10 feet is $\sin d = 5 \div 50 = .10000$, $d = 5°\ 44'$. The length of the curve is 92.75 feet, so the last chord will subtend an angle of 106 − 18′ less 9 times 11° 28′, or $d' = 1°\ 33'$ and $C' = 2.71$ feet.

| Station. | Distance. | Chord Deflection. | Total Deflection. | Remarks. |
|---|---|---|---|---|
| P. C. | 0 feet | 0° 00′ | 0° 00′ | |
| $a$ | 10 | 5 44 | 5 44 | |
| $b$ | 10 | 5 44 | 11 28 | |
| $c$ | 10 | 5 44 | 17 12 × | $c$ Transit Point |
| $d$ | 10 | 5 44 | 22 56 | |
| $e$ | 10 | 5 44 | 28 40 | |
| $f$ | 10 | 5 44 | 34 24 | |
| $g$ | 10 | 5 44 | 40 08 × | $g$ Transit Point |
| $h$ | 10 | 5 44 | 45 52 | |
| $i$ | 10 | 5 44 | 51 36 | |
| P. T. | 2.71 | 1 33 | 53 09 | |

With the transit at $g$, to take a back-sight on $c$ the vernier should be set at 17° 12′, the total deflection for $c$. The readings for $h$, $i$, and P T. are the values in the fourth column opposite those stations.

When the intersection point or points of curve or tangency are not situated so that they may be occupied by the instrument, the curve may still be located by one of the foregoing methods. If any line, as $xy$, Fig. 7, be run from a point on

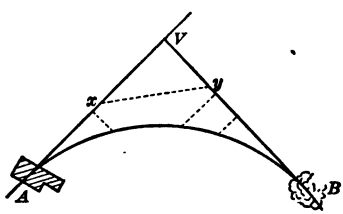


FIG. 7

one tangent to a point on the other, the sum of $Vxy$ and $Vyx$ will be the intersection angle. In the triangle $Vxy$, $Vx$ and $Vy$ may be computed when the angles and the length of $xy$ are known: then the radius having been assumed, $AV$ is computed and hence $xA$ is known. Offsets to the tangent may be computed for assumed distances from $A$ and $B$.

Prob. 4. Given $I = 85° 26'$ and $E = 21.66$ feet. Compute offsets and distances along the long chord for points 10 feet apart on the curve.

Prob. 5. If a chord $c$ subtends an arc $d$, compute $c_2$, $c_3$, $c_4$, and $c_5$ for arcs $2d$, $3d$, $4d$, and $5d$.

Prob. 6. Given: $Vx = 25$ feet, $xy = 33.5$ feet, and $E = 26.0$ feet. Make the computations necessary in laying out the curve in eight equal chords (Fig. 5).

Prob. 7. Make table of deflection angles when $I = 106° 18'$ and $R = 80$ feet, and compute the difference between the length of the curve and that of the sum of the chords.

Prob 8. With the transit at P. C. when $R = 80$ feet and $I = 150°$, how far could a point near the P. T. on a radius of 5 feet and center at the point next to the P. T., be moved without departing more than 0.01 feet from the chord P. C. — P. T.?

### ART. 8. CHANGING THE RADIUS.

(*a*) To change the Radius to agree with a given change in the Tangent.—In Fig. 8, $AO$ is the radius of the curve $AB$

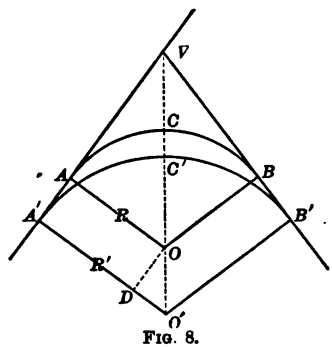


FIG. 8.

whose tangent is $AV$; it is required to find the change in the length of the radius that $A$ may be moved to $A'$. In the triangle $OO'D$, $O'D = OD \cot OO'D$, or

$$R' - R = (T' - T') \cot \frac{I}{2}.$$

The new radius will be greater or less than the first according as $VA'$ is greater or less than $VA$.

(*b*) To change the Radius to effect a given change in the External Distance.—It is required to determine a radius that shall cause the curve to pass through $C'$ instead of through $C$ in Fig. 8. The distance $VC = R \div \cos \frac{I}{2} - R$,

and $$VC' = R' \div \cos \frac{I}{2} - R',$$

then $$R' - R = \frac{VC' - VC}{\sec \frac{I}{2} - 1},$$

or $$R' - R = CC' \operatorname{exsec} \frac{I}{2}.$$

If $CC'$ be measured, the difference in radii is found numerically.

(*c*) To change the Radius so that from the same P. C. the curve shall end in a parallel tangent at a given distance from the original one.—In Fig. 9, $V'B'$ is parallel to $VB$, and the required change of radius is $OO'$. If $BD$ be parallel to $OO'$, $B'D$ and $BD$ are each equal to $OO'$, and $B$ and $B'$ are on the arc of a circle of radius $R' - R$ and center $D$;

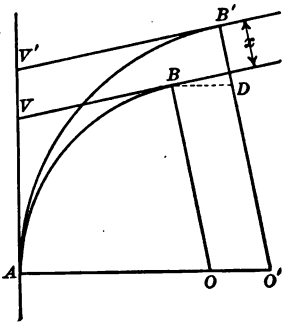


FIG. 9.

$$\text{then} \qquad (R' - R) = \frac{x}{\text{vers } I},$$

$$\text{or} \qquad (R' - R) = \frac{x}{2 \sin^2 \frac{I}{2}},$$

in which $x$ is the distance between the two positions of the tangent. The formula is equally applicable when second tangent is inside the first.

(*d*) To change the Radius so that the curve shall end in a parallel tangent directly opposite the original P. T. and a given distance from it.—In Fig. 10 the change in length of the tangents is $DV'$, or

$$(T' - T) = DV'.$$

$$(R' - R) \tan \frac{I}{2} = x \cot I;$$

$$\text{then} \qquad R' - R = x \cot I \cot \frac{I}{2},$$

$$\text{also} \qquad R' - R = \frac{x}{\text{exsec } I}.$$

The new curve will begin at $A'$, and when $R' - R$ or $O'G$ is known $AA'$ may be computed.

$$AA' = (R' - R) \tan I,$$

also $$AA' = x \cot \frac{I}{2}.$$

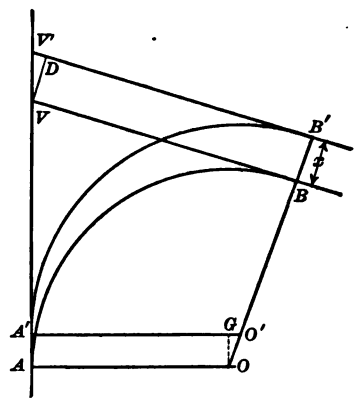


FIG 10.

The method of computation is the same whether the new tangent is outside or inside the original one.

(*e*) To change the Radius to agree with a given change in $I$.

1st. When P. C. remains unchanged.

In Fig. 11 the change in $I$ is $BVB''$ and, if the P. C. remains at $A$, $OO''$ is the required change of radius. Since $AV$ is common to both curves,

$$R' \tan \frac{AO''B''}{2} = R \tan \frac{AOB}{2};$$

$$R' = R \tan \frac{I}{2} \cot \frac{I'}{2},$$

in which $R'$ is the only unknown quantity, since $I' = I \pm$ the given change $BVB''$.

2d. When the P. T. remains unchanged.

Under this condition the vertex is moved from $V$ to $V'$ and

the given change of $I$ is $VBV'$ in Fig. 11. Since $B$ is fixed, its distance from $AV$ is

$$R' - R' \cos I' = R - R \cos I,$$

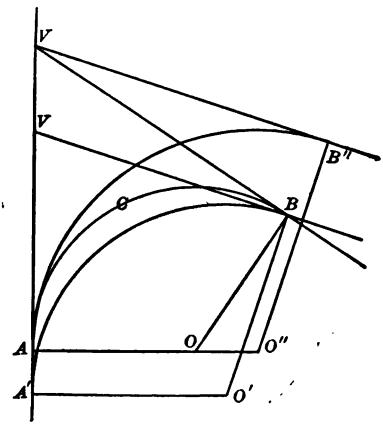


FIG. 11.

in which the algebraic sign of cosine $I'$ and of cosine $I$ must be observed,

$$\text{or} \qquad R' = R(1 - \cos I) \div (1 - \cos I'),$$

$$\text{also} \qquad R' = R\frac{\text{vers } I}{\text{vers } I'}.$$

Since $A'O'$ is parallel to $AO$, the distance from $B$ to each is

$$R' \sin I' = R \sin I,$$

$$\text{or} \qquad AA' = R' \sin I' - R \sin I,$$

which determines the position of the new P. C.

For example, let $R = 60$ feet and $I = 106° 30'$; it is required to determine the radius and P. C. of a curve that end at the same P. T. making $I' = 110°$.

$$R' = 60(1 - \cos 106° 30') \div (1 - \cos 110°),$$

$$R = 57.4 \text{ feet.}$$

$$AA' = 57.4 \sin 110° 00' - 60 \sin 106° 30' = 3.60 \text{ feet.}$$

The new P. C. will be on $AV$, 360 feet from $A$.

(*f*) To find the radius of a curve which shall pass through a given point and connect two given tangents.—Let $VA$ and $VB$ in Fig. 12 be the given tangents, and $P$ the given point lo-

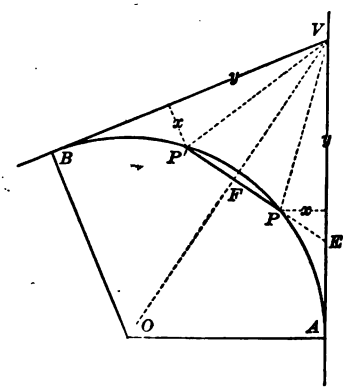


FIG. 12.

cated by an offset $x$ from $VA$ distant $y$ from the vertex. If a point $P'$ be located from $VB$ with the coordinates $x$ and $y$, the required curve will pass through it and $PP'$ will be perpendicular to the line bisecting $AVB$. Then $\tan AVP = x \div y$, and $PVF = \frac{1}{2}(180 - I) - AVP$; hence $PVF$ may be computed. $PV = \sqrt{x^2 + y^2}$, and therefore $PF$ and $VF$ may be found; also in the right triangle $EVF$, $VE = VF \div \sin\frac{I}{2}$, and $FE = VF \cot\frac{I}{2}$; from which the numerical values of $FE$ and $VE$ may be determined. Then

$$EA = \sqrt{P'E \times PE},$$

and $VA = VE + EA$, which is the tangent of the required curve whose P. C. is at $A$. The radius may then be computed, $R = VA \cot\frac{I}{2}$. If the tangents are located, $P'P$ and $PE$ may be measured instead of being computed.

For example let, $I = 120°$, $y = 80$ feet, and $x = 20$ feet, to de-

termine $T$ and $R$. Here tan $AVP = .250$, $AVP = 14° 02'$, and $PVF = 15° 58'$. $PV = \sqrt{400 + 6400} = 82.46$ feet. $PF = 82.46 \sin 15° 58' = 22.69$ feet. $VF = 82.46 \cos 15° 58' = 79.28$ feet. $VE = 79.28 \div .8660 = 91.54$ feet. Then

$$EA = \sqrt{23.09 \times 68.47} = 39.76 \text{ feet}, \quad T = 131.30,$$

and $R = 131.36 \times .5774 = 75.8$ feet.

Prob. 9. Given: $I = 75° 32'$, $R = 50$ feet. It is required to find the radius of a curve that shall connect the same tangents and pass 2 feet farther from the vertex.

Prob. 10. In Fig. 12 let $x = 15$ feet, $y = 65$ feet, and $I = 96°$. Find the radius.

Prob. 11. Given: $I = 87° 20'$ and $R = 60$ feet. What is the radius of a curve that will connect the same P. C. with a parallel tangent 4 feet outside that through the original P. T.? What will be the new tangent distances?

Prob. 12. If, in the above problem, the second P. T. be opposite the first, compute the new radius and the necessary change in position of P. C.

Prob. 13. Given: $I = 100°$, $I' = 105°$, and $R = 65$ feet. Compute the length of the new curves when P. T. is changed and when $V$ is changed.

## ART. 4. SHIFTING THE CURVE.

(*a*) To move the Point of Curve along the tangent so that the curve may end in a tangent parallel to the original one and at a certain distance from it.—Let $x$ in Fig. 13 be the distance between the two tangents, then

$$VV' = AA' = x \div \sin I,$$

because $A'O'B' = AOB$, since $AB$ is moved bodily without change of dimensions to $A'B'$.

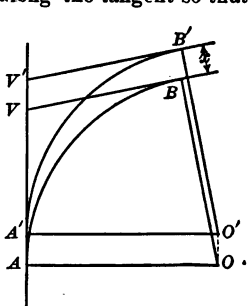


FIG. 13.

(*b*) To move the curve so that both the Points of Curve and of Tangency may be on tangents parallel to the former ones and at given distances from them.—In Fig. 14 the curve $AB$ may

be transferred to $A'B'$ by first moving $A$ along $AV$ till $B$ is on $VB'$; the curve will then be shown by the dotted line $DE$, and $E$ may be moved along $V'B'$ till $D$ is in $V'A'$, when $A'B'$

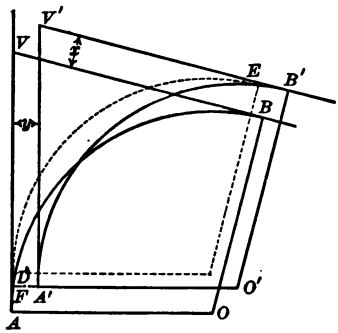


FIG. 14.

will be the required curve. Then $F$ is the point on $AV$ opposite $A'$, and

$$AF = \frac{x + y \cos I}{\sin I},$$

in which $x$ is positive when tangent through P. T. is moved out, $y$ is positive when tangent through P. C. is moved in, and $\cos I$ is negative when $I$ is greater than 90°. When the tangents are moved in opposite directions, the distances $x$ and $y$ are negative, and when $AF$ is found to be negative it means that $F$ is on $VA$ produced.

For example, in Fig. 14 let it be required to transfer the curve $AB$ to $A'B'$ when $y = 3$ feet, $x = 4.2$ feet, $I = 95°\ 22'$, and $R = 50$ feet. In this case $x$ is positive, $y$ is positive, and $\cos I$ is negative; therefore

$$AF = \frac{4.2 - 3 \times .0935}{.9956},$$

$$AF = 3.94 \text{ feet},$$

and $A'$ is located by measuring forward 3.94 feet and erecting a perpendicular offset of 3 feet at $F$.

If, in the above example, the tangent through the P. T. were

moved in, $x$ would be negative and $AF = \frac{-4.2 - 3 \times .0935}{.9956}$ $= -4.50$ feet; hence $F$ would be on $VA$ produced.

(*c*) To move the curve to agree with a given change in direction of the tangent through the P. T.—In Fig. 15 let $BVB'$

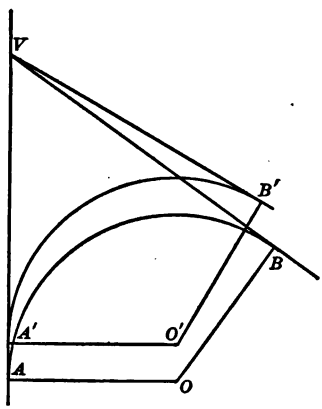


FIG. 15.

be the given change in $I$, $V$ remaining fixed; then

$$VA = R \tan \frac{I}{2},$$

$$VA' = R \tan \frac{I}{2};$$

hence $$AA' = R\left(\tan \frac{I}{2} - \tan \frac{I'}{2}\right).$$

If $AA'$ becomes negative from the above equation, $A'$ is on $VA$ produced.

When the given change in direction is from a different vertex, as shown in Fig. 16.—The change in the position of the P. C. due to the change in $I$ is, as above, $R\left(\tan \frac{I}{2} - \tan \frac{I'}{2}\right)$, and

any change in the position of the vertex will cause an equal change at the P. C.; hence

$$AA' = R\left(\tan\frac{I}{2} - \tan\frac{I'}{2}\right) + VV',$$

in which the proper algebraic signs must be observed.

For example, let $R = 60$ feet, $I = 80°$, $I' = 114°$, and $VV' = 6$ feet from $V$ toward $A$. Here $AA' = 60\ (0.83910 - 1.53986) - 6.0 = -48.05$ feet, and hence the curve will begin 48.05 feet from $A$ on $VA$ produced.

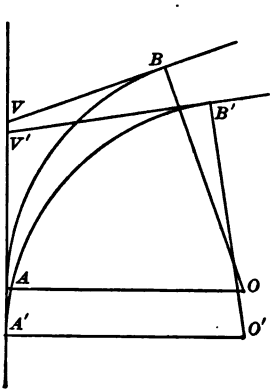


FIG. 16.

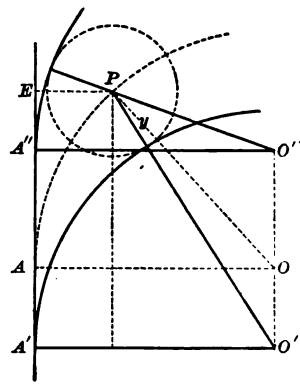


FIG. 17.

(*d*) To locate the P. C. of a curve of given radius which shall pass through a point at a given distance from the tangent.—In Fig. 17 let $P$ be the point, distant $x$ from $AE$, through which it is required to pass the curve of radius $R$. Then

$$\cos 2EAP = (R - x) \div R,$$

and

$$EA = x \cot EAP,$$

which gives the location of the point of curve.

(*e*) To locate the P. C. of a curve that shall pass at a given distance from a point whose distance from the tangent is known.—In Fig. 17 let it be required to locate a curve tangent to $AE$ that shall be a distance $y$ from $P$ which is $x$ from $AE$.

If a circle of radius $y$ be drawn with a center at $P$, it will be tangent to the required curve; therefore

$$\cos A\,O'P = (R - x) \div (R + y),$$

and $$EA' = (R + y) \sin A'O'P,$$

also $$\overline{EA}^2 = (R + y)^2 - (R - x)^2,$$

which is sufficient to fix the point of curve.

The given distance $y$ may be between $P$ and the tangent, in which case

$$\cos A''O''P = (R - x) \div (R - y),$$

and $$EA'' = (R - y) \sin A''O''P.$$

It is evident that $y$ cannot be greater than $x$ unless the curve is in the opposite direction.

For example, let the line midway between the rails be 18 feet from the curb, to locate the P. C. of a curve of 45 feet radius that shall pass 11 feet from a given curb corner. The curb corner is represented by $P$ in Fig. 17; then $\cos A''O''P = (45 - 13) \div (45 - 34)$, or $A''O''P = 37° 26'$. Then $EA'' = 34 \sin 37° 26' = 20.67$ feet.

Prob. 14. The middle line of a railroad-track is to be 20 feet from the curb; a curve is to begin opposite a point on the curb 22 feet from the curb corner, which is to be 12.8 feet from the middle line of the curve. Find the radius.

Prob. 15. The middle line of a track is 18 feet from the curb; a curve of 50 feet radius, away from the curb, is to pass 30 feet from a given point on the curb. Find the length of a chord from the P. C. to the part of the curve nearest the given point.

### ART. 5. COMPOUND CURVES.

The curve $ACB$, Fig. 18, is called compound from the fact that it is a combination of curves of different radii, as $AO$, $CO'$, and $DO''$. The points $C$ and $D$ are designated Points of Compound Curvative, P. C. C. The sum of the angles $AOC$,

$CO'D$, and $DO''B$ equals the deflection angle at $V$ and the tangent distances $VA$ and $VB$ are of unequal lengths.

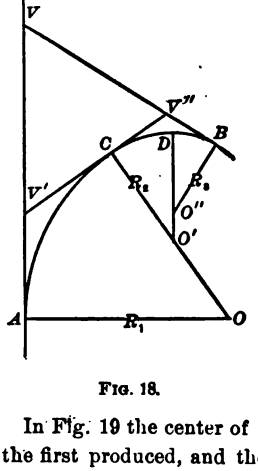


FIG. 18.

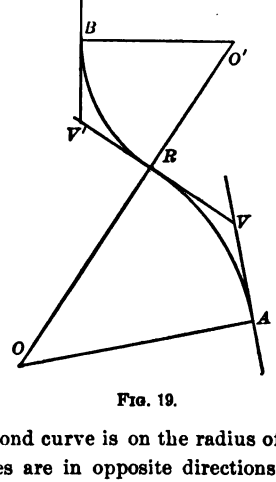


FIG. 19.

In Fig. 19 the center of the second curve is on the radius of the first produced, and the curves are in opposite directions. Such a condition is a special case of compound curves which is termed Reverse Curve. The place where the radius $AO$ changes to $RO'$ is called the Point of Reverse Curvature, P. R. C. The radii $OA$ and $O'R$ may or may not be of equal length, and the difference between the central angles, $O$ and $O'$, is that between $AV$ and $BV'$.

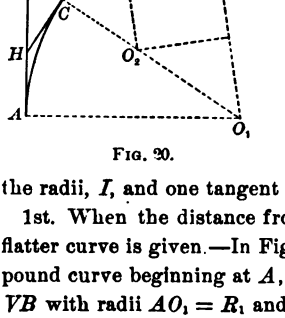


FIG. 20.

(*a*) To locate a compound curve between two given tangents when the radii, $I$, and one tangent are known.

1st. When the distance from the vertex to the P. C. of the flatter curve is given.—In Fig. 20 it is required to locate a compound curve beginning at $A$, distant $T_1$ from $V$, and ending on $VB$ with radii $AO_1 = R_1$ and $CO_2 = R_2$.

If $AC$ be produced to $F$, making $AO_1F = I$, then

$$(R_1 \tan \tfrac{1}{2}I - T_1) \sin I + R_2 + (R_1 - R_2) \cos O_2 = R_1;$$

$$\cos O_2 = 1 - \frac{(R \tan \tfrac{1}{2}I - T_1)}{R_1 - R_2} \sin I;$$

so $$\cos O_2 = \frac{T_1 \sin I + R_1 \cos I - R_2}{R_1 - R_2},$$

in which the algebraic sign of cos $I$ must be observed. If $T$ be larger than $R_1 \tan \frac{1}{2}I$, the problem is incapable of solution.

In the triangle $VHK$, $HK = R_1 \tan \frac{1}{2}O_1 + R_2 \tan \frac{1}{2}O_2$;

then $$VK = HK \frac{\sin O_1}{\sin I},$$

and $$VB = VK + R_2 \tan \tfrac{1}{2}O_2;$$

otherwise $$T_2 = VB = \frac{R_1 - (R_1 - R_2) \cos O_1}{\sin I} - R_2 \cot I,$$

which determines the location of $B$.

2d. When the distance $T_2$ from the vertex to the P. C. of the sharper curve is known.—In Fig. 21, $BC$ is produced to $F$, making $BO_2F = I$.

$$T_2 \sin I + R_2 \cos I + (R_1 - R_2) \cos O_2 = R;$$

$$\cos O_1 = \frac{R_1 - T_2 \sin I - R_2 \cos I}{R_2 - R_1}.$$

When $I$ is more than 90° the cosine of $I$ is negative.

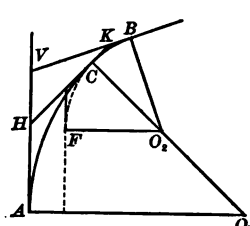


FIG. 21.

The tangent $VA = T_1$ is found as in the first case.

$$VA = VH + R_1 \tan \tfrac{1}{2}O_1,$$

also $$T_1 = \frac{R_1(\cos O_2 - \cos I) + R_2(1 - \cos O_2)}{\sin I}.$$

For example, let $T_2 = 26.2$ feet, $I = 60°$, $R_2 = 40$ feet, and $R_1 = 60$ feet; to locate the P. T. and the P. C. C. $\cos O_1 = \frac{60 - 26.2 \times .86603 - 40 \times .5}{20} = .866$, and $O_1 = 30°$; $O_2 = I - O_1 = 30°$; $T_1 = \frac{60 \times .366 + 40 \times .134}{.866} = 31.5$ feet. The P. C. C. may be located from either end of the curve after $T_1$ has been computed. The chord $BC = 2R_2 \sin \frac{O_2}{2} = 103.5$ feet.

(*b*) To locate a compound curve between two given tangents when $I$, $T_1$, $T_2$, and one radius are given.

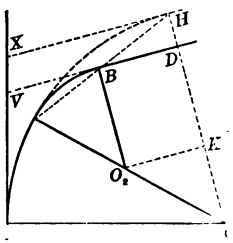


FIG. 22.

1st. When the larger radius is known.—In Fig. 22 let the arc of radius $R_1$ be continued until it ends in a tangent parallel to $VD$; then

$$HX = T = R_1 \tan \frac{I}{2}.$$

Then $HBD = VBC = \frac{O_2}{2}$, and in the triangle $HBD$

$$\cot \frac{O_2}{2} = \frac{BD}{HD},$$

or

$$\cot \frac{O_2}{2} = \frac{T - T_2 + (T - T_1) \cos I}{(T - T_1) \sin I},$$

in which $T_1 = VA$ and $T_2 = VB$.

Also from the same figure $R_2 = R_1 - HD - KO_1$,

or

$$R_2 = R_1 - (T - T_1) \frac{\sin I}{1 - \cos O_2}.$$

For example, let $I = 61°\ 56'$, $R_1 = 80$ feet, $T_1 = 38$ feet, and $T_2 = 28$ feet. Then $T = 80 \times .60007 = 48.0$ feet; $\cot \frac{O_2}{2} = \frac{20 + 10 \times .7405}{10 \times .8824} = 2.7998$, or $O_2 = 39°\ 18'$; $R_2 = 80 - 10 \frac{.88240}{.22616} = 41.0$ feet.

2d. When the smaller radius is known.—Let the curve of smaller radius be extended to a tangent parallel to $T_1$. Then, as before,

$$\cot \frac{O_1}{2} = \frac{AD}{HD}$$

$$= \frac{T_1 - T + (T_2 - T)\cos I}{(T_2 - T)\sin I},$$

in which $T$, $T_1$, and $T_2$ represent $HX$, $AV$, and $BV$ in Fig. 23.

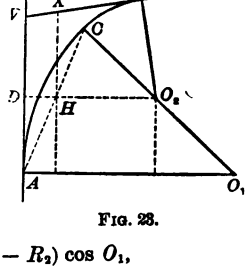


FIG. 23.

Also $$R_1 = R_2 + AD + (R_1 - R_2)\cos O_1,$$

$$R_1 = R_2 + (T_2 - T)\frac{\sin I}{1 - \cos O_1},$$

in which $R_2$ must always be selected smaller than $T_2 \cot \frac{I}{2}$, else the curve will become reverse.

For example, let $I = 119° 04'$, $T_1 = 120$ feet, $T_2 = 100$ feet, and $R_2 = 40$ feet. Then $T = 40 \times 1.69992 = 68$ feet. Since $I$ is greater than 90° its cosine is negative, and

$$\cot \frac{O_1}{2} = \frac{120 - 68 - 32 \times .48583}{32 \times .87406},$$

or $$O_1 = 75° 00'. \quad R_1 = 40 + 32 \times \frac{87406}{74118} = 77.7 \text{ feet.}$$

(*c*) To connect a given curve with a given tangent by another curve of given radius.—This is a case in which the distance between the given tangent and that to the given curve parallel to the first is known, as $DH$ in Fig. 24, but the angle $O_1$ is to be determined. In Fig. 24 let $AC$ be the given curve of radius $R_1$, and $BV$ the given tangent, making the angle $I$ with $AV$. If $AC$ be produced to $H$, making $HO_1A = I$, $HD = d$, and if $R_2$ be assumed,

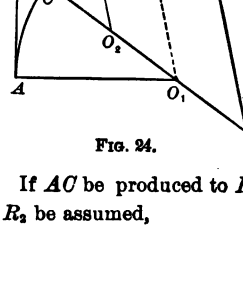


FIG. 24.

$$d = \left(R_1 \tan \frac{I}{2} - AV\right) \sin I;$$

$$\cos O_2 = 1 - \frac{d}{R_1 - R_2}.$$

The point $B$ is located by laying off from $V$ the distance $T_2$ as computed in ($a$) above.

If the required tangent is to be outside of $H$, as $V'B'$, the above formula becomes

$$\cos O_2 = 1 - \frac{d}{R_2 - R_1}.$$

The P. C. C. is found by running a tangent at any point on the original curve, as $A$, to intersect the given tangent; the angle $I$ is then measured, and $O_1 = I - O_2$. The chord $AC$ then is

$$AC = 2R_1 \sin \frac{O_1}{2},$$

which gives the location of P. C. C.

2d. When the required curve is to be in the opposite direction from the first.—Let $AB$, Fig. 25, represent the curve to

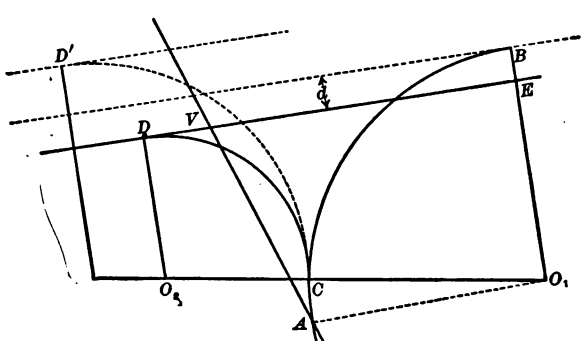


FIG. 25.

be connected with the tangent $DE$. From any point, as $A$, on the curve $AB$ a tangent is run to intersect the line $DE$, and the intersection angle $AO_1B$ is measured, $B$ in the figure be-

ing the point where the tangent to $ACB$ is parallel to $DE$. Then $O_2O_1B$ is found from

$$\cos O_2O_1B = \frac{R_1 - R_2 - d}{R_1 + R_2},$$

and $$AO_1C = I - O_2O_1B,$$

from which the distance to $C$ from the selected point may be computed. The curve $CD$ may then be located by any of the usual methods, the total deflection from a tangent at $C$ to the point $D$ being $\frac{1}{2}(180° - O_1)$.

When the required tangent is outside of the tangent through $B$, the formula is

$$\cos CO_1B = \frac{R_1 - R_2 + d}{R_1 + R_2},$$

and the P. R. C. may be fixed by measuring the computed chord $AC$ with a deflection $\frac{1}{2}(I - O_1)$ from the tangent at $A$.

For example, let it be required to connect a given curve $ACB$ of radius 50 feet, Fig. 25, with the tangent $D'E'$ by a curve of radius 100 feet. From any convenient point on $BCA$, as $A$, a tangent is produced to intersect $D'E'$ at $V$, $I$ is found to be 117° 32′, and $AV = 115.4$ feet. From which

$$d = (115.7 - 50 \tan 58° 46') \sin I = 29.5 \text{ feet.}$$

Then $\cos O_1 = \dfrac{50 - 100 + 29.5}{100 + 50} = -.13666$, or $O_1 = 97° 51'$, and $AO_1C = 19° 41'$; hence $AC = 2 \times 50 \times .1709 = 17.09$ feet, and $VAC = 9° 50'$, giving the location of the P. C. C., from which the curve may be run by deflection angles from a tangent at $C$. The deflection for $D'$ will be $\frac{1}{2}(180° - 97° 51') = 41° 04'$.

(*d*) To change the position of the P. C. C. so that the curve may end in a parallel tangent a given distance from the first.

1st. When the P. C. of flatter curve is fixed.—In Fig. 26 let $ACB$ be the given compound curve; it is required to change the position of $C$ so that the curve may end at $B_2$ in a tangent parallel to that at $B$, and $d$ feet inside of it. Let the curve $AC$ be produced to $H$, making $AO_1H = I$. Then, since the locus

of the intersections of the radii is a circle of radius $R_1 - R_2$ and center at $O_1$, from the figure

$$\cos O_3 = \cos O_2 - \frac{d}{R_1 - R_2}.$$

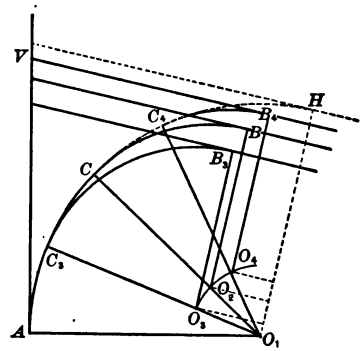


FIG. 26.

If the curve were to be moved outward the same distance to $B_4$, then

$$\cos O_4 = \cos O_2 + \frac{d}{R_1 - R_2},$$

and the new P. C. C. is located from $A$, since $AO_1C_4 = I - O_4$.

2d. When the P. C. of the sharper curve is fixed.—In Fig. 27, $ACB$ is the given compound curve ending at $B$; it is required to change the position of $C$ so that the curve may end in $B_3$, $d$ feet inside the tangent through $B$. Let the curve $AC$ be produced to $H$, making $HO_1A = I$; then

$$\cos O_3 = \cos O_2 + \frac{d}{R_2 - R_1}.$$

$$\cos O_4 = \cos O_2 - \frac{d}{R_2 - R_1}$$

when the required tangent is outside that through $B$.

For example, let $R_1 = 60$ feet, $R_2 = 40$ feet, $O_1 = 25°$, and $O_2 = 97° 52'$; it is required to so change the P. C. C. that the tangent at the end of the sharper curve may be moved out 5 feet. Then cos $O_4 = -.13687 + \frac{5}{20}$, or $O_4 = 83° 30'$ and

$O_1 = 39° 22'$. The chord from the P. C. to the P. C. C. is $2 \times 60 \times .3368 = 40.42$ feet.

Prob. 16. Given: $R_1 = 40$ feet, $R_2 = 80$ feet, $O_1 = 30°$, and $O_2 = 42° 30'$. Find the chord from the P. C. to the P. C. C. so that the compound curve shall end in a parallel tangent 4 feet inside the present one.

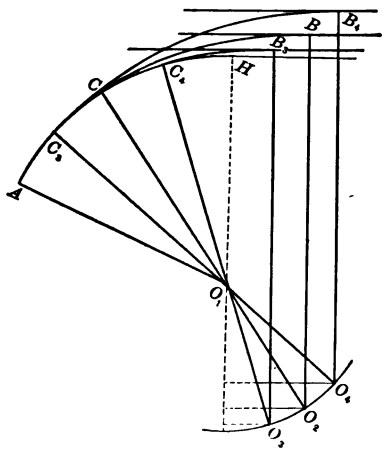


FIG. 27.

Prob. 17. In the last problem compute maximum values for $+ d$ and $- d$.

Problem 18. Given: $R_1 = 55$ feet, $R_2 = 42$ feet, $I = 64°$, and $T_1 = 33$ feet. Find the value of $T_2$ and the position of the P. C. C.

Prob. 19. Compute maximum and minimum valu s for the assumed $T_1$.

Prob. 20. Given: $I = 76°$, $T_1 = 40$ feet, $T_2 = 80$ feet, and $R_1 = 75$ feet. Make the computations necessary to locate the compound curve.

Prob. 21. It is required to connect a curve to the right, of 50 feet radius, with a line which makes an angle of 75° with a tangent to the given curve 65 feet from the point of tangency, by a curve to the left of 40 feet radius.

## CHAPTER II.

### TURNOUTS AND SIDINGS.

#### ART. 6. TURNOUTS FROM STRAIGHT TRACK.

The part of the track which is common to the outside rail of the turnout and the inner rail of the main track is called the Frog, as shown by the small shaded triangle at $F$ in Fig. 28. The distance $AF$ is the Frog Distance, the length of the frog divided by its breadth is the Number of Frog, as $\frac{cd}{ab}$ in Fig. 29, and the Frog Angle, $F$, is the angle $acb$ or that between the tangent to the outer rail of the turnout and the main track, being the same as $AOF$ in Fig. 28. The Radius of the Turnout is $AO + \frac{1}{2}G$, when $G$ is the gauge of the track or the distance between the rails. The Standard Gauge is 4′ 8½″. The Switch, Fig. 30, is the arrangement at $A$, Fig. 28, by which a car is deflected from the straight track to the curve.

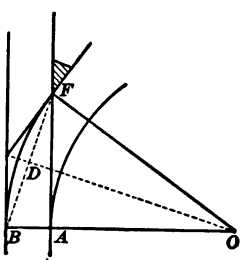


FIG. 28.

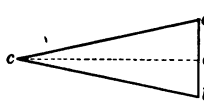


FIG. 29.

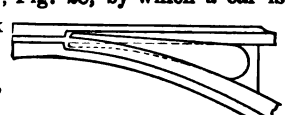

FIG. 30.

In the triangle $AFO$, Fig. 28, the Frog Distance $AF$ is

$$AF = \sqrt{2GR},$$

also from $ABF$ $AF = 2GN$.

Since $$2N = \cot\frac{F}{2} = \cot BFA,$$

then $$R = 2GN^2 = \frac{1}{2}G\cot^2\frac{F}{2},$$

by combining the two values of $AF$;

$$\text{also} \quad \cos F = \frac{2R - G}{2R + G}.$$

(*a*) When Two Turnouts of the same radius in Opposite Directions begin at the Same Point.—As shown in Fig. 31, the number, $N_1$, of the frog at the intersection of the curves may be stated in terms of that on the straight rail, for in the triangle $AOF_1$

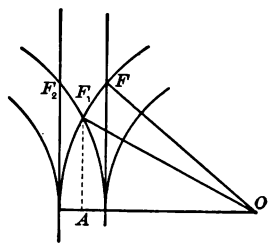


Fig. 31.

$$\overline{AF_1}^2 = G\left(R + \frac{G}{4}\right),$$

$$AF_1 = R \tan \tfrac{1}{2}F_1,$$

$$\text{also} \quad AF_1 = \frac{R}{2N_1};$$

$$\text{hence} \quad N_1 = \sqrt{\frac{4N^4}{8N^2 + 1}},$$

in which $N_1$ is the number of frog at $F_1$ and $N$ is the number of that at $F$.

The switch for a case like Fig. 31 must be so designed that a car may be made to take either of three directions, and for this reason it is called a Three throw Switch.

(*b*) When the Radii of the Turnouts are Unequal.—In the

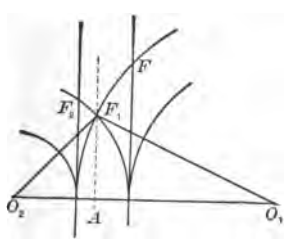


Fig. 32.

triangle $O_1O_2F_1$, Fig 32,

$$\tan \tfrac{1}{2}O_2 = \sqrt{\frac{R_1 G}{R_2(2R_1 + 2R_2 + G)}},$$

and $$\tan \tfrac{1}{2} O_2 F_1 O_1 = \sqrt{\frac{4R_1R_2}{G(2R_1 + 2R_2 + G)}};$$

the frog angle $$F_1 = 180° - O_2F_1O_1,$$

and $$N_1 = \tfrac{1}{2} \cot \tfrac{1}{2} F_1,$$

$$AF_1 = \left(R_2 + \frac{G}{2}\right) \sin O_2,$$

$$O_2A = \left(R_2 + \frac{G}{2}\right) \cos O_2,$$

from which the location of the frog $F_1$ is established.

For example, let $R_1 = 50$, $R_2 = 40$, and the gauge be the standard; then $\tan \tfrac{1}{2} O_2 = \sqrt{\frac{235.4}{40(100 + 80 + 4.708)}}$, or $O_2 = 20° \ 14'$; $\tan \tfrac{1}{2} O_2F_1O_1 = \sqrt{\frac{4 . 40 . 50}{4.708(100 + 80 + 4.708)}}$, or $O_1F_1O_2 = 143° \ 32'$, so $F_1 = 36° \ 28'$ and $N_1 = \tfrac{1}{2} \cot 18° \ 14' = 1.52$; $AF' = 42.35 \times .84584 = 14 \ 65$ feet; $O_2A = 42.35 \times .9383 = 39.74$ feet, and the point of the frog $F_1$ is 14.65 feet from the point of curve and 0.26 feet from the middle line of the track.

(*c*) When the Opposite Turnouts begin at Different Points.—A case of this kind is shown in Fig. 33. It is required to de-

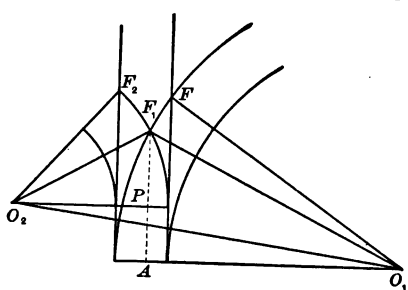


FIG. 33.

termine the frog angle $F_1$ and the frog distance $AF_1$ when $AP$, or $a$, and either the frog angles $F_2$ and $F$ or the radii of the turnouts are known. When $F_2$ and $F$ are given,

$$R_1 = \frac{G}{2}\cot^2\frac{F}{2};$$

$$O_1O_2 = \sqrt{(R_1 + R_2)^2 + a^2}.$$

Then in the triangle $O_1F_1O_2$ all the sides are known, and

$$\cos O_1 = \frac{\overline{O_1F_1}^2 + \overline{O_1O_2}^2 - \overline{O_2F_1}^2}{2O_1O_2 \cdot O_1F_1},$$

also $$\cos O_2 = \frac{\overline{O_2F_1}^2 + \overline{O_1O_2}^2 - \overline{O_1F_1}^2}{2O_1O_2 \cdot O_2F_1};$$

then $$F_1 = O_1 + O_2,$$

$$\tan O_2O_1A = \frac{a}{R_1 + R_2},$$

and $$AF_1 = \left(R_1 + \frac{G}{2}\right)\sin F_1O_1A.$$

The above expressions are readily solved when numerical values are substituted, but a formula giving $AF_1$ in terms of $R_1$, $R_2$, and $a$ directly is unwieldy.

(*d*) When the three Frog Angles are known.—It often happens that the engineer has at his disposal but a limited assortment of frogs. In such a case the turnout may be constructed as a compound curve to fit the given frogs. In Fig. 34 let

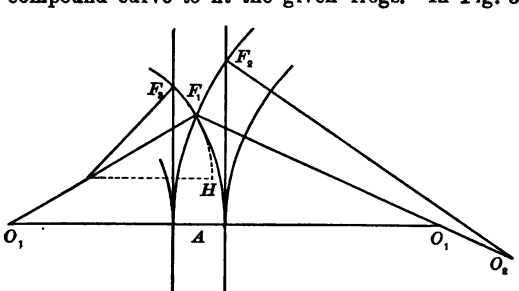


FIG. 34.

$F_1$, $F_2$, and $F_3$ represent the frog angles that are to be used, $R_1$, $R_2$ and $R_3$ the radii of middle line of track having centers

at $O_1$, $O_2$, and $O_3$. If the frog at $F_1$ be in the middle of the track,

$$O_1 = \tfrac{1}{2}F_1;$$

$$\text{then} \qquad R_1 = \frac{G \cos \tfrac{1}{2}F_1}{2(1 - \cos \tfrac{1}{2}F_1)}.$$

If $O_2H$ be made parallel to $O_1A$,

$$F_2O_2H = F_2;$$

$$\text{hence} \qquad R_2 + \frac{G}{2} = \frac{G}{2(\cos \tfrac{1}{2}F_1 - \cos F_2)},$$

$$AF_1 = R_1 \tan \tfrac{1}{2}F_1,$$

$$AF_2 = \left(R_2 + \frac{G}{2}\right) \sin F_2 + (R_1 - R_2) \sin \tfrac{1}{2}F_1,$$

and the distance $AF_2$ and radius $R_2$ may be computed in a similar manner.

$$R_2 + \frac{G}{2} = \frac{G}{2(\cos \tfrac{1}{2}F_1 - \cos F_2)},$$

$$\text{and} \qquad AF_2 = \left(R_2 + \frac{G}{2}\right) \sin F_2 - (R_2 - R_1) \sin \tfrac{1}{2}F_1.$$

The solution of nearly all problems in compound curves is simplified by continuing one of the curves until its central angle equals the total intersection angle between the tangents to the compound curve.

As an example, let it be required to locate a double turnout in opposite directions, using frog angles $F_1 = 28° 20'$, $F_2 = 17°$, and $F_3 = 21° 17'$.

$$R_1 = \frac{4.708 \times .96959}{2 \times .03041} = 75 \text{ feet},$$

$$R_3 = \frac{4.708(1 - .96959 + .93180)}{2(.96959 + .93180)} = 60 \text{ feet},$$

$$R_2 = \frac{4.708(1 - .96959 + .95630)}{2(.96959 - .95630)} = 174.8 \text{ feet};$$

$AF_1 = 75. \times .2524 = 18.93$ feet,

$AF_3 = 62.35 \times .36298 + 15 \times .24474 = 26.32$ feet, and

$AF_2 = 177.2 \times .29237 - 99.8 \times .24474 = 27.4$ feet.

It is possible to locate such a turnout having $F_1$ $F_2$, and $F_3$ equal, but $F_1$ must always be less than $F_2$ plus $F_3$.

Prob. 22. What frog number and angle and frog distance are suitable for a turnout from a straight track of "narrow gauge," or 3 feet, when the radius of middle line of turnout is 50 feet?

Prob. 23. In a double turnout in opposite directions $R_1$ and $R_2$ are each 75 feet. Find $F_1$, $F_2$, $AF_1$, and $AF_2$.

Prob. 24. A double turnout in opposite directions is composed of curves of 75 feet and 125 feet. Find frog angles and frog distances.

Prob. 25. Make the necessary computations for locating a double turnout when the three frog angles each are 17°.

Prob. 26. What must be the length of $R_2$ if $R_1$ be 100 feet when $F_1$ and $F_2$ are equal?

## ART. 7. TURNOUTS FROM A CURVED TRACK.

(*a*) To locate a turnout on the Inside of a Curve when the Frog Angle is given.—In Fig. 35, $R_1$ is the radius of the mid-

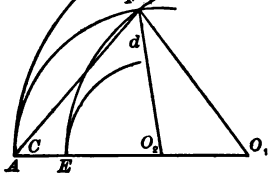


FIG. 35.

dle line of the main track, $R_2$ that of the turnout, and $F$ the frog angle or $O_1FO_2$. In the triangle $AFO_1$, $FAO_1 = 90° - \frac{1}{2}(F + O_1)$.

$$AO_1 + FO_1 : AO_1 - FO_1 = \tan \tfrac{1}{2}(AFO_1 + FAO_1) : \tan \tfrac{1}{2}(AFO_1 - FAO_1),$$

or $$\tan \tfrac{1}{2}O_1 = \frac{G}{2R_1} \cot \tfrac{1}{2}F,$$

also $$\tan \tfrac{1}{2}O_1 = \frac{GN}{R_1},$$

in which $G$ is the gauge of the track and $N$ is the number of the frog.

In the triangle $AFO_2$ the angle at $O_2$ is $F + O_1$; hence

$$AF = (2R_2 + G) \sin \tfrac{1}{2}O_2,$$

also $$EF = (2R_1 - G) \sin \tfrac{1}{2}O_1,$$

and $$cd = 2R_2 \sin \tfrac{1}{2}O_2.$$

From the triangle $FO_1O_2$,

$$R_2 = \left(R_1 - \frac{G}{2}\right) \frac{\sin O_1}{\sin O_2} - \frac{G}{2},$$

and the middle line of the turnout may be staked out in the usual manner.

For example, let the radius of the main track be 125 feet and the frog angle 17°. Then $\tan \tfrac{1}{2}O_1 = \frac{4.708}{250} \times 6.6912$, $O_1 = 14° 22'$, and $O_2 = 31° 22'$; $EF = (250 - 4.708).12504 = 30.74$ feet, and $R_2 = 122.65 \frac{24813}{52051} - 2.35 = 56.15$ feet.

(*b*) To locate a turnout on the Inside of a Curve when the Radius of the Turnout is given.—In Fig. 35 the three sides of the triangle $FO_2O_1$ are known; hence

$$\tan \tfrac{1}{2}F = \sqrt{\left(R_1 - R_2 - \frac{G}{2}\right)\frac{G}{2R_1R_2}}.$$

After $F$ is determined the other values are found as in the last paragraph (*a*).

For example, let it be required to select a frog to fit a turnout of 50 feet radius on the inside of a curve of 100 feet radius.

$\tan \tfrac{1}{2}F = \sqrt{(100 - 50 - 2.35)\frac{2.35}{5000}}$, and $F = 17°$.

(*c*) To locate a turnout on the Outside of a curve when the Frog Angle is given.—The formulas for this case are very similar to those in (*a*). In Fig. 36, $FO_1 = R_1 + \frac{G}{2}$, and $AO_1 = R_1 - \frac{G}{2}$.

Also in the triangle $AFO_1$,

$$AFO_1 = 90° - \frac{F}{2} - \frac{O_1}{2},$$

and $$FAO_1 = 90° + \frac{F}{2} - \frac{O_1}{2};$$

hence $$\tan \tfrac{1}{2}O_1 = \frac{G}{2R_1} \cot \tfrac{1}{2}F = \frac{GN}{R_1};$$

$$O_1 + O_2 = F.$$

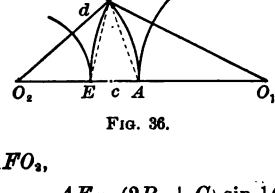


FIG. 36.

In the triangle $AFO_2$,

$$AF = (2R_2 + G) \sin \tfrac{1}{2}O_2.$$

In $EFO_1$, $$EF = (2R_1 + G) \sin \tfrac{1}{2}O_1,$$

In $O_2FO_1$, $$R_2 = \left(R_1 + \frac{G}{2}\right) \frac{\sin O_1}{\sin O_2} - \frac{G}{2}.$$

The distance from $F$ to $O_2O_1$ was given in the article on double turnouts.

(*d*) To locate a turnout on the Outside of a curve when the Radius of the Turnout is given.—In Fig. 36 the three sides of the triangle $O_1FO_2$ are known, and

$$\cos \tfrac{1}{2}F = \sqrt{\frac{R_1R_2}{\left(R_1 + \frac{G}{2}\right)\left(R_2 + \frac{G}{2}\right)}},$$

since the frog angle is the supplement of $O_2FO_1$.

For example, let the radii of the main and turnout curves be 75 and 50 feet respectively; then $\cos \tfrac{1}{2}F = \sqrt{\frac{3750}{4049.3}}$, and $F = 31° 33'$; $\tan \tfrac{1}{2}O_1 = \frac{4.708}{150} 3.5398$, or $O_1 = 12° 40'$ and $O_2 = 18°53'$; $AF = (100 + 4.708).16405 = 17.17$ feet.

Prob. 27. The radius of the main track is 80 feet, and $N$ is 3.5. Find the radius of the turnout on the inside, and the chord to the outside rail from the frog to the beginning of the turnout curve.

Prob. 28. The radius of the main track is 75 feet and that of the turnout on the inside is 45 feet. Find the chord $AF$ from the frog to the P. C. C.

Prob. 29. It is required to determine the length of the chord on the middle line of a turnout of 60 feet radius, on the outside of a curve of 100 feet radius, between a point opposite $F$ and the P. C. C.

### ART. 8. SIDINGS AND CROSSOVERS.

(*a*) To connect a Siding with a Straight Track when the Distance between the tracks and the Frog Angles are given.— The curved track $AB$ in Fig. 37 is the crossover from the

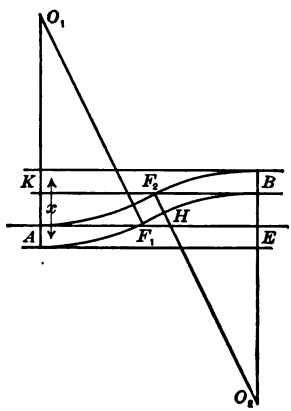


FIG. 37.

track $AE$ to the track $BK$. The radii $O_1A$ and $O_2B$ may or may not be equal and the tangent $F_1H$ may be omitted, forming a reverse curve, or the curves may be continued beyond $F_1$ and $F_2$. The same form of curve is used in passing from a main track to a siding as shown in Fig. 37 with the straight track $BK$ removed. In the figure the curves end at the frogs,

so $AO_1F_1$ is the given frog angle, and the distance between the middle lines of the straight track is $x$. Then

$$R_1 = R_2 = \frac{G}{2}\cot_2\frac{1}{2}F_1,$$

or
$$\left(R_1 + \frac{G}{2}\right) = \frac{G}{1 - \cos F},$$

$$2R_1(1 - \cos F_1) + F_1H \sin F_1 = x,$$

and
$$(2R_1 + G)(1 - \cos F_1) = 2G;$$

hence
$$F_1H = \frac{x - G(1 + \cos F_1)}{\sin F_1}.$$

When the crossover is in the form of a reverse curve $F_1H$ is zero and

$$x = G(1 + \cos F_1).$$

$$AE = 2R_1 \sin F_1 + F_1H \cos F_1,$$

from which it is seen that the length of the crossover varies with the radius and with the distance between the tracks.

For example, let it be required to design a crossover in the form of a reverse curve between two tracks which are 4 5 feet between inner rails. In this case $x = 4.708 + 4.5 = 9.208$ feet. $\cos F = \frac{4.5}{4.708} = .9558$, so $F = 17°\ 06'$. Then $R = 4.708 + .04421 - 2.354 = 104.1$ feet.

When it is convenient to have $AE$ as short as possible, still using the given frog angles and distance between tracks, the curves are continued beyond the frogs as shown, and the tangent between the curves may be assumed within certain limits. In Fig. 38 let $t$ represent the tangent $LH$ between the curves, and $P$ the middle point of $t$. Then

$$O_1P = \tfrac{1}{2}\sqrt{4R^2 + t^2},$$

and
$$\tan PO_1L = \frac{t}{2R}.$$

Since $P$ is midway between the tracks,

$$O_1P \cos PO_1A = \frac{2R_1 - x}{2O_1P},$$

$$\cos PO_1A = \frac{2R_1 - x}{\sqrt{4K^2 + t^2}},$$

and $$O_1 = PO_1A - PO_1L.$$

The tangent $t$ will be less than $F_1H$, Fig. 37, as given above. For example, it is required to locate a crossover between

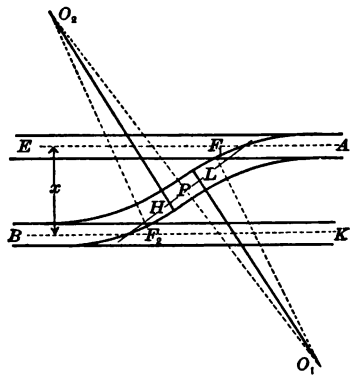


FIG. 38.

parallel tracks, 12 feet center to center, using 13° 41′ frogs and 10 feet of tangent between the curves. $R_1 = \frac{1}{2} \times 4.708 \times \overline{8.3347}^2 = 163.5$ feet; tan $PO_1L = 10 \div 327.0 = .03058$, and $PO_1L = 1° 45'$. Then $\cos PO_1A = (327 - 12.0) \div \sqrt{327^2 + 100} = .96186$, and $PO_1A = 15° 53'$. The angles $O_1$ and $O_2$ are $PO_1A - PO_1L = 14° 08'$, or 0° 27′ beyond the frogs.

(*b*) When the Radii and Distance between tracks are given. —In this case the method is the same as when the frog angles are known, since the latter may be expressed in terms of the former.

$$\cos F_1 = \frac{2R_1 - G}{2R_1 + G},$$

and the other computations are the same as above.

(*c*) To locate a Crossover between parallel Curved Tracks when the Frog Angles and Distance between tracks are given. —In Fig. 39 the curves of the turnout are extended beyond

the frogs and connected by a tangent $LH$, while the frog angles are unequal. In the triangle $O_1PO$, $OP = R + \frac{x}{2}$, $O_1O = R_1 + R$, and $O_1P = R_1 + \frac{1}{2}\sqrt{4R_1^2 + t^2}$, when $R$ is the radius of the inner main track, $R_1$ that of the curve $AL$, $x$ the

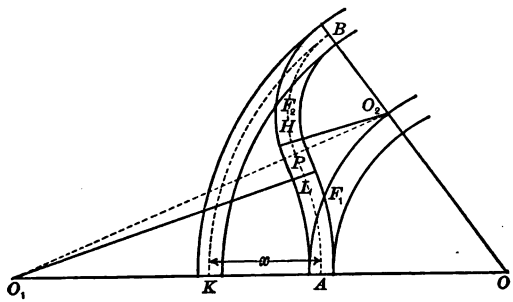


FIG. 39.

distance between tracks, and $t$ the tangent $HL$. The three sides of the triangle are then known, since $R_1$ is determined from the frog angle ; and letting $s$ represent the half sum of sides,

$$\cos PO_1O = \frac{2s(s - PO)}{O_1O \cdot O_1P} - 1,$$

$$\cos POO_1 = \frac{2s(s - O_1P)}{O_1O \cdot OP} - 1.$$

In $PO_1L$, $\tan PO_1L = \frac{t}{2R_1}$,

and $O_1 = PO_1O - PO_1L.$

In the triangle $PO_2O$, $PO_2 = \frac{1}{2}\sqrt{R_2^2 + t_2}$, $O_2O = R + x - R_2$, and $OP = R + \frac{x}{2}$ ; so the cosine of $POO_2$ may be written as before,

$$\cos PO_2O = \frac{2s(s - PO)}{OO_2 \cdot PO_2} - 1,$$

and

$$\cos POO_2 = \frac{2s(s - PO_2)}{OO_2 \cdot PO_2} - 1.$$

Then $O_2 = 180° - PO_2O - PO_2H,$

and $KB = 2(R + x) \sin \frac{1}{2}(BOP + POO_1).$

The crossover may then be run from either $A$ or $B$.

The crossover is very often located in the form of a reverse curve, as shown in Fig. 40. In the triangle $O_1O_2O$, $O_1O_2 =$

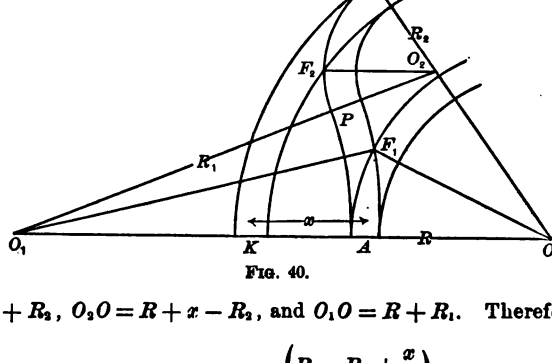


FIG. 40.

$R_1 + R_2$, $O_2O = R + x - R_2$, and $O_1O = R + R_1$. Therefore

$$\cos O_2O_1O = 1 - \frac{x\left(R - R_2 + \frac{x}{2}\right)}{(R + R_1)(R_1 + R_2)},$$

also $$\cos O_2OO_1 = 1 - \frac{x\left(R_1 + R_2 - \frac{x}{2}\right)}{(R + R_1)(R + x - R_2)}.$$

Then $PO_2B = O_2 = O_1 + O.$

The point $B$ may be located from $K$ by deflecting $\frac{1}{2}O$ from the tangent at that point and measuring the chord $KB = 2(R + x) \sin \frac{1}{2}O_2OO_1$.

For example, $F_1 = F_2 = 17°$, $R = 150$ feet, and $x = 10$ feet. Required to connect the two tracks by a crossover in the form of a compound curve. From Art. 7, (*c*), $\tan \frac{1}{2}F_1OO_1 = 4.708 \times 6.691 + 300 = .10500$, and $F_1OO_1 = 12° 00'$. $FO_1O = 17° - 12° = 5° 00'$. $R_1 = 152.35\frac{2079}{.0872} - 2.35 = 360.9$ feet. From Art. 7, (*a*), $\tan \frac{1}{2}F_2OB = 4.708 \times 6.691 \div (150 + 10)2$, and $F_2OB = 11° 14'$;

$R_2 = (160 - 2.35)\frac{.1948}{.4731} - 2.35 = 62.6$ feet; $\cos PO_1O = 1 - \frac{10(150 - 62.6 + 5)}{(150 + 360.9)(360.9 + 62.6)}$, and $PO_1O = 5°18'$; $\cos O_2OO_1 = 1 - \frac{10(360.9 + 62.6 - 5)}{(150 + 360.9)(150 + 10 - 62.6)}$, and $O_2OO_1 = 23° 40'$. Then $KB = 2(150 + 10) = 65.62$ feet.

The crossing shown in Fig. 41 is in common use and avoids

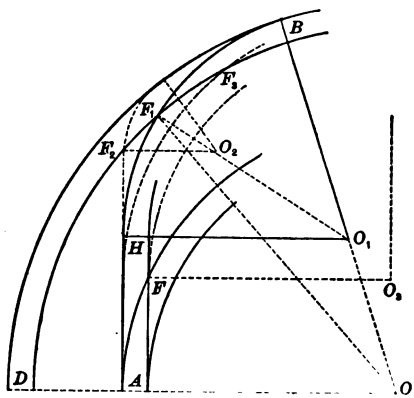


FIG. 41.

the somewhat objectionable reverse curve. The angle of the frog $F$ is constant for the curve of radius $AO$, while that for the frog on the outer track may be assumed between the limits $O_3F_3O$ and $O_2F_2O$; when $x$ is nearly $2G$, $F$ may be used in both places. Then, from Art. 6,

$$\cos F = \frac{2R - G}{2R + G},$$

in which $R$ is the radius of the inner track.

From Art. 7, (*a*),

$$\tan \frac{1}{2}BOF_1 = \frac{G}{2R} \cot \frac{1}{2}F_1,$$

and

$$BO_1F_1 = F_1 + BOF_1.$$

Then $BO_1$ or $R_2 = \left(R_1 - \frac{G}{2}\right) \frac{\sin BOF_1}{\sin BO_1F_1} - \frac{G}{2}$.

From Art. 3, (*c*),

$$\cos BOA = 1 - \frac{x}{R_1 - R_2};$$

$$AH = (R_1 - R_2) \sin BOA,$$

in which $R_1$ is the radius of the outer track. The point $B$ may be located by measuring the computed chord $DB$ deflected $\frac{1}{2}BOD$ from a tangent at $D$.

For example, let the radius of the outer track be 100 feet and $x$ be 9.21 feet. $\cos F = \frac{176.876}{186.292} = .9495$; $F = 18° 18'$; $\tan \frac{1}{2}BOF_1 = \frac{4.708}{200}6.2085$, $BOF_1 = 16° 38'$, and $BO_1F_1 = 35° 56'$. Then $R_2 = 97.65\frac{28625}{58684} - 2.35 = 45.28$; $\cos BOA = 1 - \frac{9.21}{54.72} - 2.35 = \cos 33° 44'$. The distance along the straight track to the point of the connecting curve, $H$, is $AH = 54.72 \times .55533 = 30.37$, and in this case $H$ is between the rails of the outer track, but the assumed frog angle is practically correct.

Prob. 30. Given: $F_1 = F_2 = 10° 00'$. Find the radius of the turnouts and the length of the connecting tangents.

Prob. 31. When the distance between tracks is 10 feet, find radius and frog angle necessary for a crossover in the form of a reverse curve.

Prob. 32. If the radius of the inner track be 100 feet, the distance between tracks 9.21 feet, and the frog angles 17°, find the distance $KB$ in Fig. 39.

# CHAPTER III.

## CONSTRUCTION.

### ART. 9. LOCATION.

The preliminary survey of a street railroad consists generally of a series of straight lines, run as nearly as may be to the proposed location of the road, the lengths, azimuths, and profiles of which are determined in the field. The length of the line is the principal factor of the first cost, and the grade is one of the most important items of the operating expenses of the road. The azimuths, or directions of the lines, are necessary for making a map showing the location of the road with reference to street or property lines or other objects upon which the position of the road must depend.

In cities, the streets are usually selected which will afford the largest amount of patronage regardless of other considerations, and the grades on such seldom if ever exceed that over which electric cars may be run. Examples of six per cent grades are numerous, and some of over twelve per cent are in existence. The limiting grade is that down which a car can be made to move under such perfect control of the motorman that a full stop may be effected within less than fifty feet from the place where the brakes are applied. The speed at the top of the grade of course materially affects the distance within which a car may be brought to a standstill, as will also the efficiency of the brakes and the skill of the motorman. Another important consideration in deciding whether or not a hill is suitable for a street-car line is the amount of traffic to be encountered at the base of it. If at the foot of the grade a crowded thoroughfare runs at right angles to the road, the necessity of perfect control is apparent, and the risk from any lack of it is vastly greater than where the danger is confined

to the car itself and its passengers, as is the case in the open country, where a great momentum simply has the effect of propelling the car at a high speed along an unobstructed track.

The cost of construction of a road through the country is materially lessened by allowing the use of heavy grades, since the length is decreased or the amount of excavation and embankment is reduced to a minimum. The economy in first cost is offset by the increased expense of operation, which is due not only to the extra amount of power necessary to raise the car through the given distance, but to the fact that each car must be equipped with powerful motors which are required only on the grades and add so much useless weight to be carried over the remainder of the line, and for which provision must be made in designing the bridges, culverts, and whole substructure. The following formula is given by Dr. Louis Bell as representing the number of horse-power required to drive a car at the rate of eight miles per hour on any grade:

$$\text{H. P.} = .43\,W(1 + G),$$

in which $W$ is the weight of car and load in tons, and $G$ is the per cent of grade. On straight, level track the tractive effort is considered as twenty pounds per ton.

In selecting the curves there are many things to be taken into consideration; the sharper the curves the more nearly the line may be made to conform to the contours, and hence there results but little necessary excavation; on sharp curves each rail has to be bent to the proper form at the mill and shipped in that condition at an increased cost, so it is desirable that the curve be as sharp, and therefore as short, as possible. Rails may be bent to a curve of 1000 feet radius or over by springing one end after the other has been spiked in place, and a portable bender is efficient for heavy rails on curves of radius not less than 300 feet. For sharper curves the 9″ girder rails must be run through heavy bending machines. The amount of traction on a curve as compared with that on a tangent is not very definitely known for curves of small radius; the prac-

tice on steam railroads is to reduce the amount of grade per hundred feet by from 0.03 to 0.05 feet for each degree of curvature, or about three inches on a curve of 1000 feet radius. It is stated on good authority that a curve of fifty feet radius doubles the traction and, in unfavorable circumstances, often increases it threefold. The practical effect of curves is to cause more or less discomfort to passengers, to greatly diminish the rate of safe speed, and to effect a wear and tear of track and car far in excess of that on tangent. All of these results are, to a great extent, prevented by the use of a curve of varying radius, as will be explained in a following paragraph.

As a car passes rapidly around a curve it is acted upon by gravity and by centrifugal force. The former causes a vertical pressure upon the rails, and the latter a tendency to horizontal movement of the wheel flange toward the outer rail. The latter motion is counteracted by decreasing the speed or by elevating the outer rail with reference to the inner one, thus causing a horizontal component of the weight to oppose the centrifugal force. In Fig. 42 let $o$ be the center of gravity of the load $w$ on the rails $x$ and $z$; then $c'$ and $w'$ are respectively the components of the centrifugal force and of the weight, acting in opposite directions along the inclined plane $xz$. From similar triangles,

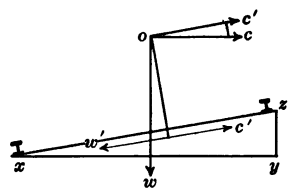

FIG. 42.

$$c' : c = xy : xz,$$

also $$w' : w = zy : xz.$$

Then $$zy = \frac{c \,.\, w' \,.\, xy}{c' \,.\, w};$$

for equilibrium $$c' = w'.$$

If $V$ be the speed in feet per second, and $R$ the radius of the curve in feet,

$$c = \frac{w V^2}{32.2 R}.$$

After substitution, $$zy = \frac{V^2 . xy}{32.2R}.$$

The distance $xy$ is very nearly the gauge $G$ of the track, and $zy = e$ is the elevation of the outer rail in feet, the expression for which becomes

$$e = \frac{V^2 G}{32.2R},$$

or $$e = .315\frac{V^2}{R},$$

when $V$ is the velocity of the car in miles per hour.

The elevation $e$ is relative to the inner rail, and not to the middle line of the track, it being the custom to depress the inner and raise the outer rail by half of $e$.

For example, let it be required to compute the proper elevation of the outer rail on a curve of 200 feet radius and speed of ten miles per hour. By substitution, $e = .315 \times 100 \div 200 = 0.16$ feet. For a radius of 50 feet $e$ would be 0.63 feet, which would be impossible in city streets, where one inch might be the maximum possible; then the speed should be reduced to correspond. $V^2 = 3.17eR$, and when $e$ is one inch the speed should not be above four miles per hour on a curve of 50 feet radius. On sharp curves the rear wheel of the car does not touch the outer rail, owing to the fact that the velocity is always low, and as a result the position of the car is cornerwise, the front outer and rear inner wheels pressing hard against the rails.

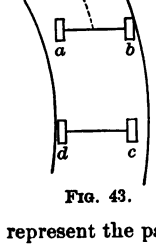

FIG. 43.

The trucks on most street cars are fixed so that the axes are parallel, and hence they cannot be on radii of the curves, but assume a position as shown in Fig. 43. $a$, $b$, $c$, and $d$ represent the part of the flanges of the wheels below the top of the track. As the car moves in the direction indicated by the arrow the flanges of the wheels $b$ and $d$ are pressed closely

against the rails, while those of $a$ and $c$ may not touch at all. Accordingly it is the custom to make the gauge on curves a fraction of an inch wider than on tangent, depending upon the length between axles $ad$ of the cars in use for the exact amount, and a guard-rail $\frac{1}{4}$ inch to $\frac{1}{2}$ inch higher than the rail is used inside the inner rail of the curve.

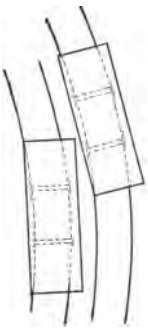

FIG. 44.

Another consideration in regard to curves is that concerning the distance between tracks if cars are to be allowed to pass at such points. The position of two cars meeting on a curve is shown in Fig. 44, and the necessity for increasing the distance between the tracks is apparent. The distance between centers should seldom be less than 10 feet on curves of 50 feet radius, and 12 feet is often specified.

Prob. 33. What is the proper elevation of the outer rail of a curve of 500 feet radius and for velocity of 15 miles per hour?

### ART. 10. TRANSITION CURVES.

The use of spiral instead of circular curves on street railways is properly becoming general, to the advantage of owners as well as of patrons of the road. Some of the reasons for adopting this form of curve have been enumerated, and to these must be added the necessity for elevation of the outer rail. On one side of the point of curve the track is straight, and hence the rails should be of equal elevation; just beyond, the outer rail, being on a curve, should be elevated as shown above, and hence, to fulfill these requirements, there would be an abrupt rise from the tangent to the curve, which is impracticable. The result with circular curves is that the rail must be improperly raised for a short distance back on the tangent, while the first part of the curve is not high enough.

In order that the elevation of the outer rail may be uniformly increasing from zero at the beginning, to the computed amount

at the end of the spiral or transition curve, the radii must diminish in the same ratio, since $R$ varies inversely as $e$. If 300 feet be assumed as the radius at the beginning of the transition curve and five feet the length of the chords, the several radii will be 300 feet into

$$1, \quad \frac{1}{2}, \quad \frac{1}{3}, \quad \frac{1}{4}, \quad \frac{1}{5}, \quad \frac{1}{6}, \quad \frac{1}{7} \cdots \cdot \frac{1}{n},$$

or 300, 150, 100. 75, 60, 50, 43 feet . . . .,

and the angles at the centers subtended by the chords may be computed.

In Fig. 45, 0, 1, 2, 3, 4, 5, 6, and 7 are stations 5 feet apart on the transition curve, and $a$, $b$, $c$, $d$, $e$, $f$, and $g$ are points on the tangent opposite them. If the distances from each of these points from 0 and the offsets $a1$, $b2$ . . . $g7$ be known, the curve may be located from $0g$. The computations are made by

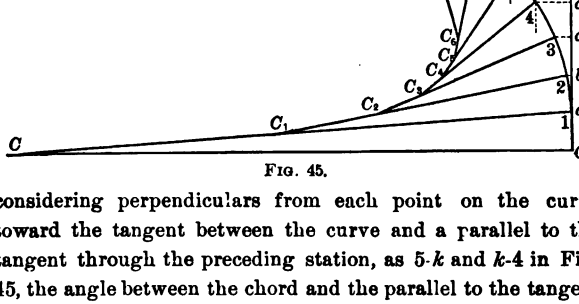


FIG. 45.

considering perpendiculars from each point on the curve toward the tangent between the curve and a parallel to the tangent through the preceding station, as 5-$k$ and $k$-4 in Fig. 45, the angle between the chord and the parallel to the tangent being the sum of the central angles subtended by the preceding chords and half the central angle subtended by the chord itself, or

$$5\text{-}4\text{-}k = C_0 + C_1 + C_2 + C_3 + \tfrac{1}{2}C_4;$$

then $$5\text{-}k = 5.0 \overset{\sin.}{\tan} 5\text{-}4\text{-}k,$$

and $$4\text{-}k = 5.0 \overset{\cos}{\cot} 5\text{-}4\text{-}k.$$

The tangent distances and offsets from the tangent, as $0e$ and 5-$e$, are found by continued additions of the distances, just computed as shown in the table.

| | Central Angle. | Chord. | Angle with Tangent. | Lati tude. | Longi- tude. | Tangent Distance. | Tangent Offset. |
|---|---|---|---|---|---|---|---|
| $C_0$ | 0° 57′.3 | 0-1 | 0° 28′.65 | 5.00 | 0.04 | 5.00 | 0.04 |
| $C_1$ | 1 54 .6 | 1-2 | 1 54 .6 | 5.00 | 0.17 | 10.00 | 0.21 |
| $C_2$ | 2 51 .9 | 2-3 | 4 17 .8 | 4.99 | 0.37 | 14.98 | 0.58 |
| $C_3$ | 3 49 .2 | 3-4 | 7 38 .4 | 4.96 | 0.66 | 19 94 | 1 25 |
| $C_4$ | 4 46 .6 | 4-5 | 11 56 .3 | 4.89 | 1.06 | 24.83 | 2.28 |
| $C_5$ | 5 43 .9 | 5-6 | 17 11 .5 | 4.78 | 1.48 | 29.61 | 3 76 |
| $C_6$ | 6 40 | 6-7 | 23 23 .5 | 4.59 | 1.99 | 34.20 | 5.75 |

The tangent of the deflection at 0 from the tangent to any station may be computed from the last two columns thus :

$$\tan d\text{-}0\text{-}4 = 1.25 \div 19.94;$$

$$\text{angle } d\text{-}0\text{-}4 = 3° 35'.$$

It often happens that from the P. C. an unobstructed view around the entire curve is impossible, and the deflections from other stations than 0 may be computed from the tangent distances and offsets. The angle between $k$-4 and 0-4 is 3° 35′, and the tangent 5-4-$k$ = 5-$k$ + $k$-4, so 5-4-$k$ = 11° 56′. Then the deflection from 0-4 produced to the chord 4-5 will be

$$h\text{-}4\text{-}5 = (k\text{-}4\text{-}5) - (d\text{-}0\text{-}4)$$

and

$$h\text{-}4\text{-}6 = (k\text{-}4\text{-}6) - (d\text{-}0\text{-}4)$$

also

$$\tan (0\text{-}4\ 2) = (d\text{-}4) - (b\text{-}2) \div (0\text{-}d)\ (0\text{-}b).$$

In like manner the deflections from each station to every other station on the curve may be computed.

In making the table of deflections for transition curves the same arrangement has been observed as in keeping the notes on simple curves; that is, the line of sight must always be in such a position that the vernier will indicate zero when the instrument is pointed toward the beginning of the spiral.

As an example of the use of the table let the location of the curve be such that only the first three stations can be seen from station zero. The deflections from the tangent at zero are given in the horrizontal line opposite "Instrument at $O$," and the stations 1, 2, and 3 are located five feet apart. The instrument is then moved to station 3, and the back-sight may be taken on either of the stakes already located, the vernier

TABLE OF DEFLECTION ANGLES FOR A SPIRAL WHEN THE CHORD = $R_1 \div 60$.

| Sta. 0 | Sta. 1 | Sta. 2 | Sta. 3 | Sta. 4 | Sta. 5 | Sta. 6 | Sta. 7 |
|---|---|---|---|---|---|---|---|
| | $R = 60C$ | $R = 30C$ | $R = 20C$ | $R = 15C$ | $R = 12C$ | $R = 10C$ | $R = 8.6C$ |
| Ins. at 0 | 0° 29′ | 1° 12′ | 2° 14′ | 3° 35′ | 5° 15′ | 7° 14′ | 9° 32′ |
| 0° 00′ | Ins. at 1 | 1 26 | 2 38 | 4 08 | 5 58 | 8 07 | 10 35 |
| 0 00 | 0 43 | Ins. at 2 | 3 06 | 4 46 | 6 46 | 9 04 | 11 42 |
| 0 00 | 0 58 | 2 04 | Ins. at 3 | 5 25 | 7 34 | 10 02 | 12 49 |
| 0 00 | 1 02 | 2 23 | 4 04 | Ins. at 4 | 8 21 | 10 59 | 13 55 |
| 0 00 | 1 12 | 2 42 | 4 32 | 6 41 | Ins. at 5 | 11 56 | 15 02 |
| 0 00 | 1 21 | 3 01 | 5 01 | 7 20 | 9 57 | Ins. at 6 | 16 09 |
| 0 00 | 1 31 | 3 21 | 5 30 | 7 58 | 10 45 | 13 51 | Ins. at 7 |

being set at the angle in the column for the station where the back-sight is taken and in the line for the instrument at the given station. In this case the vernier should be set at 2° 04′, 0° 58, or 0° 00′ according as the back-sight is to be at station 2, 1, or zero. Station 4 is fixed by setting the vernier to read 5° 25′, plunging the telescope and measuring five feet from station 3.

If the central curve to which the transition is to be applied be of large radius, the above table applies equally well by increasing the length of the chords; as for example, let the main curve be of 500 feet radius, then the radii of the spiral might be taken at 3000, 1500, 1000, 750, and 600 feet respectively, while the chords would be $5 \times 10 = 50$ feet, and intermediate points may be fixed by an ordinate to the middle of each chord.

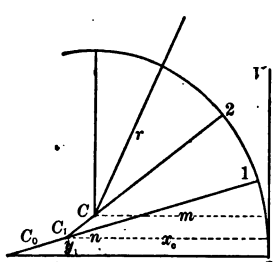


FIG. 46.

The coordinates of the centers are useful in computing the tangent and external distances of the curve, and the method of calculation is readily understood from the figure. Let $C_0$, $C_1$ . . . $C_e$ be the centers of the curves forming the spiral, and $x_0y_0$, $x_1y_1$ . . . $x_ey_e$ be the respective coordinates; also let $C$ and $m$ . $n$ represent the center and coordinates of the center of the main curve of radius $r$ between the two

transition spirals; then the latter may be computed from the following table:

| Center. | Angle with $R_0$. | Coordinates. | |
|---|---|---|---|
| | | $x$ | $y$ |
| $C_0$ | 0° 57′ 18″ | 300.00 feet | 0.00 feet |
| $C_1$ | 2 51 54 | 150.00 | 2.50 |
| $C_2$ | 5 43 48 | 100.06 | 5.00 |
| $C_3$ | 9 33 00 | 75.19 | 7.50 |
| $C_4$ | 14 19 34 | 60.41 | 9.99 |
| $C_5$ | 20 03 30 | 50.71 | 12.46 |
| $C_6$ | 26 48 28 | 44.13 | 14.86 |

The second column gives the angle that each radius (produced) makes with the first radius $C_0 - O$; in the third and fourth columns are the distances from each center to $C_0O$ and $OV$ respectively. Then the coordinates $m$ and $n$ are found.

$$m = x - (R - r)\cos\alpha,$$

and

$$n = y + (R - r)\sin\alpha,$$

in which $x$ . $y$ is the center of the arc of the spiral that joins the middle curve, $R$ is the radius of the last arc of the spiral, $r$ is the radius of the main curve, and $\alpha$ is the inclination of the radius at the end of the spiral to $R_0$ as given in column two of the above table.

Then $$T = n + m \tan \tfrac{1}{2}I,$$

and $$E = r\,\text{exsec}\,\tfrac{1}{2}I + (m - r) \div \cos \tfrac{1}{2}I.$$

These values may be found for any intersection angle when $r$ is known.

For example, let $I = 88° 30'$ and $r = 55.0$ feet; then $C_4$ is the center of the last arc of the spiral, and $\alpha = 14° 19' 34''$; $m = 60.41 - (60 - 55)\cos 14° 19' 34'' = 55.60$ feet; $n = 9.99 + 5 \sin 14° 19' 34'' = 11.23$ feet; $T = 11.23 + 55.6 \tan 44° 15' = 65.40$ feet.

It may often be desired to determine the radius of the central curve to correspond with a given tangent or external distance, and the above formulas may be stated in terms of $r$.

$$r = \frac{T - y - R(\sin\alpha - \cos\alpha \tan \tfrac{1}{2}I) - x \tan \tfrac{1}{2}I}{\cos\alpha \tan \tfrac{1}{2}I - \sin\alpha},$$

also $$r = \frac{E \cos \frac{1}{2}I - x + R \cos \alpha}{\cos \alpha - \cos \frac{1}{2}I},$$

from which $r$ may be found for assumed values of $T$ or $E$. By finding the radius of a simple curve having the given tangent, some idea may be formed as to the value of $R$, which will usually be the one on the spiral next smaller than the computed radius, $T \cot \frac{1}{2}I$. When $R$ has been determined, $x$, $y$, and $\alpha$ are taken from the table and the above equations are solved for $r$. If the value of $r$ thus determined be between the assumed $R$ and that next smaller in the spiral, the assumption is correct; if not, another value of $R$ may be chosen and corresponding values of $x$, $y$, and $\alpha$ used.

For example, let $I$ be 73° 32′ and $T = 50$ feet, to determine the value of the central radius. The radius of the simple curve, $T \cot \frac{1}{2}I = 66.9$ feet, so $R$ may be assumed as 60 feet. Then $\alpha = 14° 20'$, $x = 60\ 14$ feet, and $y = 9.99$ feet; then

$$r = \frac{50 - 9.99 - 60(.247 - .969 \times .747) - 60.14 \times .747}{.969 \times .747 - .247},$$

or $r = 49\ 71$ feet, which is so near 50 feet that the choice of $R$ may be considered as correct. If, however, the result had been, say, 45 feet, it would be better to take $R$ as 50 feet and make another computation for $r$.

Another variety of transition curve is that used to connect two grades of very different inclination; in this case the radius of the curve is in a vertical plane, and the curve is called a vertical curve. The object of the curve is to reduce

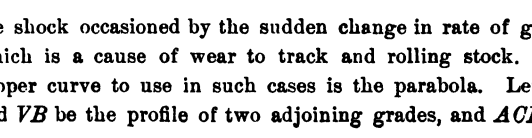


FIG. 47.

the shock occasioned by the sudden change in rate of grade, which is a cause of wear to track and rolling stock. The proper curve to use in such cases is the parabola. Let $AV$ and $VB$ be the profile of two adjoining grades, and $ACB$ the

proposed parabola; also let $VC$ be the vertical distance from the apex to the curve, and $OO_1$ any other vertical offset from the tangent to the curve. Whatever the difference in rate of grade, the horizontal distance between $A$ and $V$ is the same as that between $V$ and $B$, since the line is always so measured on the ground. Then, since $VC$ is an axis of the parabola,

$$VC = CM,$$

and $$OO_1 : VC = \overline{AK}^2 : \overline{AH}^2;$$

or, letting $E_1$, $E_2$, and $E_3$ represent the elevations of $A$, $V$, and $B$, the elevation of $M$ is $\frac{1}{2}(E_1 + E_3)$, and

$$CV = \tfrac{1}{2}(E_2 - M).$$

If $CV$ becomes negative, it indicates that the curve is above the tangents.

$$CV = \tfrac{1}{2}(E_2 - \tfrac{1}{2}E_1 - \tfrac{1}{2}E_3),$$

also $$OO_1 = \frac{VC \cdot d^2}{h^2},$$

in which $OO_1$ is any vertical offset from the tangent to the curve at a distance $d$ from $A$ or $B$, and $h$ is the horizontal distance $AV$ or $BV$.

For example, let it be required to replace the following grade by a vertical parabola. The elevations of stations 5 feet apart are $A = 100.0$; $1 = 99.5$; $2 = 99.0$; $V = 98.5$; $3 = 98.8$; $4 = 99.1$; $B = 99.4$. Then $VC = \frac{1}{2}(98.5 - \frac{1}{2} \cdot 100 - \frac{1}{2} \cdot 99.4) = -0.6$ feet. The offsets at 1 and 4 are $-0.60 \times 5^2 \div 15^2 = -0.07$ feet; at 2 and 3, $-0.27$ feet, and the elevations on the curve are 100, 99.57, 99.27, 99.10, 99.07, 99.17, and 99.40 feet.

Prob. 34. If $p$ and $p'$ represent the rise per horizontal foot of track, $d$ the distance from the beginning of the curve to any station, and $h$ half the length of curve, prove the offsets from the tangents to the vertical curve are $\frac{d^2}{4h}(p - p')$.

Prob. 35. Given: Elevation of $A = 100$; the rate of fall from $A$ to $V = 9 : 100$; the rise from $V$ to $B = 12 : 100$. Find elevations on a vertical curve of three stations 5 feet apart on each side of $V$.

Prob. 36. When $I = 91° 30'$, what central radius must be used with transition curves at each end that the external distance may be the same as with a simple curve of 75 feet radius?

Prob. 37. Make a page of field-notes for the curve found in the last problem, the instrument being placed at the point of spiral, at the P. C. and at the P. T. of the middle curve.

Prob. 38. Design a transition curve of seven stations to join a curve of 500 feet radius when $I = 88°$.

## Art. 11. Earthwork Computations.

All volumes of earth are usually considered as being composed of prisms, wedges, and pyramids. When the sides of these figures are narrow, the appearance will be that of a curved or warped surface, as a pyramid becomes a cone when the number of the sides are sufficiently increased.

The railroad engineer has to deal with two classes of excavation: that of the earth between the surface of the ground and the finished subgrade of the roadbed; and of material from borrow-pits when the amount of filling exceeds that of cutting the earth down to the proposed grade, or when ballast is taken from a gravel-pit. The computations in the two cases differ somewhat from each other, and the latter will be considered first.

In Fig. 48 the irregular curve represents the edge of the

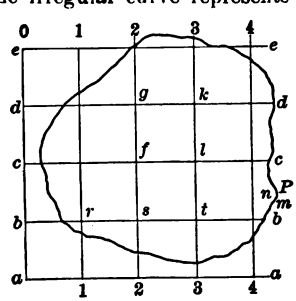


Fig. 48.

borrow-pit, and the intersections of the right lines show the points at which elevations are taken before and after the exca-

vation is completed. The points on the lines *aa* and *ae* should be so fixed that they will not be disturbed, so that all the other lines may be re-established after the first stakes and material in the pit have been removed. The squares, as *fghk*, are usually made small enough that the surface may be assumed as a plane without material error; then if the letters on the corners represent the elevations of those points before the ground is disturbed, and $f'g'k'l'$ the corresponding final elevations, the volume of $fgkl - f'g'k'l'$ will be the product of the area of a right section and the mean of the heights of the corners of the square, or

$$V = \frac{A}{4}(f - f' + g - g' + k - k' + l - l').$$

When $h \ldots hn$ are the various heights and $A$ is the area of the horizontal section of any prism of $n$ sides having plane ends, not necessarily parallel to each other,

$$V = \frac{A}{n}(h_1 + h_2 \ldots + hn),$$

in which $V$ is the volume in the same unit as the linear dimensions and the area. Usually all are in feet, and afterwards the volume is found in yards by dividing by 27.

When several prisms of equal right section adjoin each other the volume of the whole may be computed in one operation, multiplying the height at each corner by the number of entire polygons meeting at that corner, then the sum of these products into the area of a right section of one prism divided by the number of sides of the prism will be the volume, or

$$V = \frac{A}{4}(d_1 + 2d_2 + 3d_3 + 4d_4),$$

in which the prisms are of square section, $d_1$ is the sum of heights of corners common to one prism, $d_2$, $d_3$, and $d_4$ similarly are the sums of heights of corners common to two, three, and four prisms.

For example, let the corners be designated by the letters on the axes through them, as *c*2 for *f* and *d*3 for *k*. Then let, in Fig. 48, the depths of excavation at *b*1 = 6, *b*2 = 8, *b*3 = 10,

$b4 = 8$, $c1 = 5$, $c2 = 7$, $c3 = 9$, $c4 = 2$, $d1 = 4$, $d2 = 6$, $d3 = 8$, $d4 = 1$, $e2 = 0$, and $e3 = 2$. Then $b1$, $b4$, $d1$, $d4$, $e2$, and $e4$ are corners of but one prism each. Also $b2$, $b3$, $c1$, and $c4$ are common to two, $d2$ and $d3$ to three, and $c2$ and $c3$ to four prisms each. The volume will then be, if the squares be ten feet on a side, $V = 10\frac{1}{4}^0[(6 + 3 + 4 + 1 + 0 + 2) \times 1 + (8 + 10 + 5 + 2) \times 2 + (6 + 8) \times 3 + (7 + 9) \times 4] \div 27 = 159.26$ cubic yards.

It is often impossible to so locate the corners of the square that the surface so included will be a plane. However, by considering each square as two triangles the surface may usually be fairly represented by two planes intersecting on diagonals of the rectangles. The computation is the same as above, the height at each apex being taken as many times as there are triangles around that point, and the area being one half of the rectangle. Or if $A$ be the area of the rectangle and $n = 3$, the volume in cubic feet is

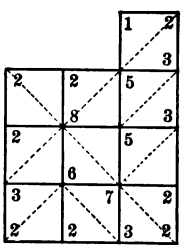

FIG. 49.

$$V = \frac{A}{6}(d_1 + d_2 + d_3 + d_4 + d_5 + d_6 + d_7 + d_8).$$

The proper direction of the diagonals is indicated in the note-book from observations in the field as shown in Fig. 49, the figures being the numbers by which the heights would be multiplied for that case.

The areas outside regular polygons are found by measurement in the field, the position of any point, as $p$, Fig. 48, being recorded $b + m$, $4 + n$, when $m$ and $n$ are the distances of $p$ from $bb$ and 44, and the volume of each irregular figure is found by taking the mean of the corner heights into the area and dividing by the number of sides. It must be constantly remembered that this rule is true only where the top and bottom surfaces are approximately plane figures.

The excavation or embankment along a line of railroad is computed from cross-sections taken at intervals of one hundred feet or less; the material being thus divided into figures

having parallel ends at the sections, and for sides the road-bed, the side slopes, and the original surface of the ground. The cross-sections are taken so near together that the ground surface may be, without material error, considered as generated by a right line touching the upper edges of the end sections. Such a figure may be divided into pyramids, wedges, and prisms having as a common height the distance between the sections, and is given the name Prismoid.

The Prismoidal Formula is an expression giving, at one operation, the volume of a group of prisms, wedges, or pyramids having a common altitude between parallel ends.

Let $A$ be the lower base, $a$ the upper base, $M$ the area of a section midway between the bases, and $l$ the distance between the bases; then the volumes will be

$$\text{Prism} = A \times l = \frac{l}{6}(A + 4M + a).$$

$$\text{Pyramid} = A \times \frac{l}{3} = \frac{l}{6}(A + 4M + a);$$

$$\text{Wedge} = A \times \frac{l}{2} = \frac{l}{6}(A + 4M + a);$$

in which $A = a$ in the prism and $a =$ zero in the wedge and pyramid; also $M = A$, $\frac{1}{4}A$, and $\frac{1}{2}A$ in the prism, pyramid, and wedge respectively.

In railroad work it is usual to compute $M$ as a section whose linear dimensions are the mean of the corresponding lines at the end sections; this is correct when the end sections are similar figures, and hence they should be taken at frequent intervals. In Fig. 50, $ABDE$ represents the finished road-bed, $AFGE$ and $BKHD$ the side slopes, $ABKF$ and $DEGH$ the end cross-sections, and $FGHK$ the original ground surface. In the field, at each section, the middle height $LN$, the side heights $Kk$ and $Ff$, and the distances of $F$ and $K$ from the middle line $fN$ and $kN$ are determined by measurement. Then the areas of the cross-sections are

$$A = \tfrac{1}{2}C(h_1 + h_2) + \tfrac{1}{2}(d_1 + d_2)b,$$

in which $C$ is the middle height, $h_1$ and $h_2$ horizontal distances from the middle to the side stakes, or $Nf$ and $Nk$, and $b$ the width of the road-bed. The middle area $M$ is computed from linear dimensions as

$$Rr = \tfrac{1}{3}(Hh + Kk)$$

substituted in the above formula, and $a$ is found from the measured dimensions of $GEDH$.

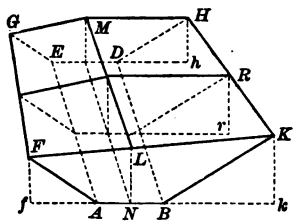


FIG. 50.

The field-work of cross-sectioning consists of setting stakes at points, as $F$, $L$, and $K$ in Fig. 50. The elevation of $N$ is previously determined from the profile, so $C$ is known when the elevation of $L$ is found, and the slopes are usually such that $3Kk = 2Bk$, or "one and one-half to one." The side height, as $Kk$, is estimated as nearly as possible, and the corresponding distance from the middle line is computed

$$Nk = \tfrac{1}{2}b + 1\tfrac{1}{2}Kk.$$

This distance is measured out and the rod held at the assumed point; then if the cut $Kk$ as determined from that rod reading is the same as that assumed in computing the distance $Nk$, the rod is at the proper place for the stake at $K$; if not, the result is a guide for a closer approximation, which should be very near the proper place.

The field-notes are kept as shown, the numerators being the side heights as $Kk$ and $Ff$, positive if cut, and negative if fill, and the denominators being the distances from the side stakes to the middle.

Here the road-bed is, for double track, twenty feet wide in cut and fifteen feet wide in fill, the extra width in cut being

for ditches each side of the track. The slopes are $1\frac{1}{2}$ to 1, and the volumes may be computed without further data if the distance between the stations, in this instance 100 feet be known.

| Station. | Left. | Middle. | Right. |
|---|---|---|---|
| 40 | $\frac{4.6}{16.9}$ | + 8.2 | + $\frac{10.8}{26.2}$ |
| 41 | $\frac{00}{10.0}$ | + 6.6 | + $\frac{7.4}{21.1}$ |
| + 20 | − $\frac{1.2}{9.3}$ | 0.0 | $\frac{3.0}{14.5}$ |
| + 60 | − $\frac{4.6}{14.4}$ | − 1.8 | $\frac{0.0}{10.0}$ |
| 42 | − $\frac{6.4}{17.1}$ | − 4.2 | − $\frac{0.8}{8.7}$ |

For example, to compute the volume between the sections at stations 40 and 41. Here $A = \frac{1}{2} \times 8.2(16.9 + 26.2) + \frac{1}{2} \times 20(4.6 + 10.8) = 253.71$ square feet; also $a = \frac{1}{2} \times 6.6(10.0 + 21.1) + \frac{1}{2} \times 20(00 + 7.4) = 139.63$ square feet. Then $M = \frac{1}{2} \times 7.4(13.45 + 23.65) + 5(2.3 + 9.1) = 194.27$ square feet. The volume is $V = \frac{100}{3}(253.71 + 777.08 + 139.63) = 19{,}507$ cubic feet = 922 5 cubic yards.

When the road-bed passes from cut to fill, as between stations 40 and 42 in the above table, it is necessary to take three cross-sections as shown in Fig. 51, where the middle line and each of the sides of the road-bed pass through the points of no cut and no fill, or at stations 41 + 00, 41 + 20, and at 41 + 60. The volume of the excavation is then *ABL-MFK* and *MFK-C*, the latter being a pyramid. Also the filling is composed of two figures, the pyramid *A-JME* and *IDC-JME*, and sufficient data are given in the notes to enable the engineer to compute the total volumes.

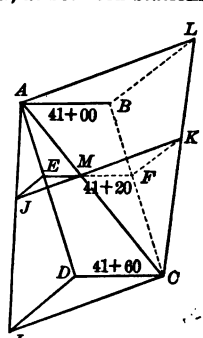


FIG. 51.

A practice which has been very general, and which is recognized by the courts of some States, is to consider the volume as the product of the length and the mean of the two end areas. The result of this method of computation is always too large, if the sections have been taken at proper points, unless the sections are exactly the same. The excuses for the custom are that the discrepancies to which it gives rise are small as compared with other unavoidable ones, such as classification of material into earth, loose and solid rock, and that the contractor should be given the benefit of the doubt.

As an extreme case let it be required to find the error involved in computing the volume of cut between stations $41 + 20$ and $41 + 60$ by the method of average end areas. Here $A = 10 \times 3.0 \times \frac{1}{2} = 15.0$ square feet, and $a =$ zero, so $V' = \frac{1}{2}(15 + 0) \times 40 = 300$ cubic feet. Since the figure is a pyramid the true volume is $V = 15 \times \frac{1}{3} \times 40 = 200$ cubic feet, and the error is hence fifty per cent. A similar error, but not so marked, will result when one section, although not zero, is considerably smaller than the other, and only when the sections are approximately equal will results sufficiently correct be obtained by this method of computation; and hence the profile as well as the cross-section should determine the frequency with which the latter should be taken.

Prob. 39. Compute the amount of excavation between stations 40 and 41 by average end areas and by the prismoidal formula.

Prob. 40. Compute the amount of excavation between stations 40 and 41 by dividing into pyramids having vertices at the side-slope stakes.

Prob. 41. Given: Two level sections, thirty feet apart, having heights of four and six feet respectively, the road-bed ten feet wide, and slopes one and a half to one; compute the volume by the prismoidal formula and by average end areas.

Prob. 42. In Fig. 49, let the figures represent the amount of excavation in the upper left four squares; compute the volume by rectangles and by triangles.

## ART. 12. ESTIMATE OF COST.

A standard statement of the cost of a street railroad, which will apply to all cases, is of course impossible. The various items only may be enumerated with accuracy since the prices are constantly changing, not only for varying dates, but for different localities, depending upon the prices of labor and upon the accessibility of the place of consumption to the source of supply. For these and other reasons the estimates of cost in this article are intended to serve only as examples giving approximate expense of the items named under given conditions.

The first cost of a line of a street electric railway may be divided properly into the following general items:

Franchise.
Right of Way.
Road-bed.
Overhead Structure.
Power Plant.
Rolling Stock.

The first item is one which, in this country, up to the present time has, at least nominally, usually been given by the people in exchange for the contemplated convenience of rapid transit, and with the provision that the railroad company shall make certain improvements, such as paving or surfacing the streets used by it. It is highly probable that in the near future, as increased skill and knowledge are attained, the operating expenses will be so reduced that interest may be earned on capital invested in electric railroads, even after paying some percentage of the receipts to the cities in return for the privilege of using the streets. When that time is reached the franchise will be salable and city governments will be prompt to take advantage of the fact.

The right of way through city streets is included in the franchise if the latter is bought. Through farming land the property must be bought outright, and near large towns and cities

the price may be from one hundred dollars to a thousand dollars per acre. The area of a mile of right of way is 0.1212 acres for each foot of width, or 1 acre for each width of 8¼ feet; so at $500 per acre and right of way 25 feet wide, the expense of this item would be $1515. Damages to abutting property not actually occupied must be considered in determining the location of the line, and reduced as much as possible by changes in grade or alignment.

In the item of road-bed are included the excavation and embankment and ballast, also the ties, rails, and fastenings. The excavation may cost from 15c. in soft loam or gravel to a dollar in rock or in broken-stone roads. Ties are worth from 30c. to 75c., and are placed approximately two feet apart. The following bill of material is for a mile of single-track road, and is taken from *The Street Railway Journal* for December 1893, since which time prices have decreased somewhat. It will be noted that 70-pound rails are quoted, whereas the present practice is to use much heavier, and oak or chestnut ties might be substituted for those of spruce.

| | |
|---|---|
| 110 tons 70-lb. rails (including freight, inspection, and hauling) @ $37.50 | $4125.00 |
| 18,000 pounds of angle-bars (360 pairs, 50 lbs. each @ $2.01 per 100 pounds) | 361.80 |
| 1700 track-bolts (½″ × 3½″) @ $3.01 per 100 pounds | 51.17 |
| 6050 spikes (5″ × $\frac{9}{16}$″) @ $2.46 per 100 pounds | 148.83 |
| 1¼ M nut locks @ $6.50 per M | 8.12 |
| 3017 spruce ties @ 55c | 1659 35 |
| 360 bonds @ 25c | 90.00 |
| 1320 tie-rods @ 20c | 264.00 |
| 2347 cubic yards excavation (8′ × 18″ deep) hauled off @ 30c | 704.10 |
| Track-laying @ $1.59 per linear foot | 1000.00 |
| Total | $8412.37 |

In many places along the side of country roads where the excavation is only to the depth of about one foot, a good substantial railroad may be built for about one dollar per linear foot.

The bill given below is from *The Street Railway Journal* of March 26, 1896, for a mile of double-track street railway in a paved street in Chicago:

| | |
|---|---|
| 283 L. tons 9″ 90-pound rails @ $33 | $9339 |
| 4224 white-oak ties 5″ × 8″ × 7′ @ 38c | 1605 |
| 352 cast welded joints @ $3.50 | 1232 |
| 1760 tie-rods @ 15c | 264 |
| 33,792 spikes ½″ × ½″ × 4½″ @ 1c | 338 |
| 42,240 feet wood-filler | 2112 |
| Labor at $1 per linear foot | 5280 |
| 10,560 square yards cedar blocks @ 30c | 3168 |
| 146 square yards sand @ $1.25 | 183 |
| 445 cubic yards broken stone @ $1.50 | 668 |
| 10,560 square yards gravel and dressing @ 8c | 845 |
| 10,560 square yards 2″ hemlock boards @ 8c. per square yard | 845 |
| Total | $25,879 |

If stone be substituted for wooden pavement, the expense will be $12,708 instead of $3168 and the total $35,419.

The poles in the overhead system are spaced about ninety to the mile. The price of wooden poles varies from $2.50 to $3.50 each, while those of iron are from $18 to $27 apiece; the cost of setting them is from $2 to $2.50 each. Sawed poles with arms may be estimated at $600 per mile. The wiring may be stated as follows: trolley wire $700; feed wire $1000; and return wire $600. The total cost of the overhead system for a mile of single track may then be given in round numbers as $3000 for wooden poles and $5000 for iron poles.

The cost of power and equipment for an electric railroad varies approximately as the number of cars operated, and the following round numbers will serve as rough checks on an estimate in a given case:

| Number Cars. | Capacity. | | Cost. | | |
|---|---|---|---|---|---|
| | H. P. | K. W. | Steam Plant. | Electrical Equipment | Cars and Equipment |
| 1 | 20 | 15 | $1000 | $700 | $3000 |

The cost for a small number of cars is, in general, somewhat larger than for many. The necessary number of cars per mile is governed by the rate of speed and the interval of time between them. Thus let the cars pass a given point in the same direction every 5 minutes, and let the average speed be 6 miles per hour; then each car must be half a mile ahead of the succeeding one; and if the same number pass in the opposite direction, the number per mile of road must be four.

Prob. 43. Make estimate of required capacity and probable cost of a power house for a single-track road over which it is proposed to run cars every 10 minutes each way at a rate of 8 miles per hour.

Prob. 44. Compute the cost per mile of excavating for an electric railway which is to follow the grade of a broken-stone street, the depth of excavation being 20 inches and the width 8 feet.

Prob. 45. Compute the cost of paving, with stone blocks, a mile of street, 80 feet between the curbs.

Prob. 46. Show that the number of gross tons of rails per mile of single track is eleven sevenths of the number of pounds per yard of rail.

### ART. 13. SUPERINTENDENCE OF CONSTRUCTION.

Before any excavation is begun the line should be staked out, and at the angles all intersection-points should be carefully referenced by measurements to firm points, or by intersecting lines, so that they may be readily replaced. On long curves the points of curve and of tangency, and also points at frequent intervals on tangent, should be preserved by reference. In city streets the most convenient way to do this is to use the curb as a reference, then the foreman may measure offsets from the same points to line off the edge of the excavation. In Fig. 52 are shown methods of locating a point so that it may be fixed

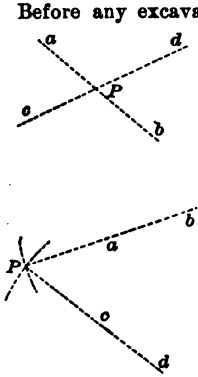

FIG. 52.

at any subsequent time. Let *P* be the given point, then from *P* the straight lines *ab* and *cd* are located outside the area to be excavated. Then at any time *P* may be relocated by finding, with the transit, the intersection of the lines *ab* and *cd*. Otherwise the same result may be attained by measuring accurately *Pa* and *Pc*; then *P* may be fixed by intersecting arcs of circles of radii *Pa* and *Pc*.

The bottom of the trench is called the subgrade of the road, and the material upon which the ties rest is the ballast. The preparation of the subgrade is an important factor in regulating the cost of future repairs. The use of the ground under the surface of the streets as a place for laying gas, sewer, water, and other pipes makes the work of securing a subgrade of uniform compactness very difficult. The trenches for the pipes should be wet and well tamped and the whole subgrade thoroughly rolled with a heavy roller to a grade very nearly parallel to the finished grade. It is usual on well-constructed roads to make the excavation about 22 inches deep, which depth leaves about seven inches of ballast under a 9-inch girder-rail and a tie six inches deep.

The best material for ballast is broken trap-rock of such size that it will just pass through a two-inch ring; this variety of stone breaks into very hard, sharp-edged fragments which do not readily wear smooth, and for this reason may be made very compact by rolling. The ballast is spread in layers in the trench, and each layer is rolled separately, preferably with a roller having a corrugated surface.

The ties may be of white oak or chestnut for best results, but yellow pine and hackmatack will often outlast the rails above them; the harder woods are preferred from the fact that they hold the spikes better. The standard size of ties is six inches deep, eight inches wide, and seven or, seldom, eight feet long; they may be either sawed or hewn with the grain, but the top and under surfaces should be nearly parallel. The ties are spaced according to the load, strength of the rail, and the width of the lower flange; the usual distance is three feet between centers, and in some cities the spacing is not uniform but the ties are placed nearer together at the rail-joints than

the middle of the rail. On the People's Traction Company's line in Philadelphia the ties are spaced as shown in Fig. 53, one being placed directly under the joint. The ties are of

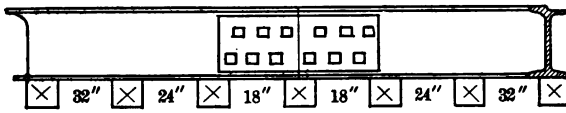


FIG. 53.

Georgia pine, seven feet long, and, except as shown in the figure, are evenly spaced, fourteen to each rail. On some roads the upper surface of the tie is protected from the rail by cast-iron tie-plates about six by ten inches in size and weighing about fifteen pounds each.

The weight and depth of rails have been constantly increasing since electricity has superseded horses as motive power on street railroads. The rail probably now most used is the ninety pound, nine-inch girder-rail, which is deep enough to allow ballast and paving over the ties. The custom of leaving a small space between rails to allow for expansion is no longer followed, but the ends are cleaned with care and laid closely together. The rails are fastened together by plates about thirty-six inches long, as shown in Fig. 53, having twelve one-inch holes. The bolts should be oiled and very firmly screwed into place, since the joint is the vulnerable point of the track. Continuous rails, made by welding after they have been laid, have been tried with considerable success, the riding qualities have been improved, and the cost of repairs lessened. There seems little doubt but this will be the practice in the future. The rails are tied together by rods every five feet, and the gauge should be kept perfect by testing at each tie as the spikes are being driven.

After the rails are spiked the final alignment and grade should be given, and the rails shifted accordingly; the tamping may be done with gravel or with broken stone, the latter material being more durable. The rails on tangent should be made the same height transversely, to prevent a rocking motion of the car, and any final error, in this respect, of more than a

hundredth of a foot should be corrected; after the bonding has been completed the remainder of the ballast is put in place and rammed or rolled till it is compact, when the paving may be begun.

The form as well as the weight of the rail is to be considered, both for its effect upon the paving of the street and as a tramway for vehicles. In order to preserve the surface of adjacent pavement, it is important that the lower flange shall not interfere with the proper setting of the paving-blocks; this is properly accomplished by using a deep rail, as shown in Fig. 54, where the lower edge of the block is above the lower flange, and there is no necessity for the bad but common practice of chipping the corner next to the rail, as illustrated in Fig. 55. This arrangement causes the stone to be easily tipped over or forced down further than the rest, owing to the wedge shape. When a rail is selected that is of less depth than the paving-blocks, the upper and lower flanges should be of nearly equal width, so that a square block will touch both. In the case of asphalt pavement a T rail is quite satisfactory; the asphalt is laid close up to both sides of the rail, and before it is completely set a car is run over the track, which makes a groove of just the proper size.

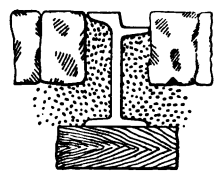

Fig. 54.

Fig. 55.

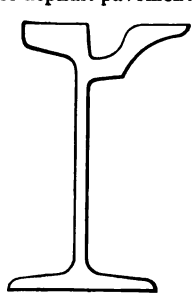

Fig. 56.

The best form for the upper flange of the rail is a subject of debate, but main points are three in number: that form may be adopted that allows carriage-wheels to enter and leave the grooves with little danger of injury, as in Fig. 56; or the groove may be of more nearly perpendicular sides, but so narrow that no ordinary tire will enter it, as shown in Fig. 57; again, the design may be such that it will not acc

modate vehicles at all, as is the case with the tee rail. These are three examples of common practice, and the best for any particular case will be determined by local considerations.

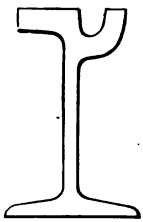

FIG. 57.

Prob. 47. Read articles on General Track Construction in the *Street Railway Journal* for October 1895, page 664; November 1895, page 736; and December 1895, page 803; also November 1896, page 697. Write an abstract of each paper.

Prob. 48. State briefly the arguments in favor of suspended rail-joints.

Prob. 49. Make a brief summary of paper and discussion of "Influence of Rails on Pavements," in *Trans. Am. Soc. C. E.*, vol. XXXVII.

# TABLES.

### TRIGONOMETRIC FUNCTIONS.

Let $A$ (Fig. 107) = angle $BAC$ = arc $BF$, and let the radius $AF = AB = AH = 1$.

We then have

$\sin A = BC$
$\cos A = AC$
$\tan A = DF$
$\cot A = HG$
$\sec A = AD$
$\operatorname{cosec} A = AG$
$\operatorname{versin} A = CF = BE$
$\operatorname{covers} A = BK = HL$
$\operatorname{exsec} A = BD$
$\operatorname{coexsec} A = BG$
$\operatorname{chord} A = BF$
$\operatorname{chord} 2A = BI = 2BC$

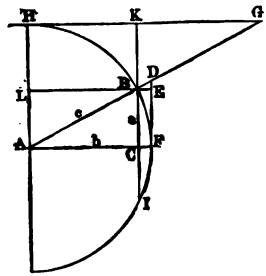


FIG. 107.

In the right-angled triangle $ABC$ (Fig. 107)
Let $AB = c$, $AC = b$, and $BC = a$.
We then have:

1. $\sin A = \frac{a}{c} = \cos B$
2. $\cos A = \frac{b}{c} = \sin B$
3. $\tan A = \frac{a}{b} = \cot B$
4. $\cot A = \frac{b}{a} = \tan B$
5. $\sec A = \frac{c}{b} = \operatorname{cosec} B$
6. $\operatorname{cosec} A = \frac{c}{a} = \sec B$
7. $\operatorname{vers} A = \frac{c-b}{c} = \operatorname{covers} B$
8. $\operatorname{exsec} A = \frac{c-b}{b} = \operatorname{coexsec} B$
9. $\operatorname{covers} A = \frac{c-a}{c} = \operatorname{versin} B$
10. $\operatorname{coexsec} A = \frac{c-a}{a} = \operatorname{exsec} B$
11. $a = c \sin A = b \tan A$
12. $b = c \cos A = a \cot A$
13. $c = \frac{a}{\sin A} = \frac{b}{\cos A}$
14. $a = c \cos B = b \cot B$
15. $b = c \sin B = a \tan B$
16. $c = \frac{a}{\cos B} = \frac{b}{\sin B}$
17. $a = \sqrt{(c+b)(c-b)}$
18. $b = \sqrt{(c+a)(c-a)}$.
19. $c = \sqrt{a^2 + b^2}$
20. $C = 90° = A + B$

21. area $= \frac{ab}{2}$

## SOLUTION OF OBLIQUE TRIANGLES.

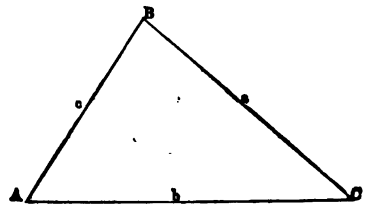


FIG. 108.

| | GIVEN. | SOUGHT. | FORMULÆ. |
|---|---|---|---|
| 22 | $A, B, a$ | $C, b, c$ | $C = 180° - (A + B)$, $\quad b = \frac{a}{\sin A} \cdot \sin B$, $\quad c = \frac{a}{\sin A} \sin (A + B)$ |
| 23 | $A, a, b$ | $B, C, c$ | $\sin B = \frac{\sin A}{a} \cdot b$, $\quad C = 180° - (A + B)$, $\quad c = \frac{a}{\sin A} \cdot \sin C.$ |
| 24 | $C, a, b$ | $\frac{1}{2}(A + B)$ | $\frac{1}{2}(A + B) = 90° - \frac{1}{2} C$ |
| 25 | | $\frac{1}{2}(A - B)$ | $\tan \frac{1}{2}(A - B) = \frac{a - b}{a + b} \tan \frac{1}{2}(A + B)$ |
| 26 | | $A, B$ | $A = \frac{1}{2}(A + B) + \frac{1}{2}(A - B)$, $B = \frac{1}{2}(A + B) - \frac{1}{2}(A - B)$ |
| 27 | | $c$ | $c = (a + b)\frac{\cos \frac{1}{2}(A + B)}{\cos \frac{1}{2}(A - B)} = (a - b)\frac{\sin \frac{1}{2}(A + B)}{\sin \frac{1}{2}(A - B)}$ |
| 28 | | area | $K = \frac{1}{2} a b \sin C.$ |
| 29 | $a, b, c$ | $A$ | Let $s = \frac{1}{2}(a + b + c)$; $\sin \frac{1}{2} A = \sqrt{\frac{(s-b)(s-c)}{b c}}$ |
| 30 | | | $\cos \frac{1}{2} A = \sqrt{\frac{s(s-a)}{b c}}$; $\tan \frac{1}{2} A = \sqrt{\frac{(s-b)(s-c)}{s(s-a)}}$ |
| 31 | | | $\sin A = \frac{2\sqrt{s(s-a)(s-b)(s-c)}}{b c}$; $\quad \text{vers } A = \frac{2(s-b)(s-c)}{b c}$ |
| 32 | | area | $K = \sqrt{s(s-a)(s-b)(s-c)}$ |
| 33 | $A, B, C, a$ | area | $K = \frac{a^2 \sin B \cdot \sin C}{2 \sin A}$ |

# Table I.—Trigonometric Formulæ.

## General Formulæ.

34 $\sin A = \frac{1}{\operatorname{cosec} A} = \sqrt{1-\cos^2 A} = \tan A \cos A$

35 $\sin A = 2 \sin \frac{1}{2} A \cos \frac{1}{2} A = \operatorname{vers} A \cot \frac{1}{2} A$

36 $\sin A = \sqrt{\frac{1}{2} \operatorname{vers} 2A} = \sqrt{\frac{1}{2}(1-\cos 2A)}$

37 $\cos A = \frac{1}{\sec A} = \sqrt{1-\sin^2 A} = \cot A \sin A$

38 $\cos A = 1 - \operatorname{vers} A = 2\cos^2 \frac{1}{2} A - 1 = 1 - 2\sin^2 \frac{1}{2} A$

39 $\cos A = \cos^2 \frac{1}{2} A - \sin^2 \frac{1}{2} A = \sqrt{\frac{1}{2} + \frac{1}{2} \cos 2A}$

40 $\tan A = \frac{1}{\cot A} = \frac{\sin A}{\cos A} = \sqrt{\sec^2 A - 1}$

41 $\tan A = \sqrt{\frac{1}{\cos^2 A} - 1} = \frac{\sqrt{1-\cos^2 A}}{\cos A} = \frac{\sin 2A}{1+\cos 2A}$

42 $\tan A = \frac{1-\cos 2A}{\sin 2A} = \frac{\operatorname{vers} 2A}{\sin 2A} = \operatorname{exsec} A \cot \frac{1}{2} A$

43 $\cot A = \frac{1}{\tan A} = \frac{\cos A}{\sin A} = \sqrt{\operatorname{cosec}^2 A - 1}$

44 $\cot A = \frac{\sin 2A}{1-\cos 2A} = \frac{\sin 2A}{\operatorname{vers} 2A} = \frac{1+\cos 2A}{\sin 2A}$

45 $\cot A = \frac{\tan \frac{1}{2} A}{\operatorname{exsec} A}$

46 $\operatorname{vers} A = 1 - \cos A = \sin A \tan \frac{1}{2} A = 2\sin^2 \frac{1}{2} A$

47 $\operatorname{vers} A = \operatorname{exsec} A \cos A$

48 $\operatorname{exsec} A = \sec A - 1 = \tan A \tan \frac{1}{2} A = \frac{\operatorname{vers} A}{\cos A}$

49 $\sin \frac{1}{2} A = \sqrt{\frac{1-\cos A}{2}} = \sqrt{\frac{\operatorname{vers} A}{2}}$

50 $\sin 2A = 2 \sin A \cos A$

51 $\cos \frac{1}{2} A = \sqrt{\frac{1+\cos A}{2}}$

52 $\cos 2A = 2\cos^2 A - 1 = \cos^2 A - \sin^2 A = 1 - 2\sin^2 A$

GENERAL FORMULÆ.

53. $\tan \frac{1}{2} A = \frac{\tan A}{1 + \sec A} = \operatorname{cosec} A - \cot A = \frac{1 - \cos A}{\sin A} = \sqrt{\frac{1 - \cos A}{1 + \cos A}}$

54. $\tan 2 A = \frac{2 \tan A}{1 - \tan^2 A}$

55. $\cot \frac{1}{2} A = \frac{\sin A}{\operatorname{vers} A} = \frac{1 + \cos A}{\sin A} = \frac{1}{\operatorname{cosec} A - \cot A}$

56. $\cot 2 A = \frac{\cot^2 A - 1}{2 \cot A}$

57. $\operatorname{vers} \frac{1}{2} A = \frac{\frac{1}{2} \operatorname{vers} A}{1 + \sqrt{1 - \frac{1}{2} \operatorname{vers} A}} = \frac{1 - \cos A}{2 + \sqrt{2 (1 + \cos A)}}$

58. $\operatorname{vers} 2 A = 2 \sin^2 A = 2 \sin A \cos A \tan A$

59. $\operatorname{exsec} \frac{1}{2} A = \frac{1 - \cos A}{(1 + \cos A) + \sqrt{2 (1 + \cos A)}}$

60. $\operatorname{exsec} 2 A = \frac{2 \tan^2 A}{1 - \tan^2 A}$

61. $\sin (A \pm B) = \sin A . \cos B \pm \sin B . \cos A$

62. $\cos (A \pm B) = \cos A . \cos B \mp \sin A . \sin B$

63. $\sin A + \sin B = 2 \sin \frac{1}{2} (A + B) \cos \frac{1}{2} (A - B)$

64. $\sin A - \sin B = 2 \cos \frac{1}{2} (A + B) \sin \frac{1}{2} (A - B)$

65. $\cos A + \cos B = 2 \cos \frac{1}{2} (A + B) \cos \frac{1}{2} (A - B)$

66. $\cos B - \cos A = 2 \sin \frac{1}{2} (A + B) \sin \frac{1}{2} (A - B)$

67. $\sin^2 A - \sin^2 B = \cos^2 B - \cos^2 A = \sin (A + B) \sin (A - B)$

68. $\cos^2 A - \sin^2 B = \cos (A + B) \cos (A - B)$

69. $\tan A + \tan B = \frac{\sin (A + B)}{\cos A . \cos B}$

70. $\tan A - \tan B = \frac{\sin (A - B)}{\cos A . \cos B}$

# Table II.—Natural Sines and Cosines.

| ′ | 0° | | 1° | | 2° | | 3° | | 4° | | ′ |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | Sine | Cosin | Sine | Cosin | Sine | Cosin | Sine | Cosin | Sine | Cosin | |
| 0 | .00000 | One. | .01745 | .99985 | .03490 | .99939 | .05234 | .99863 | .06976 | .99756 | 60 |
| 1 | .00029 | One. | .01774 | .99984 | .03519 | .99938 | .05263 | .99861 | .07005 | .99754 | 59 |
| 2 | .00058 | One. | .01803 | .99984 | .03548 | .99937 | .05292 | .99860 | .07034 | .99752 | 58 |
| 3 | .00087 | One. | .01832 | .99983 | .03577 | .99936 | .05321 | .99858 | .07063 | .99750 | 57 |
| 4 | .00116 | One. | .01862 | .99983 | .03606 | .99935 | .05350 | .99857 | .07092 | .99748 | 56 |
| 5 | .00145 | One. | .01891 | .99982 | .03635 | .99934 | .05379 | .99855 | .07121 | .99746 | 55 |
| 6 | .00175 | One. | .01920 | .99982 | .03664 | .99933 | .05408 | .99854 | .07150 | .99744 | 54 |
| 7 | .00204 | One. | .01949 | .99981 | .03693 | .99932 | .05437 | .99852 | .07179 | .99742 | 53 |
| 8 | .00233 | One. | .01978 | .99980 | .03723 | .99931 | .05466 | .99851 | .07208 | .99740 | 52 |
| 9 | .00262 | One. | .02007 | .99980 | .03752 | .99930 | .05495 | .99849 | .07237 | .99738 | 51 |
| 10 | .00291 | One. | .02036 | .99979 | .03781 | .99929 | .05524 | .99847 | .07266 | .99736 | 50 |
| 11 | .00320 | .99999 | .02065 | .99979 | .03810 | .99927 | .05553 | .99846 | .07295 | .99734 | 49 |
| 12 | .00349 | .99999 | .02094 | .99978 | .03839 | .99926 | .05582 | .99844 | .07324 | .99731 | 48 |
| 13 | .00378 | .99999 | .02123 | .99977 | .03868 | .99925 | .05611 | .99842 | .07353 | .99729 | 47 |
| 14 | .00407 | .99999 | .02152 | .99977 | .03897 | .99924 | .05640 | .99841 | .07382 | .99727 | 46 |
| 15 | .00436 | .99999 | .02181 | .99976 | .03926 | .99923 | .05669 | .99839 | .07411 | .99725 | 45 |
| 16 | .00465 | .99999 | .02211 | .99976 | .03955 | .99922 | .05698 | .99838 | .07440 | .99723 | 44 |
| 17 | .00495 | .99999 | .02240 | .99975 | .03984 | .99921 | .05727 | .99836 | .07469 | .99721 | 43 |
| 18 | .00524 | .99999 | .02269 | .99974 | .04013 | .99919 | .05756 | .99834 | .07498 | .99719 | 42 |
| 19 | .00553 | .99998 | .02298 | .99974 | .04042 | .99918 | .05785 | .99833 | .07527 | .99716 | 41 |
| 20 | .00582 | .99998 | .02327 | .99973 | .04071 | .99917 | .05814 | .99831 | .07556 | .99714 | 40 |
| 21 | .00611 | .99998 | .02356 | .99972 | .04100 | .99916 | .05844 | .99829 | .07585 | .99712 | 39 |
| 22 | .00640 | .99998 | .02385 | .99972 | .04129 | .99915 | .05873 | .99827 | .07614 | .99710 | 38 |
| 23 | .00669 | .99998 | .02414 | .99971 | .04159 | .99913 | .05902 | .99826 | .07643 | .99708 | 37 |
| 24 | .00698 | .99998 | .02443 | .99970 | .04188 | .99912 | .05931 | .99824 | .07672 | .99705 | 36 |
| 25 | .00727 | .99997 | .02472 | .99969 | .04217 | .99911 | .05960 | .99822 | .07701 | .99703 | 35 |
| 26 | .00756 | .99997 | .02501 | .99969 | .04246 | .99910 | .05989 | .99821 | .07730 | .99701 | 34 |
| 27 | .00785 | .99997 | .02530 | .99968 | .04275 | .99909 | .06018 | .99819 | .07759 | .99699 | 33 |
| 28 | .00814 | .99997 | .02560 | .99967 | .04304 | .99907 | .06047 | .99817 | .07788 | .99696 | 32 |
| 29 | .00844 | .99996 | .02589 | .99966 | .04333 | .99906 | .06076 | .99815 | .07817 | .99694 | 31 |
| 30 | .00873 | .99996 | .02618 | .99966 | .04362 | .99905 | .06105 | .99813 | .07846 | .99692 | 30 |
| 31 | .00902 | .99996 | .02647 | .99965 | .04391 | .99904 | .06134 | .99812 | .07875 | .99689 | 29 |
| 32 | .00931 | .99996 | .02676 | .99964 | .04420 | .99902 | .06163 | .99810 | .07904 | .99687 | 28 |
| 33 | .00960 | .99995 | .02705 | .99963 | .04449 | .99901 | .06192 | .99808 | .07933 | .99685 | 27 |
| 34 | .00989 | .99995 | .02734 | .99963 | .04478 | .99900 | .06221 | .99806 | .07962 | .99683 | 26 |
| 35 | .01018 | .99995 | .02763 | .99962 | .04507 | .99898 | .06250 | .99804 | .07991 | .99680 | 25 |
| 36 | .01047 | .99995 | .02792 | .99961 | .04536 | .99897 | .06279 | .99803 | .08020 | .99678 | 24 |
| 37 | .01076 | .99994 | .02821 | .99960 | .04565 | .99896 | .06308 | .99801 | .08049 | .99676 | 23 |
| 38 | .01105 | .99994 | .02850 | .99959 | .04594 | .99894 | .06337 | .99799 | .08078 | .99673 | 22 |
| 39 | .01134 | .99994 | .02879 | .99959 | .04623 | .99893 | .06366 | .99797 | .08107 | .99671 | 21 |
| 40 | .01164 | .99993 | .02908 | .99958 | .04653 | .99892 | .06395 | .99795 | .08136 | .99668 | 20 |
| 41 | .01193 | .99993 | .02938 | .99957 | .04682 | .99890 | .06424 | .99793 | .08165 | .99666 | 19 |
| 42 | .01222 | .99993 | .02967 | .99956 | .04711 | .99889 | .06453 | .99792 | .08194 | .99664 | 18 |
| 43 | .01251 | .99992 | .02996 | .99955 | .04740 | .99888 | .06482 | .99790 | .08223 | .99661 | 17 |
| 44 | .01280 | .99992 | .03025 | .99954 | .04769 | .99886 | .06511 | .99788 | .08252 | .99659 | 16 |
| 45 | .01309 | .99991 | .03054 | .99953 | .04798 | .99885 | .06540 | .99786 | .08281 | .99657 | 15 |
| 46 | .01338 | .99991 | .03083 | .99952 | .04827 | .99883 | .06569 | .99784 | .08310 | .99654 | 14 |
| 47 | .01367 | .99991 | .03112 | .99952 | .04856 | .99882 | .06598 | .99782 | .08339 | .99652 | 13 |
| 48 | .01396 | .99990 | .03141 | .99951 | .04885 | .99881 | .06627 | .99780 | .08368 | .99649 | 12 |
| 49 | .01425 | .99990 | .03170 | .99950 | .04914 | .99879 | .06656 | .99778 | .08397 | .99647 | 11 |
| 50 | .01454 | .99989 | .03199 | .99949 | .04943 | .99878 | .06685 | .99776 | .08426 | .99644 | 10 |
| 51 | .01483 | .99989 | .03228 | .99948 | .04972 | .99876 | .06714 | .99774 | .08455 | .99642 | 9 |
| 52 | .01513 | .99989 | .03257 | .99947 | .05001 | .99875 | .06743 | .99772 | .08484 | .99639 | 8 |
| 53 | .01542 | .99988 | .03286 | .99946 | .05030 | .99873 | .06773 | .99770 | .08513 | .99637 | 7 |
| 54 | .01571 | .99988 | .03316 | .99945 | .05059 | .99872 | .06802 | .99768 | .08542 | .99635 | 6 |
| 55 | .01600 | .99987 | .03345 | .99944 | .05088 | .99870 | .06831 | .99766 | .08571 | .99632 | 5 |
| 56 | .01629 | .99987 | .03374 | .99943 | .05117 | .99869 | .06860 | .99764 | .08600 | .99630 | 4 |
| 57 | .01658 | .99986 | .03403 | .99942 | .05146 | .99867 | .06889 | .99762 | .08629 | .99627 | 3 |
| 58 | .01687 | .99986 | .03432 | .99941 | .05175 | .99866 | .06918 | .99760 | .08658 | .99625 | 2 |
| 59 | .01716 | .99985 | .03461 | .99940 | .05205 | .99864 | .06947 | .99758 | .08687 | .99622 | 1 |
| 60 | .01745 | .99985 | .03490 | .99939 | .05234 | .99863 | .06976 | .99756 | .08716 | .99619 | 0 |
| ′ | Cosin | Sine | Cosin | Sine | Cosin | Sine | Cosin | Sine | Cosin | Sine | ′ |
| | 89° | | 88° | | 87° | | 86° | | 85° | | |

## TABLE II.—NATURAL SINES AND COSINES.

| ′ | 5° | | 6° | | 7° | | 8° | | 9° | | ′ |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | Sine | Cosin | Sine | Cosin | Sine | Cosin | Sine | Cosin | Sine | Cosin | |
| 0 | .08716 | .99619 | .10458 | .99452 | .12187 | .99255 | .13917 | .99027 | .15643 | .98769 | 60 |
| 1 | .08745 | .99617 | .10482 | .99449 | .12216 | .99251 | .13946 | .99023 | .15672 | .98764 | 59 |
| 2 | .08774 | .99614 | .10511 | .99446 | .12245 | .99248 | .13975 | .99019 | .15701 | .98760 | 58 |
| 3 | .08803 | .99612 | .10540 | .99443 | .12274 | .99244 | .14004 | .99015 | .15730 | .98755 | 57 |
| 4 | .08831 | .99609 | .10569 | .99440 | .12302 | .99240 | .14033 | .99011 | .15758 | .98751 | 56 |
| 5 | .08860 | .99607 | .10597 | .99437 | .12331 | .99237 | .14061 | .99006 | .15787 | .98746 | 55 |
| 6 | .08889 | .99604 | .10626 | .99434 | .12360 | .99233 | .14090 | .99002 | .15816 | .98741 | 54 |
| 7 | .08918 | .99602 | .10655 | .99431 | .12389 | .99230 | .14119 | .98998 | .15845 | .98737 | 53 |
| 8 | .08947 | .99599 | .10684 | .99428 | .12418 | .99226 | .14148 | .98994 | .15873 | .98732 | 52 |
| 9 | .08976 | .99596 | .10713 | .99424 | .12447 | .99222 | .14177 | .98990 | .15902 | .98728 | 51 |
| 10 | .09005 | .99594 | .10742 | .99421 | .12476 | .99219 | .14205 | .98986 | .15931 | .98723 | 50 |
| 11 | .09034 | .99591 | .10771 | .99418 | .12504 | .99215 | .14234 | .98982 | .15959 | .98718 | 49 |
| 12 | .09063 | .99588 | .10800 | .99415 | .12533 | .99211 | .14263 | .98978 | .15988 | .98714 | 48 |
| 13 | .09092 | .99586 | .10829 | .99412 | .12562 | .99208 | .14292 | .98973 | .16017 | .98709 | 47 |
| 14 | .09121 | .99583 | .10858 | .99409 | .12591 | .99204 | .14320 | .98969 | .16046 | .98704 | 46 |
| 15 | .09150 | .99580 | .10887 | .99406 | .12620 | .99200 | .14349 | .98965 | .16074 | .98700 | 45 |
| 16 | .09179 | .99578 | .10916 | .99402 | .12649 | .99197 | .14378 | .98961 | .16103 | .98695 | 44 |
| 17 | .09208 | .99575 | .10945 | .99399 | .12678 | .99193 | .14407 | .98957 | .16132 | .98690 | 43 |
| 18 | .09237 | .99572 | .10973 | .99396 | .12706 | .99189 | .14436 | .98953 | .16160 | .98686 | 42 |
| 19 | .09266 | .99570 | .11002 | .99393 | .12735 | .99186 | .14464 | .98948 | .16189 | .98681 | 41 |
| 20 | .09295 | .99567 | .11031 | .99390 | .12764 | .99182 | .14493 | .98944 | .16218 | .98676 | 40 |
| 21 | .09324 | .99564 | .11060 | .99386 | .12793 | .99178 | .14522 | .98940 | .16246 | .98671 | 39 |
| 22 | .09353 | .99562 | .11089 | .99383 | .12822 | .99175 | .14551 | .98936 | .16275 | .98667 | 38 |
| 23 | .09382 | .99559 | .11118 | .99380 | .12851 | .99171 | .14580 | .98931 | .16304 | .98662 | 37 |
| 24 | .09411 | .99556 | .11147 | .99377 | .12880 | .99167 | .14608 | .98927 | .16333 | .98657 | 36 |
| 25 | .09440 | .99553 | .11176 | .99374 | .12908 | .99163 | .14637 | .98923 | .16361 | .98652 | 35 |
| 26 | .09469 | .99551 | .11205 | .99370 | .12937 | .99160 | .14666 | .98919 | .16390 | .98648 | 34 |
| 27 | .09498 | .99548 | .11234 | .99367 | .12966 | .99156 | .14695 | .98914 | .16419 | .98643 | 33 |
| 28 | .09527 | .99545 | .11263 | .99364 | .12995 | .99152 | .14723 | .98910 | .16447 | .98638 | 32 |
| 29 | .09556 | .99542 | .11291 | .99360 | .13024 | .99148 | .14752 | .98906 | .16476 | .98633 | 31 |
| 30 | .09585 | .99540 | .11320 | .99357 | .13053 | .99144 | .14781 | .98902 | .16505 | .98629 | 30 |
| 31 | .09614 | .99537 | .11349 | .99354 | .13081 | .99141 | .14810 | .98897 | .16533 | .98624 | 29 |
| 32 | .09642 | .99534 | .11378 | .99351 | .13110 | .99137 | .14838 | .98893 | .16562 | .98619 | 28 |
| 33 | .09671 | .99531 | .11407 | .99347 | .13139 | .99133 | .14867 | .98889 | .16591 | .98614 | 27 |
| 34 | .09700 | .99528 | .11436 | .99344 | .13168 | .99129 | .14896 | .98884 | .16620 | .98609 | 26 |
| 35 | .09729 | .99526 | .11465 | .99341 | .13197 | .99125 | .14925 | .98880 | .16648 | .98604 | 25 |
| 36 | .09758 | .99523 | .11494 | .99337 | .13226 | .99122 | .14954 | .98876 | .16677 | .98600 | 24 |
| 37 | .09787 | .99520 | .11523 | .99334 | .13254 | .99118 | .14982 | .98871 | .16706 | .98595 | 23 |
| 38 | .09816 | .99517 | .11552 | .99331 | .13283 | .99114 | .15011 | .98867 | .16734 | .98590 | 22 |
| 39 | .09845 | .99514 | .11580 | .99327 | .13312 | .99110 | .15040 | .98863 | .16763 | .98585 | 21 |
| 40 | .09874 | .99511 | .11609 | .99324 | .13341 | .99106 | .15069 | .98858 | .16792 | .98580 | 20 |
| 41 | .09903 | .99508 | .11638 | .99320 | .13370 | .99102 | .15097 | .98854 | .16820 | .98575 | 19 |
| 42 | .09932 | .99506 | .11667 | .99317 | .13399 | .99098 | .15126 | .98849 | .16849 | .98570 | 18 |
| 43 | .09961 | .99503 | .11696 | .99314 | .13427 | .99094 | .15155 | .98845 | .16878 | .98565 | 17 |
| 44 | .09990 | .99500 | .11725 | .99310 | .13456 | .99091 | .15184 | .98841 | .16906 | .98561 | 16 |
| 45 | .10019 | .99497 | .11754 | .99307 | .13485 | .99087 | .15212 | .98836 | .16935 | .98556 | 15 |
| 46 | .10048 | .99494 | .11783 | .99303 | .13514 | .99083 | .15241 | .98832 | .16964 | .98551 | 14 |
| 47 | .10077 | .99491 | .11812 | .99300 | .13543 | .99079 | .15270 | .98827 | .16992 | .98546 | 13 |
| 48 | .10106 | .99488 | .11840 | .99297 | .13572 | .99075 | .15299 | .98823 | .17021 | .98541 | 12 |
| 49 | .10135 | .99485 | .11869 | .99293 | .13600 | .99071 | .15327 | .98818 | .17050 | .98536 | 11 |
| 50 | .10164 | .99482 | .11898 | .99290 | .13629 | .99067 | .15356 | .98814 | .17078 | .98531 | 10 |
| 51 | .10192 | .99479 | .11927 | .99286 | .13658 | .99063 | .15385 | .98809 | .17107 | .98526 | 9 |
| 52 | .10221 | .99476 | .11956 | .99283 | .13687 | .99059 | .15414 | .98805 | .17136 | .98521 | 8 |
| 53 | .10250 | .99473 | .11985 | .99279 | .13716 | .99055 | .15442 | .98800 | .17164 | .98516 | 7 |
| 54 | .10279 | .99470 | .12014 | .99276 | .13744 | .99051 | .15471 | .98796 | .17193 | .98511 | 6 |
| 55 | .10308 | .99467 | .12043 | .99272 | .13773 | .99047 | .15500 | .98791 | .17222 | .98506 | 5 |
| 56 | .10337 | .99464 | .12071 | .99269 | .13802 | .99043 | .15529 | .98787 | .17250 | .98501 | 4 |
| 57 | .10366 | .99461 | .12100 | .99265 | .13831 | .99039 | .15557 | .98782 | .17279 | .98496 | 3 |
| 58 | .10395 | .99458 | .12129 | .99262 | .13860 | .99035 | .15586 | .98778 | .17308 | .98491 | 2 |
| 59 | .10424 | .99455 | .12158 | .99258 | .13889 | .99031 | .15615 | .98773 | .17336 | .98486 | 1 |
| 60 | .10453 | .99452 | .12187 | .99255 | .13917 | .99027 | .15643 | .98769 | .17365 | .98481 | 0 |
| ′ | Cosin | Sine | Cosin | Sine | Cosin | Sine | Cosin | Sine | Cosin | Sine | ′ |
| | 84° | | 83° | | 82° | | 81° | | 80° | | |

# Table II.—Natural Sines and Cosines.

| ′ | 10° | | 11° | | 12° | | 13° | | 14° | | ′ |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | Sine | Cosin | Sine | Cosin | Sine | Cosin | Sine | Cosin | Sine | Cosin | |
| 0 | .17365 | .98481 | .19081 | .98163 | .20791 | .97815 | .22495 | .97437 | .24192 | .97030 | 60 |
| 1 | .17393 | .98476 | .19109 | .98157 | .20820 | .97809 | .22523 | .97430 | .24220 | .97023 | 59 |
| 2 | .17422 | .98471 | .19138 | .98152 | .20848 | .97803 | .22552 | .97424 | .24249 | .97015 | 58 |
| 3 | .17451 | .98466 | .19167 | .98146 | .20877 | .97797 | .22580 | .97417 | .24277 | .97008 | 57 |
| 4 | .17479 | .98461 | .19195 | .98140 | .20905 | .97791 | .22608 | .97411 | .24305 | .97001 | 56 |
| 5 | .17508 | .98455 | .19224 | .98135 | .20933 | .97784 | .22637 | .97404 | .24333 | .96994 | 55 |
| 6 | .17537 | .98450 | .19252 | .98129 | .20962 | .97778 | .22665 | .97398 | .24362 | .96987 | 54 |
| 7 | .17565 | .98445 | .19281 | .98124 | .20990 | .97772 | .22693 | .97391 | .24390 | .96980 | 53 |
| 8 | .17594 | .98440 | .19309 | .98118 | .21019 | .97766 | .22722 | .97384 | .24418 | .96973 | 52 |
| 9 | .17623 | .98435 | .19338 | .98112 | .21047 | .97760 | .22750 | .97378 | .24446 | .96966 | 51 |
| 10 | .17651 | .98430 | .19366 | .98107 | .21076 | .97754 | .22778 | .97371 | .24474 | .96959 | 50 |
| 11 | .17680 | .98425 | .19395 | .98101 | .21104 | .97748 | .22807 | .97365 | .24503 | .96952 | 49 |
| 12 | .17708 | .98420 | .19423 | .98096 | .21132 | .97742 | .22835 | .97358 | .24531 | .96945 | 48 |
| 13 | .17737 | .98414 | .19452 | .98090 | .21161 | .97735 | .22863 | .97351 | .24559 | .96937 | 47 |
| 14 | .17766 | .98409 | .19481 | .98084 | .21189 | .97729 | .22892 | .97345 | .24587 | .96930 | 46 |
| 15 | .17794 | .98404 | .19509 | .98079 | .21218 | .97723 | .22920 | .97338 | .24615 | .96923 | 45 |
| 16 | .17823 | .98399 | .19538 | .98073 | .21246 | .97717 | .22948 | .97331 | .24644 | .96916 | 44 |
| 17 | .17852 | .98394 | .19566 | .98067 | .21275 | .97711 | .22977 | .97325 | .24672 | .96909 | 43 |
| 18 | .17880 | .98389 | .19595 | .98061 | .21303 | .97705 | .23005 | .97318 | .24700 | .96902 | 42 |
| 19 | .17909 | .98383 | .19623 | .98056 | .21331 | .97698 | .23033 | .97311 | .24728 | .96894 | 41 |
| 20 | .17937 | .98378 | .19652 | .98050 | .21360 | .97692 | .23062 | .97304 | .24756 | .96887 | 40 |
| 21 | .17966 | .98373 | .19680 | .98044 | .21388 | .97686 | .23090 | .97298 | .24784 | .96880 | 39 |
| 22 | .17995 | .98368 | .19709 | .98039 | .21417 | .97680 | .23118 | .97291 | .24813 | .96873 | 38 |
| 23 | .18023 | .98362 | .19737 | .98033 | .21445 | .97673 | .23146 | .97284 | .24841 | .96866 | 37 |
| 24 | .18052 | .98357 | .19766 | .98027 | .21474 | .97667 | .23175 | .97278 | .24869 | .96858 | 36 |
| 25 | .18081 | .98352 | .19794 | .98021 | .21502 | .97661 | .23203 | .97271 | .24897 | .96851 | 35 |
| 26 | .18109 | .98347 | .19823 | .98016 | .21530 | .97655 | .23231 | .97264 | .24925 | .96844 | 34 |
| 27 | .18138 | .98341 | .19851 | .98010 | .21559 | .97648 | .23260 | .97257 | .24954 | .96837 | 33 |
| 28 | .18166 | .98336 | .19880 | .98004 | .21587 | .97642 | .23288 | .97251 | .24982 | .96829 | 32 |
| 29 | .18195 | .98331 | .19908 | .97998 | .21616 | .97636 | .23316 | .97244 | .25010 | .96822 | 31 |
| 30 | .18224 | .98325 | .19937 | .97992 | .21644 | .97630 | .23345 | .97237 | .25038 | .96815 | 30 |
| 31 | .18252 | .98320 | .19965 | .97987 | .21672 | .97623 | .23373 | .97230 | .25066 | .96807 | 29 |
| 32 | .18281 | .98315 | .19994 | .97981 | .21701 | .97617 | .23401 | .97223 | .25094 | .96800 | 28 |
| 33 | .18309 | .98310 | .20022 | .97975 | .21729 | .97611 | .23429 | .97217 | .25122 | .96793 | 27 |
| 34 | .18338 | .98304 | .20051 | .97969 | .21758 | .97604 | .23458 | .97210 | .25151 | .96786 | 26 |
| 35 | .18367 | .98299 | .20079 | .97963 | .21786 | .97598 | .23486 | .97203 | .25179 | .96778 | 25 |
| 36 | .18395 | .98294 | .20108 | .97958 | .21814 | .97592 | .23514 | .97196 | .25207 | .96771 | 24 |
| 37 | .18424 | .98288 | .20136 | .97952 | .21843 | .97585 | .23542 | .97189 | .25235 | .96764 | 23 |
| 38 | .18452 | .98283 | .20165 | .97946 | .21871 | .97579 | .23571 | .97182 | .25263 | .96756 | 22 |
| 39 | .18481 | .98277 | .20193 | .97940 | .21899 | .97573 | .23599 | .97176 | .25291 | .96749 | 21 |
| 40 | .18509 | .98272 | .20222 | .97934 | .21928 | .97566 | .23627 | .97169 | .25320 | .96742 | 20 |
| 41 | .18538 | .98267 | .20250 | .97928 | .21956 | .97560 | .23656 | .97162 | .25348 | .96734 | 19 |
| 42 | .18567 | .98261 | .20279 | .97922 | .21985 | .97553 | .23684 | .97155 | .25376 | .96727 | 18 |
| 43 | .18595 | .98256 | .20307 | .97916 | .22013 | .97547 | .23712 | .97148 | .25404 | .96719 | 17 |
| 44 | .18624 | .98250 | .20336 | .97910 | .22041 | .97541 | .23740 | .97141 | .25432 | .96712 | 16 |
| 45 | .18652 | .98245 | .20364 | .97905 | .22070 | .97534 | .23769 | .97134 | .25460 | .96705 | 15 |
| 46 | .18681 | .98240 | .20393 | .97899 | .22098 | .97528 | .23797 | .97127 | .25488 | .96697 | 14 |
| 47 | .18710 | .98234 | .20421 | .97893 | .22126 | .97521 | .23825 | .97120 | .25516 | .96690 | 13 |
| 48 | .18738 | .98229 | .20450 | .97887 | .22155 | .97515 | .23853 | .97113 | .25545 | .96682 | 12 |
| 49 | .18767 | .98223 | .20478 | .97881 | .22183 | .97508 | .23882 | .97106 | .25573 | .96675 | 11 |
| 50 | .18795 | .98218 | .20507 | .97875 | .22212 | .97502 | .23910 | .97100 | .25601 | .96667 | 10 |
| 51 | .18824 | .98212 | .20535 | .97869 | .22240 | .97496 | .23938 | .97093 | .25629 | .96660 | 9 |
| 52 | .18852 | .98207 | .20563 | .97863 | .22268 | .97489 | .23966 | .97086 | .25657 | .96653 | 8 |
| 53 | .18881 | .98201 | .20592 | .97857 | .22297 | .97483 | .23995 | .97079 | .25685 | .96645 | 7 |
| 54 | .18910 | .98196 | .20620 | .97851 | .22325 | .97476 | .24023 | .97072 | .25713 | .96638 | 6 |
| 55 | .18938 | .98190 | .20649 | .97845 | .22353 | .97470 | .24051 | .97065 | .25741 | .96630 | 5 |
| 56 | .18967 | .98185 | .20677 | .97839 | .22382 | .97463 | .24079 | .97058 | .25769 | .96623 | 4 |
| 57 | .18995 | .98179 | .20706 | .97833 | .22410 | .97457 | .24108 | .97051 | .25798 | .96615 | 3 |
| 58 | .19024 | .98174 | .20734 | .97827 | .22438 | .97450 | .24136 | .97044 | .25826 | .96608 | 2 |
| 59 | .19052 | .98168 | .20763 | .97821 | .22467 | .97444 | .24164 | .97037 | .25854 | .96600 | 1 |
| 60 | .19081 | .98163 | .20791 | .97815 | .22495 | .97437 | .24192 | .97030 | .25882 | .96593 | 0 |
| ′ | Cosin | Sine | Cosin | Sine | Cosin | Sine | Cosin | Sine | Cosin | Sine | ′ |
| | 79° | | 78° | | 77° | | 76° | | 75° | | |

# Table II.—Natural Sines and Cosines.

| ′ | 15° | | 16° | | 17° | | 18° | | 19° | | ′ |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | Sine | Cosin | Sine | Cosin | Sine | Cosin | Sine | Cosin | Sine | Cosin | |
| 0 | .25882 | .96593 | .27564 | .96126 | .29237 | .95630 | .30902 | .95106 | .32557 | .94552 | 60 |
| 1 | .25910 | .96585 | .27592 | .96118 | .29265 | .95622 | .30929 | .95097 | .32584 | .94542 | 59 |
| 2 | .25938 | .96578 | .27620 | .96110 | .29293 | .95613 | .30957 | .95088 | .32612 | .94533 | 58 |
| 3 | .25966 | .96570 | .27648 | .96102 | .29321 | .95605 | .30985 | .95079 | .32639 | .94523 | 57 |
| 4 | .25994 | .96562 | .27676 | .96094 | .29348 | .95596 | .31012 | .95070 | .32667 | .94514 | 56 |
| 5 | .26022 | .96555 | .27704 | .96086 | .29376 | .95588 | .31040 | .95061 | .32694 | .94504 | 55 |
| 6 | .26050 | .96547 | .27731 | .96078 | .29404 | .95579 | .31068 | .95052 | .32722 | .94495 | 54 |
| 7 | .26079 | .96540 | .27759 | .96070 | .29432 | .95571 | .31095 | .95043 | .32749 | .94485 | 53 |
| 8 | .26107 | .96532 | .27787 | .96062 | .29460 | .95562 | .31123 | .95033 | .32777 | .94476 | 52 |
| 9 | .26135 | .96524 | .27815 | .96054 | .29487 | .95554 | .31151 | .95024 | .32804 | .94466 | 51 |
| 10 | .26163 | .96517 | .27843 | .96046 | .29515 | .95545 | .31178 | .95015 | .32832 | .94457 | 50 |
| 11 | .26191 | .96509 | .27871 | .96037 | .29543 | .95536 | .31206 | .95006 | .32859 | .94447 | 49 |
| 12 | .26219 | .96502 | .27899 | .96029 | .29571 | .95528 | .31233 | .94997 | .32887 | .94438 | 48 |
| 13 | .26247 | .96494 | .27927 | .96021 | .29599 | .95519 | .31261 | .94988 | .32914 | .94428 | 47 |
| 14 | .26275 | .96486 | .27955 | .96013 | .29626 | .95511 | .31289 | .94979 | .32942 | .94418 | 46 |
| 15 | .26303 | .96479 | .27983 | .96005 | .29654 | .95502 | .31316 | .94970 | .32969 | .94409 | 45 |
| 16 | .26331 | .96471 | .28011 | .95997 | .29682 | .95493 | .31344 | .94961 | .32997 | .94399 | 44 |
| 17 | .26359 | .96463 | .28039 | .95989 | .29710 | .95485 | .31372 | .94952 | .33024 | .94390 | 43 |
| 18 | .26387 | .96456 | .28067 | .95981 | .29737 | .95476 | .31399 | .94943 | .33051 | .94380 | 42 |
| 19 | .26415 | .96448 | .28095 | .95972 | .29765 | .95467 | .31427 | .94933 | .33079 | .94370 | 41 |
| 20 | .26443 | .96440 | .28123 | .95964 | .29793 | .95459 | .31454 | .94924 | .33106 | .94361 | 40 |
| 21 | .26471 | .96433 | .28150 | .95956 | .29821 | .95450 | .31482 | .94915 | .33134 | .94351 | 39 |
| 22 | .26500 | .96425 | .28178 | .95948 | .29849 | .95441 | .31510 | .94906 | .33161 | .94342 | 38 |
| 23 | .26528 | .96417 | .28206 | .95940 | .29876 | .95433 | .31537 | .94897 | .33189 | .94332 | 37 |
| 24 | .26556 | .96410 | .28234 | .95931 | .29904 | .95424 | .31565 | .94888 | .33216 | .94322 | 36 |
| 25 | .26584 | .96402 | .28262 | .95923 | .29932 | .95415 | .31593 | .94878 | .33244 | .94313 | 35 |
| 26 | .26612 | .96394 | .28290 | .95915 | .29960 | .95407 | .31620 | .94869 | .33271 | .94303 | 34 |
| 27 | .26640 | .96386 | .28318 | .95907 | .29987 | .95398 | .31648 | .94860 | .33298 | .94293 | 33 |
| 28 | .26668 | .96379 | .28346 | .95898 | .30015 | .95389 | .31675 | .94851 | .33326 | .94284 | 32 |
| 29 | .26696 | .96371 | .28374 | .95890 | .30043 | .95380 | .31703 | .94842 | .33353 | .94274 | 31 |
| 30 | .26724 | .96363 | .28402 | .95882 | .30071 | .95372 | .31730 | .94832 | .33381 | .94264 | 30 |
| 31 | .26752 | .96355 | .28429 | .95874 | .30098 | .95363 | .31758 | .94823 | .33408 | .94254 | 29 |
| 32 | .26780 | .96347 | .28457 | .95865 | .30126 | .95354 | .31786 | .94814 | .33436 | .94245 | 28 |
| 33 | .26808 | .96340 | .28485 | .95857 | .30154 | .95345 | .31813 | .94805 | .33463 | .94235 | 27 |
| 34 | .26836 | .96332 | .28513 | .95849 | .30182 | .95337 | .31841 | .94795 | .33490 | .94225 | 26 |
| 35 | .26864 | .96324 | .28541 | .95841 | .30209 | .95328 | .31868 | .94786 | .33518 | .94215 | 25 |
| 36 | .26892 | .96316 | .28569 | .95832 | .30237 | .95319 | .31896 | .94777 | .33545 | .94206 | 24 |
| 37 | .26920 | .96308 | .28597 | .95824 | .30265 | .95310 | .31923 | .94768 | .33573 | .94196 | 23 |
| 38 | .26948 | .96301 | .28625 | .95816 | .30293 | .95301 | .31951 | .94758 | .33600 | .94186 | 22 |
| 39 | .26976 | .96293 | .28652 | .95807 | .30320 | .95293 | .31979 | .94749 | .33627 | .94176 | 21 |
| 40 | .27004 | .96285 | .28680 | .95799 | .30348 | .95284 | .32006 | .94740 | .33655 | .94167 | 20 |
| 41 | .27032 | .96277 | .28708 | .95791 | .30376 | .95275 | .32034 | .94730 | .33682 | .94157 | 19 |
| 42 | .27060 | .96269 | .28736 | .95782 | .30403 | .95266 | .32061 | .94721 | .33710 | .94147 | 18 |
| 43 | .27088 | .96261 | .28764 | .95774 | .30431 | .95257 | .32089 | .94712 | .33737 | .94137 | 17 |
| 44 | .27116 | .96253 | .28792 | .95766 | .30459 | .95248 | .32116 | .94702 | .33764 | .94127 | 16 |
| 45 | .27144 | .96246 | .28820 | .95757 | .30486 | .95240 | .32144 | .94693 | .33792 | .94118 | 15 |
| 46 | .27172 | .96238 | .28847 | .95749 | .30514 | .95231 | .32171 | .94684 | .33819 | .94108 | 14 |
| 47 | .27200 | .96230 | .28875 | .95740 | .30542 | .95222 | .32199 | .94674 | .33846 | .94098 | 13 |
| 48 | .27228 | .96222 | .28903 | .95732 | .30570 | .95213 | .32227 | .94665 | .33874 | .94088 | 12 |
| 49 | .27256 | .96214 | .28931 | .95724 | .30597 | .95204 | .32254 | .94656 | .33901 | .94078 | 11 |
| 50 | .27284 | .96206 | .28959 | .95715 | .30625 | .95195 | .32282 | .94646 | .33929 | .94068 | 10 |
| 51 | .27312 | .96198 | .28987 | .95707 | .30653 | .95186 | .32309 | .94637 | .33956 | .94058 | 9 |
| 52 | .27340 | .96190 | .29015 | .95698 | .30680 | .95177 | .32337 | .94627 | .33983 | .94049 | 8 |
| 53 | .27368 | .96182 | .29042 | .95690 | .30708 | .95168 | .32364 | .94618 | .34011 | .94039 | 7 |
| 54 | .27396 | .96174 | .29070 | .95681 | .30736 | .95159 | .32392 | .94609 | .34038 | .94029 | 6 |
| 55 | .27424 | .96166 | .29098 | .95673 | .30763 | .95150 | .32419 | .94599 | .34065 | .94019 | 5 |
| 56 | .27452 | .96158 | .29126 | .95664 | .30791 | .95142 | .32447 | .94590 | .34093 | .94009 | 4 |
| 57 | .27480 | .96150 | .29154 | .95656 | .30819 | .95133 | .32474 | .94580 | .34120 | .93999 | 3 |
| 58 | .27508 | .96142 | .29182 | .95647 | .30846 | .95124 | .32502 | .94571 | .34147 | .93989 | 2 |
| 59 | .27536 | .96134 | .29209 | .95639 | .30874 | .95115 | .32529 | .94561 | .34175 | .93979 | 1 |
| 60 | .27564 | .96126 | .29237 | .95630 | .30902 | .95106 | .32557 | .94552 | .34202 | .93969 | 0 |
| ′ | Cosin | Sine | Cosin | Sine | Cosin | Sine | Cosin | Sine | Cosin | Sine | ′ |
| | 74° | | 73° | | 72° | | 71° | | 70° | | |

## TABLE II.—NATURAL SINES AND COSINES.

| ′ | 20° | | 21° | | 22° | | 23° | | 24° | | ′ |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | Sine | Cosin | Sine | Cosin | Sine | Cosin | Sine | Cosin | Sine | Cosin | |
| 0 | .34202 | .93969 | .35837 | .93358 | .37461 | .92718 | .39073 | .92050 | .40674 | .91355 | 60 |
| 1 | .34229 | .93959 | .35864 | .93348 | .37488 | .92707 | .39100 | .92039 | .40700 | .91343 | 59 |
| 2 | .34257 | .93949 | .35891 | .93337 | .37515 | .92697 | .39127 | .92028 | .40727 | .91331 | 58 |
| 3 | .34284 | .93939 | .35918 | .93327 | .37542 | .92686 | .39153 | .92016 | .40753 | .91319 | 57 |
| 4 | .34311 | .93929 | .35945 | .93316 | .37569 | .92675 | .39180 | .92005 | .40780 | .91307 | 56 |
| 5 | .34339 | .93919 | .35973 | .93306 | .37596 | .92664 | .39207 | .91994 | .40806 | .91295 | 55 |
| 6 | .34366 | .93909 | .36000 | .93295 | .37622 | .92653 | .39234 | .91982 | .40833 | .91283 | 54 |
| 7 | .34393 | .93899 | .36027 | .93285 | .37649 | .92642 | .39260 | .91971 | .40860 | .91272 | 53 |
| 8 | .34421 | .93889 | .36054 | .93274 | .37676 | .92631 | .39287 | .91959 | .40886 | .91260 | 52 |
| 9 | .34448 | .93879 | .36081 | .93264 | .37703 | .92620 | .39314 | .91948 | .40913 | .91248 | 51 |
| 10 | .34475 | .93869 | .36108 | .93253 | .37730 | .92609 | .39341 | .91936 | .40939 | .91236 | 50 |
| 11 | .34503 | .93859 | .36135 | .93243 | .37757 | .92598 | .39367 | .91925 | .40966 | .91224 | 49 |
| 12 | .34530 | .93849 | .36162 | .93232 | .37784 | .92587 | .39394 | .91914 | .40992 | .91212 | 48 |
| 13 | .34557 | .93839 | .36190 | .93222 | .37811 | .92576 | .39421 | .91902 | .41019 | .91200 | 47 |
| 14 | .34584 | .93829 | .36217 | .93211 | .37838 | .92565 | .39448 | .91891 | .41045 | .91188 | 46 |
| 15 | .34612 | .93819 | .36244 | .93201 | .37865 | .92554 | .39474 | .91879 | .41072 | .91176 | 45 |
| 16 | .34639 | .93809 | .36271 | .93190 | .37892 | .92543 | .39501 | .91868 | .41098 | .91164 | 44 |
| 17 | .34666 | .93799 | .36298 | .93180 | .37919 | .92532 | .39528 | .91856 | .41125 | .91152 | 43 |
| 18 | .34694 | .93789 | .36325 | .93169 | .37946 | .92521 | .39555 | .91845 | .41151 | .91140 | 42 |
| 19 | .34721 | .93779 | .36352 | .93159 | .37973 | .92510 | .39581 | .91833 | .41178 | .91128 | 41 |
| 20 | .34748 | .93769 | .36379 | .93148 | .37999 | .92499 | .39608 | .91822 | .41204 | .91116 | 40 |
| 21 | .34775 | .93759 | .36406 | .93137 | .38026 | .92488 | .39635 | .91810 | .41231 | .91104 | 39 |
| 22 | .34803 | .93748 | .36434 | .93127 | .38053 | .92477 | .39661 | .91799 | .41257 | .91092 | 38 |
| 23 | .34830 | .93738 | .36461 | .93116 | .38080 | .92466 | .39688 | .91787 | .41284 | .91080 | 37 |
| 24 | .34857 | .93728 | .36488 | .93106 | .38107 | .92455 | .39715 | .91775 | .41310 | .91068 | 36 |
| 25 | .34884 | .93718 | .36515 | .93095 | .38134 | .92444 | .39741 | .91764 | .41337 | .91056 | 35 |
| 26 | .34912 | .93708 | .36542 | .93084 | .38161 | .92432 | .39768 | .91752 | .41363 | .91044 | 34 |
| 27 | .34939 | .93698 | .36569 | .93074 | .38188 | .92421 | .39795 | .91741 | .41390 | .91032 | 33 |
| 28 | .34966 | .93688 | .36596 | .93063 | .38215 | .92410 | .39822 | .91729 | .41416 | .91020 | 32 |
| 29 | .34993 | .93677 | .36623 | .93052 | .38241 | .92399 | .39848 | .91718 | .41443 | .91008 | 31 |
| 30 | .35021 | .93667 | .36650 | .93042 | .38268 | .92388 | .39875 | .91706 | .41469 | .90996 | 30 |
| 31 | .35048 | .93657 | .36677 | .93031 | .38295 | .92377 | .39902 | .91694 | .41496 | .90984 | 29 |
| 32 | .35075 | .93647 | .36704 | .93020 | .38322 | .92366 | .39928 | .91683 | .41522 | .90972 | 28 |
| 33 | .35102 | .93637 | .36731 | .93010 | .38349 | .92355 | .39955 | .91671 | .41549 | .90960 | 27 |
| 34 | .35130 | .93626 | .36758 | .92999 | .38376 | .92343 | .39982 | .91660 | .41575 | .90948 | 26 |
| 35 | .35157 | .93616 | .36785 | .92988 | .38403 | .92332 | .40008 | .91648 | .41602 | .90936 | 25 |
| 36 | .35184 | .93606 | .36812 | .92978 | .38430 | .92321 | .40035 | .91636 | .41628 | .90924 | 24 |
| 37 | .35211 | .93596 | .36839 | .92967 | .38456 | .92310 | .40062 | .91625 | .41655 | .90911 | 23 |
| 38 | .35239 | .93585 | .36867 | .92956 | .38483 | .92299 | .40088 | .91613 | .41681 | .90899 | 22 |
| 39 | .35266 | .93575 | .36894 | .92945 | .38510 | .92287 | .40115 | .91601 | .41707 | .90887 | 21 |
| 40 | .35293 | .93565 | .36921 | .92935 | .38537 | .92276 | .40141 | .91590 | .41734 | .90875 | 20 |
| 41 | .35320 | .93555 | .36948 | .92924 | .38564 | .92265 | .40168 | .91578 | .41760 | .90863 | 19 |
| 42 | .35347 | .93544 | .36975 | .92913 | .38591 | .92254 | .40195 | .91566 | .41787 | .90851 | 18 |
| 43 | .35375 | .93534 | .37002 | .92902 | .38617 | .92243 | .40221 | .91555 | .41813 | .90839 | 17 |
| 44 | .35402 | .93524 | .37029 | .92892 | .38644 | .92231 | .40248 | .91543 | .41840 | .90826 | 16 |
| 45 | .35429 | .93514 | .37056 | .92881 | .38671 | .92220 | .40275 | .91531 | .41866 | .90814 | 15 |
| 46 | .35456 | .93503 | .37083 | .92870 | .38698 | .92209 | .40301 | .91519 | .41892 | .90802 | 14 |
| 47 | .35484 | .93493 | .37110 | .92859 | .38725 | .92198 | .40328 | .91508 | .41919 | .90790 | 13 |
| 48 | .35511 | .93483 | .37137 | .92849 | .38752 | .92186 | .40355 | .91496 | .41945 | .90778 | 12 |
| 49 | .35538 | .93472 | .37164 | .92838 | .38778 | .92175 | .40381 | .91484 | .41972 | .90766 | 11 |
| 50 | .35565 | .93462 | .37191 | .92827 | .38805 | .92164 | .40408 | .91472 | .41998 | .90753 | 10 |
| 51 | .35592 | .93452 | .37218 | .92816 | .38832 | .92152 | .40434 | .91461 | .42024 | .90741 | 9 |
| 52 | .35619 | .93441 | .37245 | .92805 | .38859 | .92141 | .40461 | .91449 | .42051 | .90729 | 8 |
| 53 | .35647 | .93431 | .37272 | .92794 | .38886 | .92130 | .40488 | .91437 | .42077 | .90717 | 7 |
| 54 | .35674 | .93420 | .37299 | .92784 | .38912 | .92119 | .40514 | .91425 | .42104 | .90704 | 6 |
| 55 | .35701 | .93410 | .37326 | .92773 | .38939 | .92107 | .40541 | .91414 | .42130 | .90692 | 5 |
| 56 | .35728 | .93400 | .37353 | .92762 | .38966 | .92096 | .40567 | .91402 | .42156 | .90680 | 4 |
| 57 | .35755 | .93389 | .37380 | .92751 | .38993 | .92085 | .40594 | .91390 | .42183 | .90668 | 3 |
| 58 | .35782 | .93379 | .37407 | .92740 | .39020 | .92073 | .40621 | .91378 | .42209 | .90655 | 2 |
| 59 | .35810 | .93368 | .37434 | .92729 | .39046 | .92062 | .40647 | .91366 | .42235 | .90643 | 1 |
| 60 | .35837 | .93358 | .37461 | .92718 | .39073 | .92050 | .40674 | .91355 | .42262 | .90631 | 0 |
| ′ | Cosin | Sine | Cosin | Sine | Cosin | Sine | Cosin | Sine | Cosin | Sine | ′ |
| | 69° | | 68° | | 67° | | 66° | | 65° | | |

# Table II.—Natural Sines and Cosines.

| ′ | 25° | | 26° | | 27° | | 28° | | 29° | | ′ |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | Sine | Cosin | Sine | Cosin | Sine | Cosin | Sine | Cosin | Sine | Cosin | |
| 0 | .42262 | .90631 | .43837 | .89879 | .45399 | .89101 | .46947 | .88295 | .48481 | .87462 | 60 |
| 1 | .42288 | .90618 | .43863 | .89867 | .45425 | .89087 | .46973 | .88281 | .48506 | .87448 | 59 |
| 2 | .42315 | .90606 | .43889 | .89854 | .45451 | .89074 | .46999 | .88267 | .48532 | .87434 | 58 |
| 3 | .42341 | .90594 | .43916 | .89841 | .45477 | .89061 | .47024 | .88254 | .48557 | .87420 | 57 |
| 4 | .42367 | .90582 | .43942 | .89828 | .45503 | .89048 | .47050 | .88240 | .48583 | .87406 | 56 |
| 5 | .42394 | .90569 | .43968 | .89816 | .45529 | .89035 | .47076 | .88226 | .48608 | .87391 | 55 |
| 6 | .42420 | .90557 | .43994 | .89803 | .45554 | .89021 | .47101 | .88213 | .48634 | .87377 | 54 |
| 7 | .42446 | .90545 | .44020 | .89790 | .45580 | .89008 | .47127 | .88199 | .48659 | .87363 | 53 |
| 8 | .42473 | .90532 | .44046 | .89777 | .45606 | .88995 | .47153 | .88185 | .48684 | .87349 | 52 |
| 9 | .42499 | .90520 | .44072 | .89764 | .45632 | .88981 | .47178 | .88172 | .48710 | .87335 | 51 |
| 10 | .42525 | .90507 | .44098 | .89752 | .45658 | .88968 | .47204 | .88158 | .48735 | .87321 | 50 |
| 11 | .42552 | .90495 | .44124 | .89739 | .45684 | .88955 | .47229 | .88144 | .48761 | .87306 | 49 |
| 12 | .42578 | .90483 | .44151 | .89726 | .45710 | .88942 | .47255 | .88130 | .48786 | .87292 | 48 |
| 13 | .42604 | .90470 | .44177 | .89713 | .45736 | .88928 | .47281 | .88117 | .48811 | .87278 | 47 |
| 14 | .42631 | .90458 | .44203 | .89700 | .45762 | .88915 | .47306 | .88103 | .48837 | .87264 | 46 |
| 15 | .42657 | .90446 | .44229 | .89687 | .45787 | .88902 | .47332 | .88089 | .48862 | .87250 | 45 |
| 16 | .42683 | .90433 | .44255 | .89674 | .45813 | .88888 | .47358 | .88075 | .48888 | .87235 | 44 |
| 17 | .42709 | .90421 | .44281 | .89662 | .45839 | .88875 | .47383 | .88062 | .48913 | .87221 | 43 |
| 18 | .42736 | .90408 | .44307 | .89649 | .45865 | .88862 | .47409 | .88048 | .48938 | .87207 | 42 |
| 19 | .42762 | .90396 | .44333 | .89636 | .45891 | .88848 | .47434 | .88034 | .48964 | .87193 | 41 |
| 20 | .42788 | .90383 | .44359 | .89623 | .45917 | .88835 | .47460 | .88020 | .48989 | .87178 | 40 |
| 21 | .42815 | .90371 | .44385 | .89610 | .45942 | .88822 | .47486 | .88006 | .49014 | .87164 | 39 |
| 22 | .42841 | .90358 | .44411 | .89597 | .45968 | .88808 | .47511 | .87993 | .49040 | .87150 | 38 |
| 23 | .42867 | .90346 | .44437 | .89584 | .45994 | .88795 | .47537 | .87979 | .49065 | .87136 | 37 |
| 24 | .42894 | .90334 | .44464 | .89571 | .46020 | .88782 | .47562 | .87965 | .49090 | .87121 | 36 |
| 25 | .42920 | .90321 | .44490 | .89558 | .46046 | .88768 | .47588 | .87951 | .49116 | .87107 | 35 |
| 26 | .42946 | .90309 | .44516 | .89545 | .46072 | .88755 | .47614 | .87937 | .49141 | .87093 | 34 |
| 27 | .42972 | .90296 | .44542 | .89532 | .46097 | .88741 | .47639 | .87923 | .49166 | .87079 | 33 |
| 28 | .42999 | .90284 | .44568 | .89519 | .46123 | .88728 | .47665 | .87909 | .49192 | .87064 | 32 |
| 29 | .43025 | .90271 | .44594 | .89506 | .46149 | .88715 | .47690 | .87896 | .49217 | .87050 | 31 |
| 30 | .43051 | .90259 | .44620 | .89493 | .46175 | .88701 | .47716 | .87882 | .49242 | .87036 | 30 |
| 31 | .43077 | .90246 | .44646 | .89480 | .46201 | .88688 | .47741 | .87868 | .49268 | .87021 | 29 |
| 32 | .43104 | .90233 | .44672 | .89467 | .46226 | .88674 | .47767 | .87854 | .49293 | .87007 | 28 |
| 33 | .43130 | .90221 | .44698 | .89454 | .46252 | .88661 | .47793 | .87840 | .49318 | .86993 | 27 |
| 34 | .43156 | .90208 | .44724 | .89441 | .46278 | .88647 | .47818 | .87826 | .49344 | .86978 | 26 |
| 35 | .43182 | .90196 | .44750 | .89428 | .46304 | .88634 | .47844 | .87812 | .49369 | .86964 | 25 |
| 36 | .43209 | .90183 | .44776 | .89415 | .46330 | .88620 | .47869 | .87798 | .49394 | .86949 | 24 |
| 37 | .43235 | .90171 | .44802 | .89402 | .46355 | .88607 | .47895 | .87784 | .49419 | .86935 | 23 |
| 38 | .43261 | .90158 | .44828 | .89389 | .46381 | .88593 | .47920 | .87770 | .49445 | .86921 | 22 |
| 39 | .43287 | .90146 | .44854 | .89376 | .46407 | .88580 | .47946 | .87756 | .49470 | .86906 | 21 |
| 40 | .43313 | .90133 | .44880 | .89363 | .46433 | .88566 | .47971 | .87743 | .49495 | .86892 | 20 |
| 41 | .43340 | .90120 | .44906 | .89350 | .46458 | .88553 | .47997 | .87729 | .49521 | .86878 | 19 |
| 42 | .43366 | .90108 | .44932 | .89337 | .46484 | .88539 | .48022 | .87715 | .49546 | .86863 | 18 |
| 43 | .43392 | .90095 | .44958 | .89324 | .46510 | .88526 | .48048 | .87701 | .49571 | .86849 | 17 |
| 44 | .43418 | .90082 | .44984 | .89311 | .46536 | .88512 | .48073 | .87687 | .49596 | .86834 | 16 |
| 45 | .43445 | .90070 | .45010 | .89298 | .46561 | .88499 | .48099 | .87673 | .49622 | .86820 | 15 |
| 46 | .43471 | .90057 | .45036 | .89285 | .46587 | .88485 | .48124 | .87659 | .49647 | .86805 | 14 |
| 47 | .43497 | .90045 | .45062 | .89272 | .46613 | .88472 | .48150 | .87645 | .49672 | .86791 | 13 |
| 48 | .43523 | .90032 | .45088 | .89259 | .46639 | .88458 | .48175 | .87631 | .49697 | .86777 | 12 |
| 49 | .43549 | .90019 | .45114 | .89245 | .46664 | .88445 | .48201 | .87617 | .49723 | .86762 | 11 |
| 50 | .43575 | .90007 | .45140 | .89232 | .46690 | .88431 | .48226 | .87603 | .49748 | .86748 | 10 |
| 51 | .43602 | .89994 | .45166 | .89219 | .46716 | .88417 | .48252 | .87589 | .49773 | .86733 | 9 |
| 52 | .43628 | .89981 | .45192 | .89206 | .46742 | .88404 | .48277 | .87575 | .49798 | .86719 | 8 |
| 53 | .43654 | .89968 | .45218 | .89193 | .46767 | .88390 | .48303 | .87561 | .49824 | .86704 | 7 |
| 54 | .43680 | .89956 | .45243 | .89180 | .46793 | .88377 | .48328 | .87546 | .49849 | .86690 | 6 |
| 55 | .43706 | .89943 | .45269 | .89167 | .46819 | .88363 | .48354 | .87532 | .49874 | .86675 | 5 |
| 56 | .43733 | .89930 | .45295 | .89153 | .46844 | .88349 | .48379 | .87518 | .49899 | .86661 | 4 |
| 57 | .43759 | .89918 | .45321 | .89140 | .46870 | .88336 | .48405 | .87504 | .49924 | .86646 | 3 |
| 58 | .43785 | .89905 | .45347 | .89127 | .46896 | .88322 | .48430 | .87490 | .49950 | .86632 | 2 |
| 59 | .43811 | .89892 | .45373 | .89114 | .46921 | .88308 | .48456 | .87476 | .49975 | .86617 | 1 |
| 60 | .43837 | .89879 | .45399 | .89101 | .46947 | .88295 | .48481 | .87462 | .50000 | .86603 | 0 |
| ′ | Cosin | Sine | Cosin | Sine | Cosin | Sine | Cosin | Sine | Cosin | Sine | ′ |
| | 64° | | 63° | | 62° | | 61° | | 60° | | |

## Table II.—Natural Sines and Cosines.

| ′ | 30° | | 31° | | 32° | | 33° | | 34° | | ′ |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | Sine | Cosin | Sine | Cosin | Sine | Cosin | Sine | Cosin | Sine | Cosin | |
| 0 | .50000 | .86603 | .51504 | .85717 | .52992 | .84805 | .54464 | .83867 | .55919 | .82904 | 60 |
| 1 | .50025 | .86588 | .51529 | .85702 | .53017 | .84789 | .54488 | .83851 | .55943 | .82887 | 59 |
| 2 | .50050 | .86573 | .51554 | .85687 | .53041 | .84774 | .54513 | .83835 | .55968 | .82871 | 58 |
| 3 | .50076 | .86559 | .51579 | .85672 | .53066 | .84759 | .54537 | .83819 | .55992 | .82855 | 57 |
| 4 | .50101 | .86544 | .51604 | .85657 | .53091 | .84743 | .54561 | .83804 | .56016 | .82839 | 56 |
| 5 | .50126 | .86530 | .51628 | .85642 | .53115 | .84728 | .54586 | .83788 | .56040 | .82822 | 55 |
| 6 | .50151 | .86515 | .51653 | .85627 | .53140 | .84712 | .54610 | .83772 | .56064 | .82806 | 54 |
| 7 | .50176 | .86501 | .51678 | .85612 | .53164 | .84697 | .54635 | .83756 | .56088 | .82790 | 53 |
| 8 | .50201 | .86486 | .51703 | .85597 | .53189 | .84681 | .54659 | .83740 | .56112 | .82773 | 52 |
| 9 | .50227 | .86471 | .51728 | .85582 | .53214 | .84666 | .54683 | .83724 | .56136 | .82757 | 51 |
| 10 | .50252 | .86457 | .51753 | .85567 | .53238 | .84650 | .54708 | .837.8 | .56160 | .82741 | 50 |
| 11 | .50277 | .86442 | .51778 | .85551 | .53263 | .84635 | .54732 | .83692 | .56184 | .82724 | 49 |
| 12 | .50302 | .86427 | .51803 | .85536 | .53288 | .84619 | .54756 | .83676 | .56208 | .82708 | 48 |
| 13 | .50327 | .86413 | .51828 | .85521 | .53312 | .84604 | .54781 | .83660 | .56232 | .82692 | 47 |
| 14 | .50352 | .86398 | .51852 | .85506 | .53337 | .84588 | .54805 | .83645 | .56256 | .82675 | 46 |
| 15 | .50377 | .86384 | .51877 | .85491 | .53361 | .84573 | .54829 | .83629 | .56280 | .82659 | 45 |
| 16 | .50403 | .86369 | .51902 | .85476 | .53386 | .84557 | .54854 | .83613 | .56305 | .82643 | 44 |
| 17 | .50428 | .86354 | .51927 | .85461 | .53411 | .84542 | .54878 | .83597 | .56329 | .82626 | 43 |
| 18 | .50453 | .86340 | .51952 | .85446 | .53435 | .84526 | .54902 | .83581 | .56353 | .82610 | 42 |
| 19 | .50478 | .86325 | .51977 | .85431 | .53460 | .84511 | .54927 | .83565 | .56377 | .82593 | 41 |
| 20 | .50503 | .86310 | .52002 | .85416 | .53484 | .84495 | .54951 | .83549 | .56401 | .82577 | 40 |
| 21 | .50528 | .86295 | .52026 | .85401 | .53509 | .84480 | .54975 | .83533 | .56425 | .82561 | 39 |
| 22 | .50553 | .86281 | .52051 | .85385 | .53534 | .84464 | .54999 | .83517 | .56449 | .82544 | 38 |
| 23 | .50578 | .86266 | .52076 | .85370 | .53558 | .84448 | .55024 | .83501 | .56473 | .82528 | 37 |
| 24 | .50603 | .86251 | .52101 | .85355 | .53583 | .84433 | .55048 | .83485 | .56497 | .82511 | 36 |
| 25 | .50628 | .86237 | .52126 | .85340 | .53607 | .84417 | .55072 | .83469 | .56521 | .82495 | 35 |
| 26 | .50654 | .86222 | .52151 | .85325 | .53632 | .84402 | .55097 | .83453 | .56545 | .82478 | 34 |
| 27 | .50679 | .86207 | .52175 | .85310 | .53656 | .84386 | .55121 | .83437 | .56569 | .82462 | 33 |
| 28 | .50704 | .86192 | .52200 | .85294 | .53681 | .84370 | .55145 | .83421 | .56593 | .82446 | 32 |
| 29 | .50729 | .86178 | .52225 | .85279 | .53705 | .84355 | .55169 | .83405 | .56617 | .82429 | 31 |
| 30 | .50754 | .86163 | .52250 | .85264 | .53730 | .84339 | .55194 | .83389 | .56641 | .82413 | 30 |
| 31 | .50779 | .86148 | .52275 | .85249 | .53754 | .84324 | .55218 | .83373 | .56665 | .82396 | 29 |
| 32 | .50804 | .86133 | .52299 | .85234 | .53779 | .84308 | .55242 | .83356 | .56689 | .82380 | 28 |
| 33 | .50829 | .86119 | .52324 | .85218 | .53804 | .84292 | .55266 | .83340 | .56713 | .82363 | 27 |
| 34 | .50854 | .86104 | .52349 | .85203 | .53828 | .84277 | .55291 | .83324 | .56736 | .82347 | 26 |
| 35 | .50879 | .86089 | .52374 | .85188 | .53853 | .84261 | .55315 | .83308 | .56760 | .82330 | 25 |
| 36 | .50904 | .86074 | .52399 | .85173 | .53877 | .84245 | .55339 | .83292 | .56784 | .82314 | 24 |
| 37 | .50929 | .86059 | .52423 | .85157 | .53902 | .84230 | .55363 | .83276 | .56808 | .82297 | 23 |
| 38 | .50954 | .86045 | .52448 | .85142 | .53926 | .84214 | .55388 | .83260 | .56832 | .82281 | 22 |
| 39 | .50979 | .86030 | .52473 | .85127 | .53951 | .84198 | .55412 | .83244 | .56856 | .82264 | 21 |
| 40 | .51004 | .86015 | .52498 | .85112 | .53975 | .84182 | .55436 | .83228 | .56880 | .82248 | 20 |
| 41 | .51029 | .86000 | .52522 | .85096 | .54000 | .84167 | .55460 | .83212 | .56904 | .82231 | 19 |
| 42 | .51054 | .85985 | .52547 | .85081 | .54024 | .84151 | .55484 | .83195 | .56928 | .82214 | 18 |
| 43 | .51079 | .85970 | .52572 | .85066 | .54049 | .84135 | .55509 | .83179 | .56952 | .82198 | 17 |
| 44 | .51104 | .85956 | .52597 | .85051 | .54073 | .84120 | .55533 | .83163 | .56976 | .82181 | 16 |
| 45 | .51129 | .85941 | .52621 | .85035 | .54097 | .84104 | .55557 | .83147 | .57000 | .82165 | 15 |
| 46 | .51154 | .85926 | .52646 | .85020 | .54122 | .84088 | .55581 | .83131 | .57024 | .82148 | 14 |
| 47 | .51179 | .85911 | .52671 | .85005 | .54146 | .84072 | .55605 | .83115 | .57047 | .82132 | 13 |
| 48 | .51204 | .85896 | .52696 | .84989 | .54171 | .84057 | .55630 | .83098 | .57071 | .82115 | 12 |
| 49 | .51229 | .85881 | .52720 | .84974 | .54195 | .84041 | .55654 | .83082 | .57095 | .82098 | 11 |
| 50 | .51254 | .85866 | .52745 | .84959 | .54220 | .84025 | .55678 | .83066 | .57119 | .82082 | 10 |
| 51 | .51279 | .85851 | .52770 | .84943 | .54244 | .84009 | .55702 | .83050 | .57143 | .82065 | 9 |
| 52 | .51304 | .85836 | .52794 | .84928 | .54269 | .83994 | .55726 | .83034 | .57167 | .82048 | 8 |
| 53 | .51329 | .85821 | .52819 | .84913 | .54293 | .83978 | .55750 | .83017 | .57191 | .82032 | 7 |
| 54 | .51354 | .85806 | .52844 | .84897 | .54317 | .83962 | .55775 | .83001 | .57215 | .82015 | 6 |
| 55 | .51379 | .85792 | .52869 | .84882 | .54342 | .83946 | .55799 | .82985 | .57238 | .81999 | 5 |
| 56 | .51404 | .85777 | .52893 | .84866 | .54366 | .83930 | .55823 | .82969 | .57262 | .81982 | 4 |
| 57 | .51429 | .85762 | .52918 | .84851 | .54391 | .83915 | .55847 | .82953 | .57286 | .81965 | 3 |
| 58 | .51454 | .85747 | .52943 | .84836 | .54415 | .83899 | .55871 | .82936 | .57310 | .81949 | 2 |
| 59 | .51479 | .85732 | .52967 | .84820 | .54440 | .83883 | .55895 | .82920 | .57334 | .81932 | 1 |
| 60 | .51504 | .85717 | .52992 | .84805 | .54464 | .83867 | .55919 | .82904 | .57358 | .81915 | 0 |
| ′ | Cosin | Sine | Cosin | Sine | Cosin | Sine | Cosin | Sine | Cosin | Sine | ′ |
| | 59° | | 58° | | 57° | | 56° | | 55° | | |

# Table II.—Natural Sines and Cosines.

| ′ | 35° | | 36° | | 37° | | 38° | | 39° | | ′ |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | Sine | Cosin | Sine | Cosin | Sine | Cosin | Sine | Cosin | Sine | Cosin | |
| 0 | .57358 | .81915 | .58779 | .80902 | .60182 | .79864 | .61566 | .78801 | .62932 | .77715 | 60 |
| 1 | .57381 | .81899 | .58802 | .80885 | .60205 | .79846 | .61589 | .78783 | .62955 | .77696 | 59 |
| 2 | .57405 | .81882 | .58826 | .80867 | .60228 | .79829 | .61612 | .78765 | .62977 | .77678 | 58 |
| 3 | .57429 | .81865 | .58849 | .80850 | .60251 | .79811 | .61635 | .78747 | .63000 | .77660 | 57 |
| 4 | .57453 | .81848 | .58873 | .80833 | .60274 | .79793 | .61658 | .78729 | .63022 | .77641 | 56 |
| 5 | .57477 | .81832 | .58896 | .80816 | .60298 | .79776 | .61681 | .78711 | .63045 | .77623 | 55 |
| 6 | .57501 | .81815 | .58920 | .80799 | .60321 | .79758 | .61704 | .78694 | .63068 | .77605 | 54 |
| 7 | .57524 | .81798 | .58943 | .80782 | .60344 | .79741 | .61726 | .78676 | .63090 | .77586 | 53 |
| 8 | .57548 | .81782 | .58967 | .80765 | .60367 | .79723 | .61749 | .78658 | .63113 | .77568 | 52 |
| 9 | .57572 | .81765 | .58990 | .80748 | .60390 | .79706 | .61772 | .78640 | .63135 | .77550 | 51 |
| 10 | .57596 | .81748 | .59014 | .80730 | .60414 | .79688 | .61795 | .78622 | .63158 | .77531 | 50 |
| 11 | .57619 | .81731 | .59037 | .80713 | .60437 | .79671 | .61818 | .78604 | .63180 | .77513 | 49 |
| 12 | .57643 | .81714 | .59061 | .80696 | .60460 | .79653 | .61841 | .78586 | .63203 | .77494 | 48 |
| 13 | .57667 | .81698 | .59084 | .80679 | .60483 | .79635 | .61864 | .78568 | .63225 | .77476 | 47 |
| 14 | .57691 | .81681 | .59108 | .80662 | .60506 | .79618 | .61887 | .78550 | .63248 | .77458 | 46 |
| 15 | .57715 | .81664 | .59131 | .80644 | .60529 | .79600 | .61909 | .78532 | .63271 | .77439 | 45 |
| 16 | .57738 | .81647 | .59154 | .80627 | .60553 | .79583 | .61932 | .78514 | .63293 | .77421 | 44 |
| 17 | .57762 | .81631 | .59178 | .80610 | .60576 | .79565 | .61955 | .78496 | .63316 | .77402 | 43 |
| 18 | .57786 | .81614 | .59201 | .80593 | .60599 | .79547 | .61978 | .78478 | .63338 | .77384 | 42 |
| 19 | .57810 | .81597 | .59225 | .80576 | .60622 | .79530 | .62001 | .78460 | .63361 | .77366 | 41 |
| 20 | .57833 | .81580 | .59248 | .80558 | .60645 | .79512 | .62024 | .78442 | .63383 | .77347 | 40 |
| 21 | .57857 | .81563 | .59272 | .80541 | .60668 | .79494 | .62046 | .78424 | .63406 | .77329 | 39 |
| 22 | .57881 | .81546 | .59295 | .80524 | .60691 | .79477 | .62069 | .78405 | .63428 | .77310 | 38 |
| 23 | .57904 | .81530 | .59318 | .80507 | .60714 | .79459 | .62092 | .78387 | .63451 | .77292 | 37 |
| 24 | .57928 | .81513 | .59342 | .80489 | .60738 | .79441 | .62115 | .78369 | .63473 | .77273 | 36 |
| 25 | .57952 | .81496 | .59365 | .80472 | .60761 | .79424 | .62138 | .78351 | .63496 | .77255 | 35 |
| 26 | .57976 | .81479 | .59389 | .80455 | .60784 | .79406 | .62160 | .78333 | .63518 | .77236 | 34 |
| 27 | .57999 | .81462 | .59412 | .80438 | .60807 | .79388 | .62183 | .78315 | .63540 | .77218 | 33 |
| 28 | .58023 | .81445 | .59436 | .80420 | .60830 | .79371 | .62206 | .78297 | .63563 | .77199 | 32 |
| 29 | .58047 | .81428 | .59459 | .80403 | .60853 | .79353 | .62229 | .78279 | .63585 | .77181 | 31 |
| 30 | .58070 | .81412 | .59482 | .80386 | .60876 | .79335 | .62251 | .78261 | .63608 | .77162 | 30 |
| 31 | .58094 | .81395 | .59506 | .80368 | .60899 | .79318 | .62274 | .78243 | .63630 | .77144 | 29 |
| 32 | .58118 | .81378 | .59529 | .80351 | .60922 | .79300 | .62297 | .78225 | .63653 | .77125 | 28 |
| 33 | .58141 | .81361 | .59552 | .80334 | .60945 | .79282 | .62320 | .78206 | .63675 | .77107 | 27 |
| 34 | .58165 | .81344 | .59576 | .80316 | .60968 | .79264 | .62342 | .78188 | .63698 | .77088 | 26 |
| 35 | .58189 | .81327 | .59599 | .80299 | .60991 | .79247 | .62365 | .78170 | .63720 | .77070 | 25 |
| 36 | .58212 | .81310 | .59622 | .80282 | .61015 | .79229 | .62388 | .78152 | .63742 | .77051 | 24 |
| 37 | .58236 | .81293 | .59646 | .80264 | .61038 | .79211 | .62411 | .78134 | .63765 | .77033 | 23 |
| 38 | .58260 | .81276 | .59669 | .80247 | .61061 | .79193 | .62433 | .78116 | .63787 | .77014 | 22 |
| 39 | .58283 | .81259 | .59693 | .80230 | .61084 | .79176 | .62456 | .78098 | .63810 | .76996 | 21 |
| 40 | .58307 | .81242 | .59716 | .80212 | .61107 | .79158 | .62479 | .78079 | .63832 | .76977 | 20 |
| 41 | .58330 | .81225 | .59739 | .80195 | .61130 | .79140 | .62502 | .78061 | .63854 | .76959 | 19 |
| 42 | .58354 | .81208 | .59763 | .80178 | .61153 | .79122 | .62524 | .78043 | .63877 | .76940 | 18 |
| 43 | .58378 | .81191 | .59786 | .80160 | .61176 | .79105 | .62547 | .78025 | .63899 | .76921 | 17 |
| 44 | .58401 | .81174 | .59809 | .80143 | .61199 | .79087 | .62570 | .78007 | .63922 | .76903 | 16 |
| 45 | .58425 | .81157 | .59832 | .80125 | .61222 | .79069 | .62592 | .77988 | .63944 | .76884 | 15 |
| 46 | .58449 | .81140 | .59856 | .80108 | .61245 | .79051 | .62615 | .77970 | .63966 | .76866 | 14 |
| 47 | .58472 | .81123 | .59879 | .80091 | .61268 | .79033 | .62638 | .77952 | .63989 | .76847 | 13 |
| 48 | .58496 | .81106 | .59902 | .80073 | .61291 | .79016 | .62660 | .77934 | .64011 | .76828 | 12 |
| 49 | .58519 | .81089 | .59926 | .80056 | .61314 | .78998 | .62683 | .77916 | .64033 | .76810 | 11 |
| 50 | .58543 | .81072 | .59949 | .80038 | .61337 | .78980 | .62706 | .77897 | .64056 | .76791 | 10 |
| 51 | .58567 | .81055 | .59972 | .80021 | .61360 | .78962 | .62728 | .77879 | .64078 | .76772 | 9 |
| 52 | .58590 | .81038 | .59995 | .80003 | .61383 | .78944 | .62751 | .77861 | .64100 | .76754 | 8 |
| 53 | .58614 | .81021 | .60019 | .79986 | .61406 | .78926 | .62774 | .77843 | .64123 | .76735 | 7 |
| 54 | .58637 | .81004 | .60042 | .79968 | .61429 | .78908 | .62796 | .77824 | .64145 | .76717 | 6 |
| 55 | .58661 | .80987 | .60065 | .79951 | .61451 | .78891 | .62819 | .77806 | .64167 | .76698 | 5 |
| 56 | .58684 | .80970 | .60089 | .79934 | .61474 | .78873 | .62842 | .77788 | .64190 | .76679 | 4 |
| 57 | .58708 | .80953 | .60112 | .79916 | .61497 | .78855 | .62864 | .77769 | .64212 | .76661 | 3 |
| 58 | .58731 | .80936 | .60135 | .79899 | .61520 | .78837 | .62887 | .77751 | .64234 | .76642 | 2 |
| 59 | .58755 | .80919 | .60158 | .79881 | .61543 | .78819 | .62909 | .77733 | .64256 | .76623 | 1 |
| 60 | .58779 | .80902 | .60182 | .79864 | .61566 | .78801 | .62932 | .77715 | .64279 | .76604 | 0 |
| ′ | Cosin | Sine | Cosin | Sine | Cosin | Sine | Cosin | Sine | Cosin | Sine | ′ |
| | 54° | | 53° | | 52° | | 51° | | 50° | | |

# Table II.—Natural Sines and Cosines.

| ′ | 40° | | 41° | | 42° | | 43° | | 44° | | ′ |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | Sine | Cosin | Sine | Cosin | Sine | Cosin | Sine | Cosin | Sine | Cosin | |
| 0 | .64279 | .76604 | .65606 | .75471 | .66913 | .74314 | .68200 | .73135 | .69466 | .71934 | 60 |
| 1 | .64301 | .76586 | .65628 | .75452 | .66935 | .74295 | .68221 | .73116 | .69487 | .71914 | 59 |
| 2 | .64323 | .76567 | .65650 | .75433 | .66956 | .74276 | .68242 | .73096 | .69508 | .71894 | 58 |
| 3 | .64346 | .76548 | .65672 | .75414 | .66978 | .74256 | .68264 | .73076 | .69529 | .71873 | 57 |
| 4 | .64368 | .76530 | .65694 | .75395 | .66999 | .74237 | .68285 | .73056 | .69549 | .71853 | 56 |
| 5 | .64390 | .76511 | .65716 | .75375 | .67021 | .74217 | .68306 | .73036 | .69570 | .71833 | 55 |
| 6 | .64412 | .76492 | .65738 | .75356 | .67043 | .74198 | .68327 | .73016 | .69591 | .71813 | 54 |
| 7 | .64435 | .76473 | .65759 | .75337 | .67064 | .74178 | .68349 | .72996 | .69612 | .71792 | 53 |
| 8 | .64457 | .76455 | .65781 | .75318 | .67086 | .74159 | .68370 | .72976 | .69633 | .71772 | 52 |
| 9 | .64479 | .76436 | .65803 | .75299 | .67107 | .74139 | .68391 | .72957 | .69654 | .71752 | 51 |
| 10 | .64501 | .76417 | .65825 | .75280 | .67129 | .74120 | .68412 | .72937 | .69675 | .71732 | 50 |
| 11 | .64524 | .76398 | .65847 | .75261 | .67151 | .74100 | .68434 | .72917 | .69696 | .71711 | 49 |
| 12 | .64546 | .76380 | .65869 | .75241 | .67172 | .74080 | .68455 | .72897 | .69717 | .71691 | 48 |
| 13 | .64568 | .76361 | .65891 | .75222 | .67194 | .74061 | .68476 | .72877 | .69737 | .71671 | 47 |
| 14 | .64590 | .76342 | .65913 | .75203 | .67215 | .74041 | .68497 | .72857 | .69758 | .71650 | 46 |
| 15 | .64612 | .76323 | .65935 | .75184 | .67237 | .74022 | .68518 | .72837 | .69779 | .71630 | 45 |
| 16 | .64635 | .76304 | .65956 | .75165 | .67258 | .74002 | .68539 | .72817 | .69800 | .71610 | 44 |
| 17 | .64657 | .76286 | .65978 | .75146 | .67280 | .73983 | .68561 | .72797 | .69821 | .71590 | 43 |
| 18 | .64679 | .76267 | .66000 | .75126 | .67301 | .73963 | .68582 | .72777 | .69842 | .71569 | 42 |
| 19 | .64701 | .76248 | .66022 | .75107 | .67323 | .73944 | .68603 | .72757 | .69862 | .71549 | 41 |
| 20 | .64723 | .76229 | .66044 | .75088 | .67344 | .73924 | .68624 | .72737 | .69883 | .71529 | 40 |
| 21 | .64746 | .76210 | .66066 | .75069 | .67366 | .73904 | .68645 | .72717 | .69904 | .71508 | 39 |
| 22 | .64768 | .76192 | .66088 | .75050 | .67387 | .73885 | .68666 | .72697 | .69925 | .71488 | 38 |
| 23 | .64790 | .76173 | .66109 | .75030 | .67409 | .73865 | .68688 | .72677 | .69946 | .71468 | 37 |
| 24 | .64812 | .76154 | .66131 | .75011 | .67430 | .73846 | .68709 | .72657 | .69966 | .71447 | 36 |
| 25 | .64834 | .76135 | .66153 | .74992 | .67452 | .73826 | .68730 | .72637 | .69987 | .71427 | 35 |
| 26 | .64856 | .76116 | .66175 | .74973 | .67473 | .73806 | .68751 | .72617 | .70008 | .71407 | 34 |
| 27 | .64878 | .76097 | .66197 | .74953 | .67495 | .73787 | .68773 | .72597 | .70029 | .71386 | 33 |
| 28 | .64901 | .76078 | .66218 | .74934 | .67516 | .73767 | .68793 | .72577 | .70049 | .71366 | 32 |
| 29 | .64923 | .76059 | .66240 | .74915 | .67538 | .73747 | .68814 | .72557 | .70070 | .71345 | 31 |
| 30 | .64945 | .76041 | .66262 | .74896 | .67559 | .73728 | .68835 | .72537 | .70091 | .71325 | 30 |
| 31 | .64967 | .76022 | .66284 | .74876 | .67580 | .73708 | .68857 | .72517 | .70112 | .71305 | 29 |
| 32 | .64989 | .76003 | .66306 | .74857 | .67602 | .73688 | .68878 | .72497 | .70132 | .71284 | 28 |
| 33 | .65011 | .75984 | .66327 | .74838 | .67623 | .73669 | .68899 | .72477 | .70153 | .71264 | 27 |
| 34 | .65033 | .75965 | .66349 | .74818 | .67645 | .73649 | .68920 | .72457 | .70174 | .71243 | 26 |
| 35 | .65055 | .75946 | .66371 | .74799 | .67666 | .73629 | .68941 | .72437 | .70195 | .71222 | 25 |
| 36 | .65077 | .75927 | .66393 | .74780 | .67688 | .73610 | .68962 | .72417 | .70215 | .71203 | 24 |
| 37 | .65100 | .75908 | .66414 | .74760 | .67709 | .73590 | .68983 | .72397 | .70236 | .71182 | 23 |
| 38 | .65122 | .75889 | .66436 | .74741 | .67730 | .73570 | .69004 | .72377 | .70257 | .71162 | 22 |
| 39 | .65144 | .75870 | .66458 | .74722 | .67752 | .73551 | .69025 | .72357 | .70277 | .71141 | 21 |
| 40 | .65166 | .75851 | .66480 | .74703 | .67773 | .73531 | .69046 | .72337 | .70298 | .71121 | 20 |
| 41 | .65188 | .75832 | .66501 | .74683 | .67795 | .73511 | .69067 | .72317 | .70319 | .71100 | 19 |
| 42 | .65210 | .75813 | .66523 | .74664 | .67816 | .73491 | .69088 | .72297 | .70339 | .71080 | 18 |
| 43 | .65232 | .75794 | .66545 | .74644 | .67837 | .73472 | .69109 | .72277 | .70360 | .71059 | 17 |
| 44 | .65254 | .75775 | .66566 | .74625 | .67859 | .73452 | .69130 | .72257 | .70381 | .71039 | 16 |
| 45 | .65276 | .75756 | .66588 | .74606 | .67880 | .73432 | .69151 | .72236 | .70401 | .71019 | 15 |
| 46 | .65298 | .75738 | .66610 | .74586 | .67901 | .73413 | .69172 | .72216 | .70422 | .70998 | 14 |
| 47 | .65320 | .75719 | .66632 | .74567 | .67923 | .73393 | .69193 | .72196 | .70443 | .70978 | 13 |
| 48 | .65342 | .75700 | .66653 | .74548 | .67944 | .73373 | .69214 | .72176 | .70463 | .70957 | 12 |
| 49 | .65364 | .75680 | .66675 | .74528 | .67965 | .73353 | .69235 | .72156 | .70484 | .70937 | 11 |
| 50 | .65386 | .75661 | .66697 | .74509 | .67987 | .73333 | .69256 | .72136 | .70505 | .70916 | 10 |
| 51 | .65408 | .75642 | .66718 | .74489 | .68008 | .73314 | .69277 | .72116 | .70525 | .70896 | 9 |
| 52 | .65430 | .75623 | .66740 | .74470 | .68029 | .73294 | .69298 | .72095 | .70546 | .70875 | 8 |
| 53 | .65452 | .75604 | .66762 | .74451 | .68051 | .73274 | .69319 | .72075 | .70567 | .70855 | 7 |
| 54 | .65474 | .75585 | .66783 | .74431 | .68072 | .73254 | .69340 | .72055 | .70587 | .70834 | 6 |
| 55 | .65496 | .75566 | .66805 | .74412 | .68093 | .73234 | .69361 | .72035 | .70608 | .70813 | 5 |
| 56 | .65518 | .75547 | .66827 | .74392 | .68115 | .73215 | .69382 | .72015 | .70628 | .70793 | 4 |
| 57 | .65540 | .75528 | .66848 | .74373 | .68136 | .73195 | .69403 | .71995 | .70649 | .70772 | 3 |
| 58 | .65562 | .75509 | .66870 | .74352 | .68157 | .73175 | .69424 | .71974 | .70670 | .70752 | 2 |
| 59 | .65584 | .75490 | .66891 | .74334 | .68179 | .73155 | .69445 | .71954 | .70690 | .70731 | 1 |
| 60 | .65606 | .75471 | .66913 | .74314 | .68200 | .73135 | .69466 | .71934 | .70711 | .70711 | 0 |
| ′ | Cosin | Sine | Cosin | Sine | Cosin | Sine | Cosin | Sine | Cosin | Sine | ′ |
| | 49° | | 48° | | 47° | | 46° | | 45° | | |

# Table III.—Natural Tangents and Cotangents.

| ′ | 0° | | 1° | | 2° | | 3° | | ′ |
|---|---|---|---|---|---|---|---|---|---|
| | Tang | Cotang | Tang | Cotang | Tang | Cotang | Tang | Cotang | |
| 0 | .00000 | Infinite. | .01746 | 57.2900 | .03492 | 28.6363 | .05241 | 19.0811 | 60 |
| 1 | .00029 | 3437.75 | .01775 | 56.3506 | .03521 | 28.3994 | .05270 | 18.9755 | 59 |
| 2 | .00058 | 1718.87 | .01804 | 55.4415 | .03550 | 28.1664 | .05299 | 18.8711 | 58 |
| 3 | .00087 | 1145.92 | .01833 | 54.5613 | .03579 | 27.9372 | .05328 | 18.7678 | 57 |
| 4 | .00116 | 859.436 | .01862 | 53.7086 | .03609 | 27.7117 | .05357 | 18.6656 | 56 |
| 5 | .00145 | 687.549 | .01891 | 52.8821 | .03638 | 27.4899 | .05387 | 18.5645 | 55 |
| 6 | .00175 | 572.957 | .01920 | 52.0807 | .03667 | 27.2715 | .05416 | 18.4645 | 54 |
| 7 | .00204 | 491.106 | .01949 | 51.3032 | .03696 | 27.0566 | .05445 | 18.3655 | 53 |
| 8 | .00233 | 429.718 | .01978 | 50.5485 | .03725 | 26.8450 | .05474 | 18.2677 | 52 |
| 9 | .00262 | 381.971 | .02007 | 49.8157 | .03754 | 26.6367 | .05508 | 18.1708 | 51 |
| 10 | .00291 | 343.774 | .02036 | 49.1039 | .03783 | 26.4316 | .05533 | 18.0750 | 50 |
| 11 | .00320 | 312.521 | .02066 | 48.4121 | .03812 | 26.2296 | .05562 | 17.9802 | 49 |
| 12 | .00349 | 286.478 | .02095 | 47.7395 | .03842 | 26.0307 | .05591 | 17.8863 | 48 |
| 13 | .00378 | 264.441 | .02124 | 47.0853 | .03871 | 25.8348 | .05620 | 17.7934 | 47 |
| 14 | .00407 | 245.552 | .02153 | 46.4489 | .03900 | 25.6418 | .05649 | 17.7015 | 46 |
| 15 | .00436 | 229.182 | .02182 | 45.8294 | .03929 | 25.4517 | .05678 | 17.6106 | 45 |
| 16 | .00465 | 214.858 | .02211 | 45.2261 | .03958 | 25.2644 | .05706 | 17.5205 | 44 |
| 17 | .00495 | 202.219 | .02240 | 44.6386 | .03987 | 25.0798 | .05737 | 17.4314 | 43 |
| 18 | .00524 | 190.984 | .02269 | 44.0661 | .04016 | 24.8978 | .05766 | 17.3432 | 42 |
| 19 | .00553 | 180.932 | .02298 | 43.5081 | .04046 | 24.7185 | .05795 | 17.2558 | 41 |
| 20 | .00582 | 171.885 | .02328 | 42.9641 | .04075 | 24.5418 | .05824 | 17.1693 | 40 |
| 21 | .00611 | 163.700 | .02357 | 42.4335 | .04104 | 24.3675 | .05854 | 17.0837 | 39 |
| 22 | .00640 | 156.259 | .02386 | 41.9158 | .04133 | 24.1957 | .05883 | 16.9990 | 38 |
| 23 | .00669 | 149.465 | .02415 | 41.4106 | .04162 | 24.0263 | .05912 | 16.9150 | 37 |
| 24 | .00698 | 143.237 | .02444 | 40.9174 | .04191 | 23.8593 | .05941 | 16.8319 | 36 |
| 25 | .00727 | 137.507 | .02473 | 40.4358 | .04220 | 23.6945 | .05970 | 16.7496 | 35 |
| 26 | .00756 | 132.219 | .02502 | 39.9655 | .04250 | 23.5321 | .05999 | 16.6681 | 34 |
| 27 | .00785 | 127.321 | .02531 | 39.5059 | .04279 | 23.3718 | .06029 | 16.5874 | 33 |
| 28 | .00815 | 122.774 | .02560 | 39.0568 | .04308 | 23.2137 | .06058 | 16.5075 | 32 |
| 29 | .00844 | 118.540 | .02589 | 38.6177 | .04337 | 23.0577 | .06087 | 16.4283 | 31 |
| 30 | .00873 | 114.589 | .02619 | 38.1885 | .04366 | 22.9038 | .06116 | 16.3499 | 30 |
| 31 | .00902 | 110.892 | .02648 | 37.7686 | .04395 | 22.7519 | .06145 | 16.2722 | 29 |
| 32 | .00931 | 107.426 | .02677 | 37.3579 | .04424 | 22.6020 | .06175 | 16.1952 | 28 |
| 33 | .00960 | 104.171 | .02706 | 36.9560 | .04454 | 22.4541 | .06204 | 16.1190 | 27 |
| 34 | .00989 | 101.107 | .02735 | 36.5627 | .04483 | 22.3081 | .06233 | 16.0435 | 26 |
| 35 | .01018 | 98.2179 | .02764 | 36.1776 | .04512 | 22.1640 | .06262 | 15.9687 | 25 |
| 36 | .01047 | 95.4895 | .02793 | 35.8006 | .04541 | 22.0217 | .06291 | 15.8945 | 24 |
| 37 | .01076 | 92.9085 | .02822 | 35.4313 | .04570 | 21.8813 | .06321 | 15.8211 | 23 |
| 38 | .01105 | 90.4633 | .02851 | 35.0695 | .04599 | 21.7426 | .06350 | 15.7483 | 22 |
| 39 | .01135 | 88.1436 | .02881 | 34.7151 | .04628 | 21.6056 | .06379 | 15.6762 | 21 |
| 40 | .01164 | 85.9398 | .02910 | 34.3678 | .04658 | 21.4704 | .06408 | 15.6048 | 20 |
| 41 | .01193 | 83.8435 | .02939 | 34.0273 | .04687 | 21.3369 | .06437 | 15.5340 | 19 |
| 42 | .01222 | 81.8470 | .02968 | 33.6935 | .04716 | 21.2049 | .06467 | 15.4638 | 18 |
| 43 | .01251 | 79.9434 | .02997 | 33.3662 | .04745 | 21.0747 | .06496 | 15.3943 | 17 |
| 44 | .01280 | 78.1263 | .03026 | 33.0452 | .04774 | 20.9460 | .06525 | 15.3254 | 16 |
| 45 | .01309 | 76.3900 | .03055 | 32.7303 | .04803 | 20.8188 | .06554 | 15.2571 | 15 |
| 46 | .01338 | 74.7292 | .03084 | 32.4213 | .04833 | 20.6932 | .06584 | 15.1893 | 14 |
| 47 | .01367 | 73.1390 | .03114 | 32.1181 | .04862 | 20.5691 | .06613 | 15.1222 | 13 |
| 48 | .01396 | 71.6151 | .03143 | 31.8205 | .04891 | 20.4465 | .06642 | 15.0557 | 12 |
| 49 | .01425 | 70.1533 | .03172 | 31.5284 | .04920 | 20.3253 | .06671 | 14.9898 | 11 |
| 50 | .01455 | 68.7501 | .03201 | 31.2416 | .04949 | 20.2056 | .06700 | 14.9244 | 10 |
| 51 | .01484 | 67.4019 | .03230 | 30.9599 | .04978 | 20.0872 | .06730 | 14.8596 | 9 |
| 52 | .01513 | 66.1055 | .03259 | 30.6833 | .05007 | 19.9702 | .06759 | 14.7954 | 8 |
| 53 | .01542 | 64.8580 | .03288 | 30.4116 | .05037 | 19.8546 | .06788 | 14.7317 | 7 |
| 54 | .01571 | 63.6567 | .03317 | 30.1446 | .05066 | 19.7403 | .06817 | 14.6685 | 6 |
| 55 | .01600 | 62.4992 | .03346 | 29.8823 | .05095 | 19.6273 | .06847 | 14.6059 | 5 |
| 56 | .01629 | 61.3829 | .03376 | 29.6245 | .05124 | 19.5156 | .06876 | 14.5438 | 4 |
| 57 | .01658 | 60.3058 | .03405 | 29.3711 | .05153 | 19.4051 | .06905 | 14.4823 | 3 |
| 58 | .01687 | 59.2659 | .03434 | 29.1220 | .05182 | 19.2959 | .06934 | 14.4212 | 2 |
| 59 | .01716 | 58.2612 | .03463 | 28.8771 | .05212 | 19.1879 | .06963 | 14.3607 | 1 |
| 60 | .01746 | 57.2900 | .03492 | 28.6363 | .05241 | 19.0811 | .06993 | 14.3007 | 0 |
| ′ | Cotang | Tang | Cotang | Tang | Cotang | Tang | Cotang | Tang | ′ |
| | 89° | | 88° | | 87° | | 86° | | |

# Table III.—Natural Tangents and Cotangents.

| ′ | 4° | | 5° | | 6° | | 7° | | ′ |
|---|---|---|---|---|---|---|---|---|---|
| | Tang | Cotang | Tang | Cotang | Tang | Cotang | Tang | Cotang | |
| 0 | .06993 | 14.3007 | .08749 | 11.4301 | .10510 | 9.51436 | .12278 | 8.14435 | 60 |
| 1 | .07022 | 14.2411 | .08778 | 11.3919 | .10540 | 9.48781 | .12308 | 8.12481 | 59 |
| 2 | .07051 | 14.1821 | .08807 | 11.3540 | .10569 | 9.46141 | .12338 | 8.10536 | 58 |
| 3 | .07080 | 14.1235 | .08837 | 11.3163 | .10599 | 9.43515 | .12367 | 8.08600 | 57 |
| 4 | .07110 | 14.0655 | .08866 | 11.2789 | .10628 | 9.40904 | .12397 | 8.06674 | 56 |
| 5 | .07139 | 14.0079 | .08895 | 11.2417 | .10657 | 9.38307 | .12426 | 8.04756 | 55 |
| 6 | .07168 | 13.9507 | .08925 | 11.2048 | .10687 | 9.35724 | .12456 | 8.02848 | 54 |
| 7 | .07197 | 13.8940 | .08954 | 11.1681 | .10716 | 9.33155 | .12485 | 8.00948 | 53 |
| 8 | .07227 | 13.8378 | .08983 | 11.1316 | .10746 | 9.30599 | .12515 | 7.99058 | 52 |
| 9 | .07256 | 13.7821 | .09013 | 11.0954 | .10775 | 9.28058 | .12544 | 7.97176 | 51 |
| 10 | .07285 | 13.7267 | .09042 | 11.0594 | .10605 | 9.25530 | .12574 | 7.95302 | 50 |
| 11 | .07314 | 13.6719 | .09071 | 11.0237 | .10834 | 9.23016 | .12603 | 7.93438 | 49 |
| 12 | .07344 | 13.6174 | .09101 | 10.9882 | .10863 | 9.20516 | .12633 | 7.91582 | 48 |
| 13 | .07373 | 13.5634 | .09130 | 10.9529 | .10893 | 9.18028 | .12662 | 7.89734 | 47 |
| 14 | .07402 | 13.5098 | .09159 | 10.9178 | .10922 | 9.15554 | .12692 | 7.87895 | 46 |
| 15 | .07431 | 13.4566 | .09189 | 10.8829 | .10952 | 9.13093 | .12722 | 7.86064 | 45 |
| 16 | .07461 | 13.4039 | .09218 | 10.8483 | .10981 | 9.10646 | .12751 | 7.84242 | 44 |
| 17 | .07490 | 13.3515 | .09247 | 10.8139 | .11011 | 9.08211 | .12781 | 7.82428 | 43 |
| 18 | .07519 | 13.2996 | .09277 | 10.7797 | .11040 | 9.05789 | .12810 | 7.80622 | 42 |
| 19 | .07548 | 13.2480 | .09306 | 10.7457 | .11070 | 9.03379 | .12840 | 7.78825 | 41 |
| 20 | .07578 | 13.1969 | .09335 | 10.7119 | .11099 | 9.00983 | .12869 | 7.77035 | 40 |
| 21 | .07607 | 13.1461 | .09365 | 10.6783 | .11128 | 8.98598 | .12899 | 7.75254 | 39 |
| 22 | .07636 | 13.0958 | .09394 | 10.6450 | .11158 | 8.96227 | .12929 | 7.73480 | 38 |
| 23 | .07665 | 13.0458 | .09423 | 10.6118 | .11187 | 8.93867 | .12958 | 7.71715 | 37 |
| 24 | .07695 | 12.9962 | .09453 | 10.5789 | .11217 | 8.91520 | .12988 | 7.69957 | 36 |
| 25 | .07724 | 12.9469 | .09482 | 10.5462 | .11246 | 8.89185 | .13017 | 7.68208 | 35 |
| 26 | .07753 | 12.8981 | .09511 | 10.5136 | .11276 | 8.86862 | .13047 | 7.66466 | 34 |
| 27 | .07782 | 12.8496 | .09541 | 10.4813 | .11305 | 8.84551 | .13076 | 7.64732 | 33 |
| 28 | .07812 | 12.8014 | .09570 | 10.4491 | .11335 | 8.82252 | .13106 | 7.63005 | 32 |
| 29 | .07841 | 12.7536 | .09600 | 10.4172 | .11364 | 8.79964 | .13136 | 7.61287 | 31 |
| 30 | .07870 | 12.7062 | .09629 | 10.3854 | .11394 | 8.77689 | .13165 | 7.59575 | 30 |
| 31 | .07899 | 12.6591 | .09658 | 10.3538 | .11423 | 8.75425 | .13195 | 7.57872 | 29 |
| 32 | .07929 | 12.6124 | .09688 | 10.3224 | .11452 | 8.73172 | .13224 | 7.56176 | 28 |
| 33 | .07958 | 12.5660 | .09717 | 10.2913 | .11482 | 8.70931 | .13254 | 7.54487 | 27 |
| 34 | .07987 | 12.5199 | .09746 | 10.2602 | .11511 | 8.68701 | .13284 | 7.52806 | 26 |
| 35 | .08017 | 12.4742 | .09776 | 10.2294 | .11541 | 8.66482 | .13313 | 7.51132 | 25 |
| 36 | .08046 | 12.4288 | .09805 | 10.1988 | .11570 | 8.64275 | .13343 | 7.49465 | 24 |
| 37 | .08075 | 12.3838 | .09834 | 10.1683 | .11600 | 8.62078 | .13372 | 7.47806 | 23 |
| 38 | .08104 | 12.3390 | .09864 | 10.1381 | .11629 | 8.59893 | .13402 | 7.46154 | 22 |
| 39 | .08134 | 12.2946 | .09893 | 10.1080 | .11659 | 8.57718 | .13432 | 7.44509 | 21 |
| 40 | .08163 | 12.2505 | .09923 | 10.0780 | .11688 | 8.55555 | .13461 | 7.42871 | 20 |
| 41 | .08192 | 12.2067 | .09952 | 10.0483 | .11718 | 8.53402 | .13491 | 7.41240 | 19 |
| 42 | .08221 | 12.1632 | .09981 | 10.0187 | .11747 | 8.51259 | .13521 | 7.39616 | 18 |
| 43 | .08251 | 12.1201 | .10011 | 9.98931 | .11777 | 8.49128 | .13550 | 7.37999 | 17 |
| 44 | .08280 | 12.0772 | .10040 | 9.96007 | .11806 | 8.47007 | .13580 | 7.36389 | 16 |
| 45 | .08309 | 12.0346 | .10069 | 9.93101 | .11836 | 8.44896 | .13609 | 7.34786 | 15 |
| 46 | .08339 | 11.9923 | .10099 | 9.90211 | .11865 | 8.42795 | .13639 | 7.33190 | 14 |
| 47 | .08368 | 11.9504 | .10128 | 9.87338 | .11895 | 8.40705 | .13669 | 7.31600 | 13 |
| 48 | .08397 | 11.9087 | .10158 | 9.84482 | .11924 | 8.38625 | .13698 | 7.30018 | 12 |
| 49 | .08427 | 11.8673 | .10187 | 9.81641 | .11954 | 8.36555 | .13728 | 7.28442 | 11 |
| 50 | .08456 | 11.8262 | .10216 | 9.78817 | .11983 | 8.34496 | .13758 | 7.26873 | 10 |
| 51 | .08485 | 11.7853 | .10246 | 8.76009 | .12013 | 8.32446 | .13787 | 7.25310 | 9 |
| 52 | .08514 | 11.7448 | .10275 | 9.73217 | .12042 | 8.30406 | .13817 | 7.23754 | 8 |
| 53 | .08544 | 11.7045 | .10305 | 9.70441 | .12072 | 8.28376 | .13846 | 7.22204 | 7 |
| 54 | .08573 | 11.6645 | .10334 | 9.67680 | .12101 | 8.26355 | .13876 | 7.20661 | 6 |
| 55 | .08602 | 11.6248 | .10363 | 9.64935 | .12131 | 8.24345 | .13906 | 7.19125 | 5 |
| 56 | .08632 | 11.5853 | .10393 | 9.62205 | .12160 | 8.22344 | .13935 | 7.17594 | 4 |
| 57 | .08661 | 11.5461 | .10422 | 9.59490 | .12190 | 8.20352 | .13965 | 7.16071 | 3 |
| 58 | .08690 | 11 5072 | .10452 | 9.56791 | .12219 | 8.18370 | .13995 | 7.14553 | 2 |
| 59 | .08720 | 11.4685 | .10481 | 9.54106 | .12249 | 8.16398 | .14024 | 7.13042 | 1 |
| 60 | .08749 | 11.4301 | .10510 | 9.51436 | .12278 | 8.14435 | .14054 | 7.11537 | 0 |
| ′ | Cotang | Tang | Cotang | Tang | Cotang | Tang | Cotang | Tang | ′ |
| | 85° | | 84° | | 83° | | 82° | | |

| ′ | 8° | | 9° | | 10° | | 11° | | ′ |
|---|---|---|---|---|---|---|---|---|---|
| | Tang | Cotang | Tang | Cotang | Tang | Cotang | Tang | Cotang | |
| 0 | .14054 | 7.11537 | .15838 | 6.31375 | .17633 | 5.67128 | .19438 | 5.14455 | 60 |
| 1 | .14084 | 7.10038 | .15868 | 6.30189 | .17663 | 5.66165 | .19468 | 5.13658 | 59 |
| 2 | .14113 | 7.08546 | .15898 | 6.29007 | .17693 | 5.65205 | .19498 | 5.12862 | 58 |
| 3 | .14143 | 7.07059 | .15928 | 6.27829 | .17723 | 5.64248 | .19529 | 5.12069 | 57 |
| 4 | .14173 | 7.05579 | .15958 | 6.26655 | .17753 | 5.63295 | .19559 | 5.11279 | 56 |
| 5 | .14202 | 7.04105 | .15988 | 6.25486 | .17783 | 5.62344 | .19589 | 5.10490 | 55 |
| 6 | .14232 | 7.02637 | .16017 | 6.24321 | .17813 | 5.61397 | .19619 | 5.09704 | 54 |
| 7 | .14262 | 7.01174 | .16047 | 6.23160 | .17843 | 5.60452 | .19649 | 5.08921 | 53 |
| 8 | .14291 | 6.99718 | .16077 | 6.22003 | .17873 | 5.59511 | .19680 | 5.08139 | 52 |
| 9 | .14321 | 6.98268 | .16107 | 6.20851 | .17903 | 5.58573 | .19710 | 5.07360 | 51 |
| 10 | .14351 | 6.96823 | .16137 | 6.19703 | .17933 | 5.57638 | .19740 | 5.06584 | 50 |
| 11 | .14381 | 6.95385 | .16167 | 6.18559 | .17963 | 5.56706 | .19770 | 5.05809 | 49 |
| 12 | .14410 | 6.93952 | .16196 | 6.17419 | .17993 | 5.55777 | .19801 | 5.05037 | 48 |
| 13 | .14440 | 6.92525 | .16226 | 6.16283 | .18023 | 5.54851 | .19831 | 5.04267 | 47 |
| 14 | .14470 | 6.91104 | .16256 | 6.15151 | .18053 | 5.53927 | .19861 | 5.03499 | 46 |
| 15 | .14499 | 6.89688 | .16286 | 6.14023 | .18083 | 5.53007 | .19891 | 5.02734 | 45 |
| 16 | .14529 | 6.88278 | .16316 | 6.12899 | .18113 | 5.52090 | .19921 | 5.01971 | 44 |
| 17 | .14559 | 6.86874 | .16346 | 6.11779 | .18143 | 5.51176 | .19952 | 5.01210 | 43 |
| 18 | .14588 | 6.85475 | .16376 | 6.10664 | .18173 | 5.50264 | .19982 | 5.00451 | 42 |
| 19 | .14618 | 6.84082 | .16405 | 6.09552 | .18203 | 5.49356 | .20012 | 4.99695 | 41 |
| 20 | .14648 | 6.82694 | .16435 | 6.08444 | .18233 | 5.48451 | .20042 | 4.98940 | 40 |
| 21 | .14678 | 6.81312 | .16465 | 6.07340 | .18263 | 5.47548 | .20073 | 4.98188 | 39 |
| 22 | .14707 | 6.79936 | .16495 | 6.06240 | .18293 | 5.46648 | .20103 | 4.97438 | 38 |
| 23 | .14737 | 6.78564 | .16525 | 6.05143 | .18323 | 5.45751 | .20133 | 4.96690 | 37 |
| 24 | .14767 | 6.77199 | .16555 | 6.04051 | .18353 | 5.44857 | .20164 | 4.95945 | 36 |
| 25 | .14796 | 6.75838 | .16585 | 6.02963 | .18384 | 5.43966 | .20194 | 4.95201 | 35 |
| 26 | .14826 | 6.74483 | .16615 | 6.01878 | .18414 | 5.43077 | .20224 | 4.94460 | 34 |
| 27 | .14856 | 6.73133 | .16645 | 6.00797 | .18444 | 5.42192 | .20254 | 4.93721 | 33 |
| 28 | .14886 | 6.71789 | .16674 | 5.99720 | .18474 | 5.41309 | .20285 | 4.92984 | 32 |
| 29 | .14915 | 6.70450 | .16704 | 5.98646 | .18504 | 5.40429 | .20315 | 4.92249 | 31 |
| 30 | .14945 | 6.69116 | .16734 | 5.97576 | .18534 | 5.39552 | .20345 | 4.91516 | 30 |
| 31 | .14975 | 6.67787 | .16764 | 5.96510 | .18564 | 5.38677 | .20376 | 4.90785 | 29 |
| 32 | .15005 | 6.66463 | .16794 | 5.95448 | .18594 | 5.37805 | .20406 | 4.90056 | 28 |
| 33 | .15034 | 6.65144 | .16824 | 5.94390 | .18624 | 5.36936 | .20436 | 4.89330 | 27 |
| 34 | .15064 | 6.63831 | .16854 | 5.93335 | .18654 | 5.36070 | .20466 | 4.88605 | 26 |
| 35 | .15094 | 6.62523 | .16884 | 5.92283 | .18684 | 5.35206 | .20497 | 4.87882 | 25 |
| 36 | .15124 | 6.61219 | .16914 | 5.91236 | .18714 | 5.34345 | .20527 | 4.87162 | 24 |
| 37 | .15153 | 6.59921 | .16944 | 5.90191 | .18745 | 5.33487 | .20557 | 4.86444 | 23 |
| 38 | .15183 | 6.58627 | .16974 | 5.89151 | .18775 | 5.32631 | .20588 | 4.85727 | 22 |
| 39 | .15213 | 6.57339 | .17004 | 5.88114 | .18805 | 5.31778 | .20618 | 4.85013 | 21 |
| 40 | .15243 | 6.56055 | .17033 | 5.87080 | .18835 | 5.30928 | .20648 | 4.84300 | 20 |
| 41 | .15272 | 6.54777 | .17063 | 5.86051 | .18865 | 5.30080 | .20679 | 4.83590 | 19 |
| 42 | .15302 | 6.53503 | .17093 | 5.85024 | .18895 | 5.29235 | .20709 | 4.82882 | 18 |
| 43 | .15332 | 6.52234 | .17123 | 5.84001 | .18925 | 5.28393 | .20739 | 4.82175 | 17 |
| 44 | .15362 | 6.50970 | .17153 | 5.82982 | .18955 | 5.27553 | .20770 | 4.81471 | 16 |
| 45 | .15391 | 6.49710 | .17183 | 5.81966 | .18986 | 5.26715 | .20800 | 4.80769 | 15 |
| 46 | .15421 | 6.48456 | .17213 | 5.80953 | .19016 | 5.25880 | .20830 | 4.80068 | 14 |
| 47 | .15451 | 6.47206 | .17243 | 5.79944 | .19046 | 5.25048 | .20861 | 4.79370 | 13 |
| 48 | .15481 | 6.45961 | .17273 | 5.78938 | .19076 | 5.24218 | .20891 | 4.78673 | 12 |
| 49 | .15511 | 6.44720 | .17303 | 5.77936 | .19106 | 5.23391 | .20921 | 4.77978 | 11 |
| 50 | .15540 | 6.43484 | .17333 | 5.76937 | .19136 | 5.22566 | .20952 | 4.77286 | 10 |
| 51 | .15570 | 6.42253 | .17363 | 5.75941 | .19166 | 5.21744 | .20982 | 4.76595 | 9 |
| 52 | .15600 | 6.41026 | .17393 | 5.74949 | .19197 | 5.20925 | .21013 | 4.75906 | 8 |
| 53 | .15630 | 6.39804 | .17423 | 5.73960 | .19227 | 5.20107 | .21043 | 4.75219 | 7 |
| 54 | .15660 | 6.38587 | .17453 | 5.72974 | .19257 | 5.19293 | .21073 | 4.74534 | 6 |
| 55 | .15689 | 6.37374 | .17483 | 5.71992 | .19287 | 5.18480 | .21104 | 4.73851 | 5 |
| 56 | .15719 | 6.36165 | .17513 | 5.71013 | .19317 | 5.17671 | .21134 | 4.73170 | 4 |
| 57 | .15749 | 6.34961 | .17543 | 5.70037 | .19347 | 5.16863 | .21164 | 4.72490 | 3 |
| 58 | .15779 | 6.33761 | .17573 | 5.69064 | .19378 | 5.16058 | .21195 | 4.71813 | 2 |
| 59 | .15809 | 6.32566 | .17603 | 5.68094 | .19408 | 5.15256 | .21225 | 4.71137 | 1 |
| 60 | .15838 | 6.31375 | .17633 | 5.67128 | .19438 | 5.14455 | .21256 | 4.70463 | 0 |
| ′ | Cotang | Tang | Cotang | Tang | Cotang | Tang | Cotang | Tang | ′ |
| | 81° | | 80° | | 79° | | 78° | | |

## Table III.—Natural Tangents and Cotangents.

| ′ | 12° | | 13° | | 14° | | 15° | | ′ |
|---|---|---|---|---|---|---|---|---|---|
| | Tang | Cotang | Tang | Cotang | Tang | Cotang | Tang | Cotang | |
| 0 | .21256 | 4.70463 | .23087 | 4.33148 | .24933 | 4.01078 | .26795 | 3.73205 | 60 |
| 1 | .21286 | 4.69791 | .23117 | 4.32573 | .24964 | 4.00582 | .26826 | 3.72771 | 59 |
| 2 | .21316 | 4.69121 | .23148 | 4.32001 | .24995 | 4.00086 | .26857 | 3.72338 | 58 |
| 3 | .21347 | 4.68452 | .23179 | 4.31430 | .25026 | 3.99592 | .26888 | 3.71907 | 57 |
| 4 | .21377 | 4.67786 | .23209 | 4.30860 | .25056 | 3.99099 | .26920 | 3.71476 | 56 |
| 5 | .21408 | 4.67121 | .23240 | 4.30291 | .25087 | 3.98607 | .26951 | 3.71046 | 55 |
| 6 | .21438 | 4.66458 | .23271 | 4.29724 | .25118 | 3.98117 | .26982 | 3.70616 | 54 |
| 7 | .21469 | 4.65797 | .23301 | 4.29159 | .25149 | 3.97627 | .27013 | 3.70188 | 53 |
| 8 | .21499 | 4.65138 | .23332 | 4.28595 | .25180 | 3.97139 | .27044 | 3.69761 | 52 |
| 9 | .21529 | 4.64480 | .23363 | 4.28032 | .25211 | 3.96651 | .27076 | 3.69335 | 51 |
| 10 | .21560 | 4.63825 | .23393 | 4.27471 | .25242 | 3.96165 | .27107 | 3.68909 | 50 |
| 11 | .21590 | 4.63171 | .23424 | 4.26911 | .25273 | 3.95680 | .27138 | 3.68485 | 49 |
| 12 | .21621 | 4.62518 | .23455 | 4.26352 | .25304 | 3.95196 | .27169 | 3.68061 | 48 |
| 13 | .21651 | 4.61868 | .23485 | 4.25795 | .25335 | 3.94713 | .27201 | 3.67638 | 47 |
| 14 | .21682 | 4.61219 | .23516 | 4.25239 | .25366 | 3.94232 | .27232 | 3.67217 | 46 |
| 15 | .21712 | 4.60572 | .23547 | 4.24685 | .25397 | 3.93751 | .27263 | 3.66796 | 45 |
| 16 | .21743 | 4.59927 | .23578 | 4.24132 | .25428 | 3.93271 | .27294 | 3.66376 | 44 |
| 17 | .21773 | 4.59283 | .23608 | 4.23580 | .25459 | 3.92793 | .27326 | 3.65957 | 43 |
| 18 | .21804 | 4.58641 | .23639 | 4.23030 | .25490 | 3.92316 | .27357 | 3.65538 | 42 |
| 19 | .21834 | 4.58001 | .23670 | 4.22481 | .25521 | 3.91839 | .27388 | 3.65121 | 41 |
| 20 | .21864 | 4.57363 | .23700 | 4.21933 | .25552 | 3.91364 | .27419 | 3.64705 | 40 |
| 21 | .21895 | 4.56726 | .23731 | 4.21387 | .25583 | 3.90890 | .27451 | 3.64289 | 39 |
| 22 | .21925 | 4.56091 | .23762 | 4.20842 | .25614 | 3.90417 | .27482 | 3.63874 | 38 |
| 23 | .21956 | 4.55458 | .23793 | 4.20298 | .25645 | 3.89945 | .27513 | 3.63461 | 37 |
| 24 | .21986 | 4.54826 | .23823 | 4.19756 | .25676 | 3.89474 | .27545 | 3.63048 | 36 |
| 25 | .22017 | 4.54196 | .23854 | 4.19215 | .25707 | 3.89004 | .27576 | 3.62636 | 35 |
| 26 | .22047 | 4.53568 | .23885 | 4.18675 | .25738 | 3.88536 | .27607 | 3.62224 | 34 |
| 27 | .22078 | 4.52941 | .23916 | 4.18137 | .25769 | 3.88068 | .27638 | 3.61814 | 33 |
| 28 | .22108 | 4.52316 | .23946 | 4.17600 | .25800 | 3.87601 | .27670 | 3.61405 | 32 |
| 29 | .22139 | 4.51693 | .23977 | 4.17064 | .25831 | 3.87136 | .27701 | 3.60996 | 31 |
| 30 | .22169 | 4.51071 | .24008 | 4.16530 | .25862 | 3.86671 | .27732 | 3.60588 | 30 |
| 31 | .22200 | 4.50451 | .24039 | 4.15997 | .25893 | 3.86208 | .27764 | 3.60181 | 29 |
| 32 | .22231 | 4.49832 | .24069 | 4.15465 | .25924 | 3.85745 | .27795 | 3.59775 | 28 |
| 33 | .22261 | 4.49215 | .24100 | 4.14934 | .25955 | 3.85284 | .27826 | 3.59370 | 27 |
| 34 | .22292 | 4.48600 | .24131 | 4.14405 | .25986 | 3.84824 | .27858 | 3.58966 | 26 |
| 35 | .22323 | 4.47986 | .24162 | 4.13877 | .26017 | 3.84364 | .27889 | 3.58562 | 25 |
| 36 | .22353 | 4.47374 | .24193 | 4.13350 | .26048 | 3.83906 | .27921 | 3.58160 | 24 |
| 37 | .22383 | 4.46764 | .24223 | 4.12825 | .26079 | 3.83449 | .27952 | 3.57758 | 23 |
| 38 | .22414 | 4.46155 | .24254 | 4.12301 | .26110 | 3.82992 | .27983 | 3.57357 | 22 |
| 39 | .22444 | 4.45548 | .24285 | 4.11778 | .26141 | 3.82537 | .28015 | 3.56957 | 21 |
| 40 | .22475 | 4.44942 | .24316 | 4.11256 | .26172 | 3.82083 | .28046 | 3.56557 | 20 |
| 41 | .22505 | 4.44338 | .24347 | 4.10736 | .26203 | 3.81630 | .28077 | 3.56159 | 19 |
| 42 | .22536 | 4.43735 | .24377 | 4.10216 | .26235 | 3.81177 | .28109 | 3.55761 | 18 |
| 43 | .22567 | 4.43134 | .24408 | 4.09699 | .26266 | 3.80726 | .28140 | 3.55364 | 17 |
| 44 | .22597 | 4.42534 | .24439 | 4.09182 | .26297 | 3.80276 | .28172 | 3.54968 | 16 |
| 45 | .22628 | 4.41936 | .24470 | 4.08666 | .26328 | 3.79827 | .28203 | 3.54573 | 15 |
| 46 | .22658 | 4.41340 | .24501 | 4.08152 | .26359 | 3.79378 | .28234 | 3.54179 | 14 |
| 47 | .22689 | 4.40745 | .24532 | 4.07639 | .26390 | 3.78931 | .28266 | 3.53785 | 13 |
| 48 | .22719 | 4.40152 | .24562 | 4.07127 | .26421 | 3.78485 | .28297 | 3.53393 | 12 |
| 49 | .22750 | 4.39560 | .24593 | 4.06616 | .26452 | 3.78040 | .28329 | 3.53001 | 11 |
| 50 | .22781 | 4.38969 | .24624 | 4.06107 | .26483 | 3.77595 | .28360 | 3.52609 | 10 |
| 51 | .22811 | 4.38381 | .24655 | 4.05599 | .26515 | 3.77152 | .28391 | 3.52219 | 9 |
| 52 | .22842 | 4.37793 | .24686 | 4.05092 | .26546 | 3.76709 | .28423 | 3.51829 | 8 |
| 53 | .22872 | 4.37207 | .24717 | 4.04586 | .26577 | 3.76268 | .28454 | 3.51441 | 7 |
| 54 | .22903 | 4.36623 | .24747 | 4.04081 | .26608 | 3.75828 | .28486 | 3.51053 | 6 |
| 55 | .22934 | 4.36040 | .24778 | 4.03578 | .26639 | 3.75388 | .28517 | 3.50666 | 5 |
| 56 | .22964 | 4.35459 | .24809 | 4.03076 | .26670 | 3.74950 | .28549 | 3.50279 | 4 |
| 57 | .22995 | 4.34879 | .24840 | 4.02574 | .26701 | 3.74512 | .28580 | 3.49894 | 3 |
| 58 | .23026 | 4.34300 | .24871 | 4.02074 | .26733 | 3.74075 | .28612 | 3.49509 | 2 |
| 59 | .23056 | 4.33723 | .24902 | 4.01576 | .26764 | 3.73640 | .28643 | 3.49125 | 1 |
| 60 | .23087 | 4.33148 | .24933 | 4.01078 | .26795 | 3.73205 | .28675 | 3.48741 | 0 |
| ′ | Cotang | Tang | Cotang | Tang | Cotang | Tang | Cotang | Tang | ′ |
| | 77° | | 76° | | 75° | | 74° | | |

| ′ | 16° | | 17° | | 18° | | 19° | | ′ |
|---|---|---|---|---|---|---|---|---|---|
| | Tang | Cotang | Tang | Cotang | Tang | Cotang | Tang | Cotang | |
| 0 | .28675 | 3.48741 | .30573 | 3.27085 | .32492 | 3.07768 | .34433 | 2.90421 | 60 |
| 1 | .28706 | 3.48359 | .30605 | 3.26745 | .32524 | 3.07464 | .34465 | 2.90147 | 59 |
| 2 | .28738 | 3.47977 | .30637 | 3.26406 | .32556 | 3.07160 | .34498 | 2.89878 | 58 |
| 3 | .28769 | 3.47596 | .30669 | 3.26067 | .32588 | 3.06857 | .34530 | 2.89600 | 57 |
| 4 | .28800 | 3.47216 | .30700 | 3.25729 | .32621 | 3.06554 | .34563 | 2.89327 | 56 |
| 5 | .28832 | 3.46837 | .30732 | 3.25392 | .32653 | 3.06252 | .34596 | 2.89055 | 55 |
| 6 | .28864 | 3.46458 | .30764 | 3.25055 | .32685 | 3.05950 | .34628 | 2.88783 | 54 |
| 7 | .28895 | 3.46080 | .30796 | 3.24719 | .32717 | 3.05649 | .34661 | 2.88511 | 53 |
| 8 | .28927 | 3.45703 | .30828 | 3.24383 | .32749 | 3.05349 | .34693 | 2.88240 | 52 |
| 9 | .28958 | 3.45327 | .30860 | 3.24049 | .32782 | 3.05049 | .34726 | 2.87970 | 51 |
| 10 | .28990 | 3.44951 | .30891 | 3.23714 | .32814 | 3.04749 | .34758 | 2.87700 | 50 |
| 11 | .29021 | 3.44576 | .30923 | 3.23381 | .32846 | 3.04450 | .34791 | 2.87430 | 49 |
| 12 | .29053 | 3.44202 | .30955 | 3.23048 | .32878 | 3.04152 | .34824 | 2.87161 | 48 |
| 13 | .29084 | 3.43829 | .30987 | 3.22715 | .32911 | 3.03854 | .34856 | 2.86892 | 47 |
| 14 | .29116 | 3.43456 | .31019 | 3.22384 | .32943 | 3.03556 | .34889 | 2.86624 | 46 |
| 15 | .29147 | 3.43084 | .31051 | 3.22053 | .32975 | 3.03260 | .34922 | 2.86356 | 45 |
| 16 | .29179 | 3.42713 | .31083 | 3.21722 | .33007 | 3.02963 | .34954 | 2.86089 | 44 |
| 17 | .29210 | 3.42343 | .31115 | 3.21392 | .33040 | 3.02667 | .34987 | 2.85822 | 43 |
| 18 | .29242 | 3.41973 | .31147 | 3.21063 | .33072 | 3.02372 | .35020 | 2.85555 | 42 |
| 19 | .29274 | 3.41604 | .31178 | 3.20734 | .33104 | 3.02077 | .35052 | 2.85289 | 41 |
| 20 | .29305 | 3.41236 | .31210 | 3.20406 | .33136 | 3.01783 | .35085 | 2.85023 | 40 |
| 21 | .29337 | 3.40869 | .31242 | 3.20079 | .33169 | 3.01489 | .35118 | 2.84758 | 39 |
| 22 | .29368 | 3.40502 | .31274 | 3.19752 | .33201 | 3.01196 | .35150 | 2.84494 | 38 |
| 23 | .29400 | 3.40136 | .31306 | 3.19426 | .33233 | 3.00903 | .35183 | 2.84229 | 37 |
| 24 | .29432 | 3.39771 | .31338 | 3.19100 | .33266 | 3.00611 | .35216 | 2.83965 | 36 |
| 25 | .29463 | 3.39406 | .31370 | 3.18775 | .33298 | 3.00319 | .35248 | 2.83702 | 35 |
| 26 | .29495 | 3.39042 | .31402 | 3.18451 | .33330 | 3.00028 | .35281 | 2.83439 | 34 |
| 27 | .29526 | 3.38679 | .31434 | 3.18127 | .33363 | 2.99738 | .35314 | 2.83176 | 33 |
| 28 | .29558 | 3.38317 | .31466 | 3.17804 | .33395 | 2.99447 | .35346 | 2.82914 | 32 |
| 29 | .29590 | 3.37955 | .31498 | 3.17481 | .33427 | 2.99158 | .35379 | 2.82653 | 31 |
| 30 | .29621 | 3.37594 | .31530 | 3.17159 | .33460 | 2.98868 | .35413 | 2.82391 | 30 |
| 31 | .29653 | 3.37234 | .31562 | 3.16838 | .33492 | 2.98580 | .35445 | 2.82130 | 29 |
| 32 | .29685 | 3.36875 | .31594 | 3.16517 | .33524 | 2.98292 | .35477 | 2.81870 | 28 |
| 33 | .29716 | 3.36516 | .31626 | 3.16197 | .33557 | 2.98004 | .35510 | 2.81610 | 27 |
| 34 | .29748 | 3.36158 | .31658 | 3.15877 | .33589 | 2.97717 | .35543 | 2.81350 | 26 |
| 35 | .29780 | 3.35800 | .31690 | 3.15558 | .33621 | 2.97430 | .35576 | 2.81091 | 25 |
| 36 | .29811 | 3.35443 | .31722 | 3.15240 | .33654 | 2.97144 | .35608 | 2.80833 | 24 |
| 37 | .29843 | 3.35087 | .31754 | 3.14922 | .33686 | 2.96858 | .35641 | 2.80574 | 23 |
| 38 | .29875 | 3.34732 | .31786 | 3.14605 | .33718 | 2.96573 | .35674 | 2.80316 | 22 |
| 39 | .29906 | 3.34377 | .31818 | 3.14288 | .33751 | 2.96288 | .35707 | 2.80059 | 21 |
| 40 | .29938 | 3.34023 | .31850 | 3.13972 | .33783 | 2.96004 | .35740 | 2.79802 | 20 |
| 41 | .29970 | 3.33670 | .31882 | 3.13656 | .33816 | 2.95721 | .35772 | 2.79545 | 19 |
| 42 | .30001 | 3.33317 | .31914 | 3.13341 | .33848 | 2.95437 | .35805 | 2.79289 | 18 |
| 43 | .30033 | 3.32965 | .31946 | 3.13027 | .33881 | 2.95155 | .35838 | 2.79033 | 17 |
| 44 | .30065 | 3.32614 | .31978 | 3.12713 | .33913 | 2.94872 | .35871 | 2.78778 | 16 |
| 45 | .30097 | 3.32264 | .32010 | 3.12400 | .33945 | 2.94591 | .35904 | 2.78523 | 15 |
| 46 | .30128 | 3.31914 | .32042 | 3.12087 | .33978 | 2.94309 | .35937 | 2.78269 | 14 |
| 47 | .30160 | 3.31565 | .32074 | 3.11775 | .34010 | 2.94028 | .35969 | 2.78014 | 13 |
| 48 | .30192 | 3.31216 | .32106 | 3.11464 | .34043 | 2.93748 | .36002 | 2.77761 | 12 |
| 49 | .30224 | 3.30868 | .32139 | 3.11153 | .34075 | 2.93468 | .36035 | 2.77507 | 11 |
| 50 | .30255 | 3.30521 | .32171 | 3.10842 | .34108 | 2.93189 | .36068 | 2.77254 | 10 |
| 51 | .30287 | 3.30174 | .32203 | 3.10532 | .34140 | 2.92910 | .36101 | 2.77002 | 9 |
| 52 | .30319 | 3.29829 | .32235 | 3.10223 | .34173 | 2.92632 | .36134 | 2.76750 | 8 |
| 53 | .30351 | 3.29483 | .32267 | 3.09914 | .34205 | 2.92354 | .36167 | 2.76498 | 7 |
| 54 | .30382 | 3.29139 | .32299 | 3.09606 | .34238 | 2.92076 | .36199 | 2.76247 | 6 |
| 55 | .30414 | 3.28795 | .32331 | 3.09298 | .34270 | 2.91799 | .36232 | 2.75996 | 5 |
| 56 | .30446 | 3.28452 | .32363 | 3.08991 | .34303 | 2.91523 | .36265 | 2.75746 | 4 |
| 57 | .30478 | 3.28109 | .32396 | 3.08685 | .34335 | 2.91246 | .36298 | 2.75496 | 3 |
| 58 | .30509 | 3.27767 | .32428 | 3.08379 | .34368 | 2.90971 | .36331 | 2.75246 | 2 |
| 59 | .30541 | 3.27426 | .32460 | 3.08073 | .34400 | 2.90696 | .36364 | 2.74997 | 1 |
| 60 | .30573 | 3.27085 | .32492 | 3.07768 | .34433 | 2.90421 | .36397 | 2.74748 | 0 |
| ′ | Cotang | Tang | Cotang | Tang | Cotang | Tang | Cotang | Tang | |
| | 73° | | 72° | | 71° | | 70° | | |

# Table III.—Natural Tangents and Cotangents.

| ′ | 20° | | 21° | | 22° | | 23° | | ′ |
|---|---|---|---|---|---|---|---|---|---|
| | Tang | Cotang | Tang | Cotang | Tang | Cotang | Tang | Cotang | |
| 0 | .36397 | 2.74748 | .38386 | 2.60509 | .40403 | 2.47509 | .42447 | 2.35585 | 60 |
| 1 | .36420 | 2.74499 | .38420 | 2.60283 | .40436 | 2.47302 | .42482 | 2.35395 | 59 |
| 2 | .36463 | 2.74251 | .38453 | 2.60057 | .40470 | 2.47095 | .42516 | 2.35205 | 58 |
| 3 | .36496 | 2.74004 | .38487 | 2.59831 | .40504 | 2.46888 | .42551 | 2.35015 | 57 |
| 4 | .36529 | 2.73756 | .38520 | 2.59606 | .40538 | 2.46682 | .42585 | 2.34825 | 56 |
| 5 | .36562 | 2.73509 | .38553 | 2.59381 | .40572 | 2.46476 | .42619 | 2.34636 | 55 |
| 6 | .36595 | 2.73263 | .38587 | 2.59156 | .40606 | 2.46270 | .42654 | 2.34447 | 54 |
| 7 | .36628 | 2.73017 | .38620 | 2.58932 | .40640 | 2.46065 | .42688 | 2.34258 | 53 |
| 8 | .36661 | 2.72771 | .38654 | 2.58708 | .40674 | 2.45860 | .42722 | 2.34069 | 52 |
| 9 | .36694 | 2.72526 | .38687 | 2.58484 | .40707 | 2.45655 | .42757 | 2.33881 | 51 |
| 10 | .36727 | 2.72281 | .38721 | 2.58261 | .40741 | 2.45451 | .42791 | 2.33693 | 50 |
| 11 | .36760 | 2.72036 | .38754 | 2.58038 | .40775 | 2.45246 | .42826 | 2.33505 | 49 |
| 12 | .36793 | 2.71792 | .38787 | 2.57815 | .40809 | 2.45043 | .42860 | 2.33317 | 48 |
| 13 | .36826 | 2.71548 | .38821 | 2.57593 | .40843 | 2.44839 | .42894 | 2.33130 | 47 |
| 14 | .36859 | 2.71305 | .38854 | 2.57371 | .40877 | 2.44636 | .42929 | 2.32943 | 46 |
| 15 | .36892 | 2.71062 | .38888 | 2.57150 | .40911 | 2.44433 | .42963 | 2.32756 | 45 |
| 16 | .36925 | 2.70819 | .38921 | 2.56928 | .40945 | 2.44230 | .42998 | 2.32570 | 44 |
| 17 | .36958 | 2.70577 | .38955 | 2.56707 | .40979 | 2.44027 | .43032 | 2.32383 | 43 |
| 18 | .36991 | 2.70335 | .38988 | 2.56487 | .41013 | 2.43825 | .43067 | 2.32197 | 42 |
| 19 | .37024 | 2.70094 | .39022 | 2.56266 | .41047 | 2.43623 | .43101 | 2.32012 | 41 |
| 20 | .37057 | 2.69853 | .39055 | 2.56046 | .41081 | 2.43422 | .43136 | 2.31826 | 40 |
| 21 | .37090 | 2.69612 | .39089 | 2.55827 | .41115 | 2.43220 | .43170 | 2.31641 | 39 |
| 22 | .37123 | 2.69371 | .39122 | 2.55608 | .41149 | 2.43019 | .43205 | 2.31456 | 38 |
| 23 | .37157 | 2.69131 | .39156 | 2.55389 | .41183 | 2.42819 | .43239 | 2.31271 | 37 |
| 24 | .37190 | 2.68892 | .39190 | 2.55170 | .41217 | 2.42618 | .43274 | 2.31086 | 36 |
| 25 | .37223 | 2.68653 | .39223 | 2.54952 | .41251 | 2.42418 | .43308 | 2.30902 | 35 |
| 26 | .37256 | 2.68414 | .39257 | 2.54734 | .41285 | 2.42218 | .43343 | 2.30718 | 34 |
| 27 | .37289 | 2.68175 | .39290 | 2.54516 | .41319 | 2.42019 | .43378 | 2.30534 | 33 |
| 28 | .37322 | 2.67937 | .39324 | 2.54299 | .41353 | 2.41819 | .43412 | 2.30351 | 32 |
| 29 | .37355 | 2.67700 | .39357 | 2.54082 | .41387 | 2.41020 | .43447 | 2.30167 | 31 |
| 30 | .37388 | 2.67462 | .39291 | 2.53865 | .41421 | 2.41421 | .43481 | 2.29984 | 30 |
| 31 | .37422 | 2.67225 | .39425 | 2.53648 | .41455 | 2.41222 | .43516 | 2.29801 | 29 |
| 32 | .37455 | 2.66989 | .39458 | 2.53432 | .41490 | 2.41025 | .43550 | 2.29619 | 28 |
| 33 | .37488 | 2.66752 | .39492 | 2.53217 | .41524 | 2.40827 | .43585 | 2.29437 | 27 |
| 34 | .37521 | 2.66516 | .39526 | 2.53001 | .41558 | 2.40629 | .43620 | 2.29254 | 26 |
| 35 | .37554 | 2.66281 | .39559 | 2.52786 | .41592 | 2.40433 | .43654 | 2.29073 | 25 |
| 36 | .37588 | 2.66046 | .39593 | 2.52571 | .41626 | 2.40235 | .43689 | 2.28891 | 24 |
| 37 | .37621 | 2.65811 | .39626 | 2.52357 | .41660 | 2.40038 | .43724 | 2.28710 | 23 |
| 38 | .37654 | 2.65576 | .39660 | 2.52143 | .41694 | 2.39841 | .43758 | 2.28528 | 22 |
| 39 | .37687 | 2.65342 | .39694 | 2.519.9 | .41728 | 2.39645 | .43793 | 2.28348 | 21 |
| 40 | .37720 | 2.65109 | .39727 | 2.51715 | .41763 | 2.39449 | .43828 | 2.28167 | 20 |
| 41 | .37754 | 2.64875 | .39761 | 2.51502 | .41797 | 2.39253 | .43862 | 2.27987 | 19 |
| 42 | .37787 | 2.64642 | .39795 | 2.51289 | .41831 | 2.39058 | .43897 | 2.27806 | 18 |
| 43 | .37820 | 2.64410 | .39829 | 2.51076 | .41865 | 2.38863 | .43932 | 2.27626 | 17 |
| 44 | .37853 | 2.64177 | .39862 | 2.50864 | 41899 | 2.38668 | .43966 | 2.27447 | 16 |
| 45 | .37887 | 2.63945 | .39896 | 2.50652 | .41933 | 2.38473 | .44001 | 2.27267 | 15 |
| 46 | .37920 | 2.63714 | .39930 | 2.50440 | .41968 | 2.38279 | .44036 | 2.27088 | 14 |
| 47 | .37953 | 2.63483 | .39963 | 2.50229 | .42002 | 2.38084 | .44071 | 2.26909 | 13 |
| 48 | .37986 | 2.63252 | .39997 | 2.50018 | .42036 | 2.37891 | .44105 | 2.26730 | 12 |
| 49 | .38020 | 2.63021 | .40031 | 2.49807 | .42070 | 2.37697 | .44140 | 2.26552 | 11 |
| 50 | .38053 | 2.62791 | .40065 | 2.49597 | .42105 | 2.37504 | .44175 | 2.26374 | 10 |
| 51 | .38086 | 2.62561 | .40098 | 2.49386 | .42139 | 2.37311 | .44210 | 2.26196 | 9 |
| 52 | .38120 | 2.62332 | .40132 | 2.49177 | .42173 | 2.37118 | .44244 | 2.26018 | 8 |
| 53 | .38153 | 2.62103 | .40166 | 2.48967 | .42207 | 2.36925 | .44279 | 2.25840 | 7 |
| 54 | .38186 | 2.61874 | .40200 | 2.48758 | .42242 | 2.36733 | .44314 | 2.25663 | 6 |
| 55 | .38220 | 2.61646 | .40234 | 2.48549 | .42276 | 2.36541 | .44349 | 2.25486 | 5 |
| 56 | .38253 | 2.61418 | .40267 | 2 48340 | .42310 | 2.36349 | .44384 | 2.25309 | 4 |
| 57 | .38286 | 2.61190 | .40301 | 2.48132 | .42345 | 2.36158 | .44418 | 2.25132 | 3 |
| 58 | .38320 | 2.60963 | .40335 | 2.47924 | .42379 | 2.35967 | .44453 | 2.24956 | 2 |
| 59 | .38353 | 2.60736 | .40369 | 2.47716 | .42413 | 2.35776 | .44488 | 2 24780 | 1 |
| 60 | .38386 | 2.60509 | .40403 | 2.47509 | .42447 | 2.35585 | .44523 | 2.24604 | 0 |
| ′ | Cotang | Tang | Cotang | Tang | Cotang | Tang | Cotang | Tang | ′ |
| | 69° | | 68° | | 67° | | 66° | | |

## TABLE III.—NATURAL TANGENTS AND COTANGENTS.

| ′ | 24° | | 25° | | 26° | | 27° | | ′ |
|---|---|---|---|---|---|---|---|---|---|
| | Tang | Cotang | Tang | Cotang | Tang | Cotang | Tang | Cotang | |
| 0 | .44523 | 2.24604 | .46631 | 2.14451 | .48773 | 2.05030 | .50953 | 1.96261 | 60 |
| 1 | .44558 | 2.24428 | .46666 | 2.14288 | .48809 | 2.04879 | .50989 | 1.96120 | 59 |
| 2 | .44593 | 2.24252 | .46702 | 2.14125 | .48845 | 2.04728 | .51026 | 1.95979 | 58 |
| 3 | .44627 | 2.24077 | .46737 | 2.13963 | .48881 | 2.04577 | .51063 | 1.95838 | 57 |
| 4 | .44662 | 2.23902 | .46772 | 2.13801 | .48917 | 2.04426 | .51099 | 1.95698 | 56 |
| 5 | .44697 | 2.23727 | .46808 | 2.13639 | .48953 | 2.04276 | .51136 | 1.95557 | 55 |
| 6 | .44732 | 2.23553 | .46843 | 2.13477 | .48989 | 2.04125 | .51173 | 1.95417 | 54 |
| 7 | .44767 | 2.23378 | .46879 | 2.13316 | .49026 | 2.03975 | .51209 | 1.95277 | 53 |
| 8 | .44802 | 2.23204 | .46914 | 2.13154 | .49062 | 2.03825 | .51246 | 1.95137 | 52 |
| 9 | .44837 | 2.23030 | .46950 | 2.12993 | .49098 | 2.03675 | .51283 | 1.94997 | 51 |
| 10 | .44872 | 2.22857 | .46985 | 2.12832 | .49134 | 2.03526 | .51319 | 1.94858 | 50 |
| 11 | .44907 | 2.22683 | .47021 | 2.12671 | .49170 | 2.03376 | .51356 | 1.94718 | 49 |
| 12 | .44942 | 2.22510 | .47056 | 2.12511 | .49206 | 2.03227 | .51393 | 1.94579 | 48 |
| 13 | .44977 | 2.22337 | .47092 | 2.12350 | .49242 | 2.03078 | .51430 | 1.94440 | 47 |
| 14 | .45012 | 2.22164 | .47128 | 2.12190 | .49278 | 2.02929 | .51467 | 1.94301 | 46 |
| 15 | .45047 | 2.21992 | .47163 | 2.12030 | .49315 | 2.02780 | .51503 | 1.94162 | 45 |
| 16 | .45082 | 2.21819 | .47199 | 2.11871 | .49351 | 2.02631 | .51540 | 1.94023 | 44 |
| 17 | .45117 | 2.21647 | .47234 | 2.11711 | .49387 | 2.02483 | .51577 | 1.93885 | 43 |
| 18 | .45152 | 2.21475 | .47270 | 2.11552 | .49423 | 2.02335 | .51614 | 1.93746 | 42 |
| 19 | .45187 | 2.21304 | .47305 | 2.11392 | .49459 | 2.02187 | .51651 | 1.93608 | 41 |
| 20 | .45222 | 2.21132 | .47341 | 2.11233 | .49495 | 2.02039 | .51688 | 1.93470 | 40 |
| 21 | .45257 | 2.20961 | .47377 | 2.11075 | .49532 | 2.01891 | .51724 | 1.93332 | 39 |
| 22 | .45292 | 2.20790 | .47412 | 2.10916 | .49568 | 2.01743 | .51761 | 1.93195 | 38 |
| 23 | .45327 | 2.20619 | .47448 | 2.10758 | .49604 | 2.01596 | .51798 | 1.93057 | 37 |
| 24 | .45362 | 2.20449 | .47488 | 2.10600 | .49640 | 2.01449 | .51835 | 1.92920 | 36 |
| 25 | .45397 | 2.20278 | .47519 | 2.10442 | .49677 | 2.01302 | .51872 | 1.92782 | 35 |
| 26 | .45432 | 2.20108 | .47555 | 2.10284 | .49713 | 2.01155 | .51909 | 1.92645 | 34 |
| 27 | .45467 | 2.19938 | .47590 | 2.10126 | .49749 | 2.01008 | .51946 | 1.92508 | 33 |
| 28 | .45502 | 2.19769 | .47626 | 2.09969 | .49786 | 2.00862 | .51983 | 1.92371 | 32 |
| 29 | .45538 | 2.19599 | .47662 | 2.09811 | .49822 | 2.00715 | .52020 | 1.92235 | 31 |
| 30 | .45573 | 2.19430 | .47698 | 2.09654 | .49858 | 2.00569 | .52057 | 1.92098 | 30 |
| 31 | .45608 | 2.19261 | .47733 | 2.09498 | .49894 | 2.00423 | .52094 | 1.91962 | 29 |
| 32 | .45643 | 2.19092 | .47769 | 2.09341 | .49931 | 2.00277 | .52131 | 1.91826 | 28 |
| 33 | .45678 | 2.18923 | .47805 | 2.09184 | .49967 | 2.00131 | .52168 | 1.91690 | 27 |
| 34 | .45713 | 2.18755 | .47840 | 2.09028 | .50004 | 1.99986 | .52205 | 1.91554 | 26 |
| 35 | .45748 | 2.18587 | .47876 | 2.08872 | .50040 | 1.99841 | .52242 | 1.91418 | 25 |
| 36 | .45784 | 2.18419 | .47912 | 2.08716 | .50076 | 1.99695 | .52279 | 1.91282 | 24 |
| 37 | .45819 | 2.18251 | .47948 | 2.08560 | .50113 | 1.99550 | .52316 | 1.91147 | 23 |
| 38 | .45854 | 2.18084 | .47984 | 2.08405 | .50149 | 1.99406 | .52353 | 1.91012 | 22 |
| 39 | .45889 | 2.17916 | .48019 | 2.08250 | .50185 | 1.99261 | .52390 | 1.90876 | 21 |
| 40 | .45924 | 2.17749 | .48055 | 2.08094 | .50222 | 1.99116 | .52427 | 1.90741 | 20 |
| 41 | .45960 | 2.17582 | .48091 | 2.07939 | .50258 | 1.98972 | .52464 | 1.90607 | 19 |
| 42 | .45995 | 2.17416 | .48127 | 2.07785 | .50295 | 1.98828 | .52501 | 1.90472 | 18 |
| 43 | .46030 | 2.17249 | .48163 | 2.07630 | .50331 | 1.98684 | .52538 | 1.90337 | 17 |
| 44 | .46065 | 2.17083 | .48198 | 2.07476 | .50368 | 1.98540 | .52575 | 1.90203 | 16 |
| 45 | .46101 | 2.16917 | .48234 | 2.07321 | .50404 | 1.98396 | .52613 | 1.90069 | 15 |
| 46 | .46136 | 2.16751 | .48270 | 2.07167 | .50441 | 1.98253 | .52650 | 1.89935 | 14 |
| 47 | .46171 | 2.16585 | .48306 | 2.07014 | .50477 | 1.98110 | .52687 | 1.89801 | 13 |
| 48 | .46206 | 2.16420 | .48342 | 2.06860 | .50514 | 1.97966 | .52724 | 1.89667 | 12 |
| 49 | .46242 | 2.16255 | .48378 | 2.06706 | .50550 | 1.97823 | .52761 | 1.89533 | 11 |
| 50 | .46277 | 2.16090 | .48414 | 2.06553 | .50587 | 1.97681 | .52798 | 1.89400 | 10 |
| 51 | .46312 | 2.15925 | .48450 | 2.06400 | .50623 | 1.97538 | .52836 | 1.89266 | 9 |
| 52 | .46348 | 2.15760 | .48486 | 2.06247 | .50660 | 1.97395 | .52873 | 1.89133 | 8 |
| 53 | .46383 | 2.15596 | .48521 | 2.06094 | .50696 | 1.97253 | .52910 | 1.89000 | 7 |
| 54 | .46418 | 2.15432 | .48557 | 2.05942 | .50733 | 1.97111 | .52947 | 1.88867 | 6 |
| 55 | .46454 | 2.15268 | .48593 | 2.05790 | .50769 | 1.96969 | .52985 | 1.88734 | 5 |
| 56 | .46489 | 2.15104 | .48629 | 2.05637 | .50806 | 1.96827 | .53022 | 1.88602 | 4 |
| 57 | .46525 | 2.14940 | .48665 | 2.05485 | .50843 | 1.96685 | .53059 | 1.88469 | 3 |
| 58 | .46560 | 2.14777 | .48701 | 2.05333 | .50879 | 1.96544 | .53096 | 1.88337 | 2 |
| 59 | .46595 | 2.14614 | .48737 | 2.05182 | .50916 | 1.96402 | .53134 | 1.88205 | 1 |
| 60 | .46631 | 2.14451 | .48773 | 2.05030 | .50953 | 1.96261 | .53171 | 1.88073 | 0 |
| ′ | Cotang | Tang | Cotang | Tang | Cotang | Tang | Cotang | Tang | ′ |
| | 65° | | 64° | | 63° | | 62° | | |

# Table III.—Natural Tangents and Cotangents.

| ′ | 28° | | 29° | | 30° | | 31° | | ′ |
|---|---|---|---|---|---|---|---|---|---|
| | Tang | Cotang | Tang | Cotang | Tang | Cotang | Tang | Cotang | |
| 0 | .53171 | 1.88073 | .55431 | 1.80405 | .57735 | 1.73205 | .60086 | 1.66428 | 60 |
| 1 | .53208 | 1.87941 | .55469 | 1.80281 | .57774 | 1.73089 | .60126 | 1.66318 | 59 |
| 2 | .53246 | 1.87809 | .55507 | 1.80158 | .57813 | 1.72973 | .60165 | 1.66209 | 58 |
| 3 | .53283 | 1.87677 | .55545 | 1.80034 | .57851 | 1.72857 | .60205 | 1.66099 | 57 |
| 4 | .53320 | 1.87546 | .55583 | 1.79911 | .57890 | 1.72741 | .60245 | 1.65990 | 56 |
| 5 | .53358 | 1.87415 | .55621 | 1.79788 | .57929 | 1.72625 | .60284 | 1.65881 | 55 |
| 6 | .53395 | 1.87283 | .55659 | 1.79665 | .57968 | 1.72509 | .60324 | 1.65772 | 54 |
| 7 | .53432 | 1.87152 | .55697 | 1.79542 | .58007 | 1.72393 | .60364 | 1.65663 | 53 |
| 8 | .53470 | 1.87021 | .55736 | 1.79419 | .58046 | 1.72278 | .60403 | 1.65554 | 52 |
| 9 | .53507 | 1.86891 | .55774 | 1.79296 | .58085 | 1.72163 | .60443 | 1.65445 | 51 |
| 10 | .53545 | 1.86760 | .55812 | 1.79174 | .58124 | 1.72047 | .60483 | 1.65337 | 50 |
| 11 | .53582 | 1.86630 | .55850 | 1.79051 | .58162 | 1.71932 | .60522 | 1.65228 | 49 |
| 12 | .53620 | 1.86499 | .55888 | 1.78929 | .58201 | 1.71817 | .60562 | 1.65120 | 48 |
| 13 | .53657 | 1.86369 | .55926 | 1.78807 | .58240 | 1.71702 | .60602 | 1.65011 | 47 |
| 14 | .53694 | 1.86239 | .55964 | 1.78685 | .58279 | 1.71588 | .60642 | 1.64903 | 46 |
| 15 | .53732 | 1.86109 | .56003 | 1.78563 | .58318 | 1.71473 | .60681 | 1.64795 | 45 |
| 16 | .53769 | 1.85979 | .56041 | 1.78441 | .58357 | 1.71358 | .60721 | 1.64687 | 44 |
| 17 | .53807 | 1.85850 | .56079 | 1.78319 | .58396 | 1.71244 | .60761 | 1.64579 | 43 |
| 18 | .53844 | 1.85720 | .56117 | 1.78198 | .58435 | 1.71129 | .60801 | 1.64471 | 42 |
| 19 | .53882 | 1.85591 | .56156 | 1.78077 | .58474 | 1.71015 | .60841 | 1.64363 | 41 |
| 20 | .53920 | 1.85462 | .56194 | 1.77955 | .58513 | 1.70901 | .60881 | 1.64256 | 40 |
| 21 | .53957 | 1.85333 | .56232 | 1.77834 | .58552 | 1.70787 | .60921 | 1.64148 | 39 |
| 22 | .53995 | 1.85204 | .56270 | 1.77713 | .58591 | 1.70673 | .60960 | 1.64041 | 38 |
| 23 | .54032 | 1.85075 | .56309 | 1.77592 | .58631 | 1.70560 | .61000 | 1.63934 | 37 |
| 24 | .54070 | 1.84946 | .56347 | 1.77471 | .58670 | 1.70446 | .61040 | 1.63826 | 36 |
| 25 | .54107 | 1.84818 | .56385 | 1.77351 | .58709 | 1.70332 | .61080 | 1.63719 | 35 |
| 26 | .54145 | 1.84689 | .56424 | 1.77230 | .58748 | 1.70219 | .61120 | 1.63612 | 34 |
| 27 | .54183 | 1.84561 | .56462 | 1.77110 | .58787 | 1.70106 | .61160 | 1.63505 | 33 |
| 28 | .54220 | 1.84433 | .56501 | 1.76990 | .58826 | 1.69992 | .61200 | 1.63398 | 32 |
| 29 | .54258 | 1.84305 | .56539 | 1.76869 | .58865 | 1.69879 | .61240 | 1.63292 | 31 |
| 30 | .54296 | 1.84177 | .56577 | 1.76749 | .58905 | 1.69766 | .61280 | 1.63185 | 30 |
| 31 | .54333 | 1.84049 | .56616 | 1.76629 | .58944 | 1.69653 | .61320 | 1.63079 | 29 |
| 32 | .54371 | 1.83922 | .56654 | 1.76510 | .58983 | 1.69541 | .61360 | 1.62972 | 28 |
| 33 | .54409 | 1.83794 | .56693 | 1.76390 | .59022 | 1.69428 | .61400 | 1.62866 | 27 |
| 34 | .54446 | 1.83667 | .56731 | 1.76271 | .59061 | 1.69316 | .61440 | 1.62760 | 26 |
| 35 | .54484 | 1.83540 | .56769 | 1.76151 | .59101 | 1.69203 | .61480 | 1.62654 | 25 |
| 36 | .54522 | 1.83413 | .56808 | 1.76032 | .59140 | 1.69091 | .61520 | 1.62548 | 24 |
| 37 | .54560 | 1.83286 | .56846 | 1.75913 | .59179 | 1.68979 | .61561 | 1.62442 | 23 |
| 38 | .54597 | 1.83159 | .56885 | 1.75794 | .59218 | 1.68866 | .61601 | 1.62336 | 22 |
| 39 | .54635 | 1.83033 | .56923 | 1.75675 | .59258 | 1.68754 | .61641 | 1.62230 | 21 |
| 40 | .54673 | 1.82906 | .56962 | 1.75556 | .59297 | 1.68643 | .61681 | 1.62125 | 20 |
| 41 | .54711 | 1.82780 | .57000 | 1.75437 | .59336 | 1.68531 | .61721 | 1.62019 | 19 |
| 42 | .54748 | 1.82654 | .57039 | 1.75319 | .59376 | 1.68419 | .61761 | 1.61914 | 18 |
| 43 | .54786 | 1.82528 | .57078 | 1.75200 | .59415 | 1.68308 | .61801 | 1.61808 | 17 |
| 44 | .54824 | 1.82402 | .57116 | 1.75082 | .59454 | 1.68196 | .61842 | 1.61703 | 16 |
| 45 | .54862 | 1.82276 | .57155 | 1.74964 | .59494 | 1.68085 | .61882 | 1.61598 | 15 |
| 46 | .54900 | 1.82150 | .57193 | 1.74846 | .59533 | 1.67974 | .61922 | 1.61493 | 14 |
| 47 | .54938 | 1.82025 | .57232 | 1.74728 | .59573 | 1.67863 | .61962 | 1.61388 | 13 |
| 48 | .54975 | 1.81899 | .57271 | 1.74610 | .59612 | 1.67752 | .62003 | 1.61283 | 12 |
| 49 | .55013 | 1.81774 | .57309 | 1.74492 | .59651 | 1.67641 | .62043 | 1.61179 | 11 |
| 50 | .55051 | 1.81649 | .57348 | 1.74375 | .59691 | 1.67530 | .62083 | 1.61074 | 10 |
| 51 | .55089 | 1.81524 | .57386 | 1.74257 | .59730 | 1.67419 | .62124 | 1.60970 | 9 |
| 52 | .55127 | 1.81399 | .57425 | 1.74140 | .59770 | 1.67309 | .62164 | 1.60865 | 8 |
| 53 | .55165 | 1.81274 | .57464 | 1.74022 | .59809 | 1.67198 | .62204 | 1.60761 | 7 |
| 54 | .55203 | 1.81150 | .57503 | 1.73905 | .59849 | 1.67088 | .62245 | 1.60657 | 6 |
| 55 | .55241 | 1.81025 | .57541 | 1.73788 | .59888 | 1.66978 | .62285 | 1.60553 | 5 |
| 56 | .55279 | 1.80901 | .57580 | 1.73671 | .59928 | 1.66867 | .62325 | 1.60449 | 4 |
| 57 | .55317 | 1.80777 | .57619 | 1.73555 | .59967 | 1.66757 | .62366 | 1.60345 | 3 |
| 58 | .55355 | 1.80653 | .57657 | 1.73438 | .60007 | 1.66647 | .62406 | 1.60241 | 2 |
| 59 | .55393 | 1.80529 | .57696 | 1.73321 | .60046 | 1.66538 | .62446 | 1.60137 | 1 |
| 60 | .55431 | 1.80405 | .57735 | 1.73205 | .60086 | 1.66428 | .62487 | 1.60033 | 0 |
| ′ | Cotang | Tang | Cotang | Tang | Cotang | Tang | Cotang | Tang | ′ |
| | 61° | | 60° | | 59° | | 58° | | |

# Table III.—Natural Tangents and Cotangents.

| ′ | 32° Tang | 32° Cotang | 33° Tang | 33° Cotang | 34° Tang | 34° Cotang | 35° Tang | 35° Cotang | ′ |
|---|---|---|---|---|---|---|---|---|---|
| 0 | .62487 | 1.60033 | .64941 | 1.53986 | .67451 | 1.48256 | .70021 | 1.42815 | 60 |
| 1 | .62527 | 1.59930 | .64982 | 1.53888 | .67493 | 1.48163 | .70064 | 1.42726 | 59 |
| 2 | .62568 | 1.59826 | .65024 | 1.53791 | .67536 | 1.48070 | .70107 | 1.42638 | 58 |
| 3 | .62608 | 1.59723 | .65065 | 1.53693 | .67578 | 1.47977 | .70151 | 1.42550 | 57 |
| 4 | .62649 | 1.59620 | .65106 | 1.53595 | .67620 | 1.47885 | .70194 | 1.42462 | 56 |
| 5 | .62689 | 1.59517 | .65148 | 1.53497 | .67663 | 1.47792 | .70238 | 1.42374 | 55 |
| 6 | .62730 | 1.59414 | .65189 | 1.53400 | .67705 | 1.47699 | .70281 | 1.42286 | 54 |
| 7 | .62770 | 1.59311 | .65231 | 1.53302 | .67748 | 1.47607 | .70325 | 1.42198 | 53 |
| 8 | .62811 | 1.59208 | .65272 | 1.53205 | .67790 | 1.47514 | .70368 | 1.42110 | 52 |
| 9 | .62852 | 1.59105 | .65314 | 1.53107 | .67832 | 1.47422 | .70412 | 1.42022 | 51 |
| 10 | .62892 | 1.59002 | .65355 | 1.53010 | .67875 | 1.47330 | .70455 | 1.41934 | 50 |
| 11 | .62933 | 1.58900 | .65397 | 1.52913 | .67917 | 1.47238 | .70499 | 1.41847 | 49 |
| 12 | .62973 | 1.58797 | .65438 | 1.52816 | .67960 | 1.47146 | .70542 | 1.41759 | 48 |
| 13 | .63014 | 1.58695 | .65480 | 1.52719 | .68002 | 1.47053 | .70586 | 1.41672 | 47 |
| 14 | .63055 | 1.58593 | .65521 | 1.52622 | .68045 | 1.46962 | .70629 | 1.41584 | 46 |
| 15 | .63095 | 1.58490 | .65563 | 1.52525 | .68088 | 1.46870 | .70673 | 1.41497 | 45 |
| 16 | .63136 | 1.58388 | .65604 | 1.52429 | .68130 | 1.46778 | .70717 | 1.41409 | 44 |
| 17 | .63177 | 1.58286 | .65646 | 1.52332 | .68173 | 1.46686 | .70760 | 1.41322 | 43 |
| 18 | .63217 | 1.58184 | .65688 | 1.52235 | .68215 | 1.46595 | .70804 | 1.41235 | 42 |
| 19 | .63258 | 1.58083 | .65729 | 1.52139 | .68258 | 1.46503 | .70848 | 1.41148 | 41 |
| 20 | .63299 | 1.57981 | .65771 | 1.52043 | .68301 | 1.46411 | .70891 | 1.41061 | 40 |
| 21 | .63340 | 1.57879 | .65813 | 1.51946 | .68343 | 1.46320 | .70935 | 1.40974 | 39 |
| 22 | .63380 | 1.57778 | .65854 | 1.51850 | .68386 | 1.46229 | .70979 | 1.40887 | 38 |
| 23 | .63421 | 1.57676 | .65896 | 1.51754 | .68429 | 1.46137 | .71023 | 1.40800 | 37 |
| 24 | .63462 | 1.57575 | .65938 | 1.51658 | .68471 | 1.46046 | .71066 | 1.40714 | 36 |
| 25 | .63503 | 1.57474 | .65980 | 1.51562 | .68514 | 1.45955 | .71110 | 1.40627 | 35 |
| 26 | .63544 | 1.57372 | .66021 | 1.51466 | .68557 | 1.45864 | .71154 | 1.40540 | 34 |
| 27 | .63584 | 1.57271 | .66063 | 1.51370 | .68600 | 1.45773 | .71198 | 1.40454 | 33 |
| 28 | .63625 | 1.57170 | .66105 | 1.51275 | .68642 | 1.45682 | .71242 | 1.40367 | 32 |
| 29 | .63666 | 1.57069 | .66147 | 1.51179 | .68685 | 1.45592 | .71285 | 1.40281 | 31 |
| 30 | .63707 | 1.56969 | .66189 | 1.51084 | .68728 | 1.45501 | .71329 | 1.40195 | 30 |
| 31 | .63748 | 1.56868 | .66230 | 1.50988 | .68771 | 1.45410 | .71373 | 1.40109 | 29 |
| 32 | .63789 | 1.56767 | .66272 | 1.50893 | .68814 | 1.45320 | .71417 | 1.40022 | 28 |
| 33 | .63830 | 1.56667 | .66314 | 1.50797 | .68857 | 1.45229 | .71461 | 1.39936 | 27 |
| 34 | .63871 | 1.56566 | .66356 | 1.50702 | .68900 | 1.45139 | .71505 | 1.39850 | 26 |
| 35 | .63912 | 1.56466 | .66398 | 1.50607 | .68942 | 1.45049 | .71549 | 1.39764 | 25 |
| 36 | .63953 | 1.56366 | .66440 | 1.50512 | .68985 | 1.44958 | .71593 | 1.39679 | 24 |
| 37 | .63994 | 1.56265 | .66482 | 1.50417 | .69028 | 1.44868 | .71637 | 1.39593 | 23 |
| 38 | .64035 | 1.56165 | .66524 | 1.50322 | .69071 | 1.44778 | .71681 | 1.39507 | 22 |
| 39 | .64076 | 1.56065 | .66566 | 1.50228 | .69114 | 1.44688 | .71725 | 1.39421 | 21 |
| 40 | .64117 | 1.55966 | .66608 | 1.50133 | .69157 | 1.44598 | .71769 | 1.39336 | 20 |
| 41 | .64158 | 1.55866 | .66650 | 1.50038 | .69200 | 1.44508 | .71813 | 1.39250 | 19 |
| 42 | .64199 | 1.55766 | .66692 | 1.49944 | .69243 | 1.44418 | .71857 | 1.39165 | 18 |
| 43 | .64240 | 1.55666 | .66734 | 1.49849 | .69286 | 1.44329 | .71901 | 1.39079 | 17 |
| 44 | .64281 | 1.55567 | .66776 | 1.49755 | .69329 | 1.44239 | .71946 | 1.38994 | 16 |
| 45 | .64322 | 1.55467 | .66818 | 1.49661 | .69372 | 1.44149 | .71990 | 1.38909 | 15 |
| 46 | .64363 | 1.55368 | .66860 | 1.49566 | .69416 | 1.44060 | .72034 | 1.38824 | 14 |
| 47 | .64404 | 1.55269 | .66902 | 1.49472 | .69459 | 1.43970 | .72078 | 1.38738 | 13 |
| 48 | .64446 | 1.55170 | .66944 | 1.49378 | .69502 | 1.43881 | .72122 | 1.38653 | 12 |
| 49 | .64487 | 1.55071 | .66986 | 1.49284 | .69545 | 1.43792 | .72167 | 1.38568 | 11 |
| 50 | .64528 | 1.54972 | .67028 | 1.49190 | .69588 | 1.43703 | .72211 | 1.38484 | 10 |
| 51 | .64569 | 1.54873 | .67071 | 1.49097 | .69631 | 1.43614 | .72255 | 1.38399 | 9 |
| 52 | .64610 | 1.54774 | .67113 | 1.49003 | .69675 | 1.43525 | .72299 | 1.38314 | 8 |
| 53 | .64652 | 1.54675 | .67155 | 1.48909 | .69718 | 1.43436 | .72344 | 1.38229 | 7 |
| 54 | .64693 | 1.54576 | .67197 | 1.48816 | .69761 | 1.43347 | .72388 | 1.38145 | 6 |
| 55 | .64734 | 1.54478 | .67239 | 1.48722 | .69804 | 1.43258 | .72432 | 1.38060 | 5 |
| 56 | .64775 | 1.54379 | .67282 | 1.48629 | .69847 | 1.43169 | .72477 | 1.37976 | 4 |
| 57 | .64817 | 1.54281 | .67324 | 1.48536 | .69891 | 1.43080 | .72521 | 1.37891 | 3 |
| 58 | .64858 | 1.54183 | .67366 | 1.48442 | .69934 | 1.42992 | .72565 | 1.37807 | 2 |
| 59 | .64899 | 1.54085 | .67409 | 1.48349 | .69977 | 1.42903 | .72610 | 1.37722 | 1 |
| 60 | .64941 | 1.53986 | .67451 | 1.48256 | .70021 | 1.42815 | .72654 | 1.37638 | 0 |
| ′ | Cotang | Tang | Cotang | Tang | Cotang | Tang | Cotang | Tang | |
| | 57° | | 56° | | 55° | | 54° | | |

## Table III.—Natural Tangents and Cotangents.

| ′ | 36° | | 37° | | 38° | | 39° | | ′ |
|---|---|---|---|---|---|---|---|---|---|
| | Tang | Cotang | Tang | Cotang | Tang | Cotang | Tang | Cotang | |
| 0 | .72654 | 1.37638 | .75355 | 1.32704 | .78129 | 1.27994 | .80978 | 1.23490 | 60 |
| 1 | .72699 | 1.37554 | .75401 | 1.32624 | .78175 | 1.27917 | .81027 | 1.23416 | 59 |
| 2 | .72743 | 1.37470 | .75447 | 1.32544 | .78222 | 1.27841 | .81075 | 1.23343 | 58 |
| 3 | .72788 | 1.37386 | .75492 | 1.32464 | .78269 | 1.27764 | .81123 | 1.23270 | 57 |
| 4 | .72832 | 1.37302 | .75538 | 1.32384 | .78316 | 1.27688 | .81171 | 1.23196 | 56 |
| 5 | .72877 | 1.37218 | .75584 | 1.32304 | .78363 | 1.27611 | .81220 | 1.23123 | 55 |
| 6 | .72921 | 1.37134 | .75629 | 1.32224 | .78410 | 1.27535 | .81268 | 1.23050 | 54 |
| 7 | .72966 | 1.37050 | .75675 | 1.32144 | .78457 | 1.27458 | .81316 | 1.22977 | 53 |
| 8 | .73010 | 1.36967 | .75721 | 1.32064 | .78504 | 1.27382 | .81364 | 1.22904 | 52 |
| 9 | .73055 | 1.36883 | .75767 | 1.31984 | .78551 | 1.27306 | .81413 | 1.22831 | 51 |
| 10 | .73100 | 1.36800 | .75812 | 1.31904 | .78598 | 1.27230 | .81461 | 1.22758 | 50 |
| 11 | .73144 | 1.36716 | .75858 | 1.31825 | .78645 | 1.27153 | .81510 | 1.22685 | 49 |
| 12 | .73189 | 1.36633 | .75904 | 1.31745 | .78692 | 1.27077 | .81558 | 1.22612 | 48 |
| 13 | .73234 | 1.36549 | .75950 | 1.31666 | .78739 | 1.27001 | .81606 | 1.22539 | 47 |
| 14 | .73278 | 1.36466 | .75996 | 1.31586 | .78786 | 1.26925 | .81655 | 1.22467 | 46 |
| 15 | .73323 | 1.36383 | .76042 | 1.31507 | .78834 | 1.26849 | .81703 | 1.22394 | 45 |
| 16 | .73368 | 1.36300 | .76088 | 1.31427 | .78881 | 1.26774 | .81752 | 1.22321 | 44 |
| 17 | .73413 | 1.36217 | .76134 | 1.31348 | .78928 | 1.26698 | .81800 | 1.22249 | 43 |
| 18 | .73457 | 1.36134 | .76180 | 1.31269 | .78975 | 1.26622 | .81849 | 1.22176 | 42 |
| 19 | .73502 | 1.36051 | .76226 | 1.31190 | .79022 | 1.26546 | .81898 | 1.22104 | 41 |
| 20 | .73547 | 1.35968 | .76272 | 1.31110 | .79070 | 1.26471 | .81946 | 1.22031 | 40 |
| 21 | .73592 | 1.35885 | .76318 | 1.31031 | .79117 | 1.26395 | .81995 | 1.21959 | 39 |
| 22 | .73637 | 1.35802 | .76364 | 1.30952 | .79164 | 1.26319 | .82044 | 1.21886 | 38 |
| 23 | .73681 | 1.35719 | .76410 | 1.30873 | .79212 | 1.26244 | .82092 | 1.21814 | 37 |
| 24 | .73726 | 1.35637 | .76456 | 1.30795 | .79259 | 1.26169 | .82141 | 1.21742 | 36 |
| 25 | .73771 | 1.35554 | .76502 | 1.30716 | .79306 | 1.26093 | .82190 | 1.21670 | 35 |
| 26 | .73816 | 1.35472 | .76548 | 1.30637 | .79354 | 1.26018 | .82238 | 1.21598 | 34 |
| 27 | .73861 | 1.35389 | .76594 | 1.30558 | .79401 | 1.25943 | .82287 | 1.21526 | 33 |
| 28 | .73906 | 1.35307 | .76640 | 1.30480 | .79449 | 1.25867 | .82336 | 1.21454 | 32 |
| 29 | .73951 | 1.35224 | .76686 | 1.30401 | .79496 | 1.25792 | .82385 | 1.21382 | 31 |
| 30 | .73996 | 1.35142 | .76733 | 1.30323 | .79544 | 1.25717 | .82434 | 1.21310 | 30 |
| 31 | .74041 | 1.35060 | .76779 | 1.30244 | .79591 | 1.25642 | .82483 | 1.21238 | 29 |
| 32 | .74086 | 1.34978 | .76825 | 1.30166 | .79639 | 1.25567 | .82531 | 1.21166 | 28 |
| 33 | .74131 | 1.34896 | .76871 | 1.30087 | .79686 | 1.25492 | .82580 | 1.21094 | 27 |
| 34 | .74176 | 1.34814 | .76918 | 1.30009 | .79734 | 1.25417 | .82629 | 1.21023 | 26 |
| 35 | .74221 | 1.34732 | .76964 | 1.29931 | .79781 | 1.25343 | .82678 | 1.20951 | 25 |
| 36 | .74267 | 1.34650 | .77010 | 1.29853 | .79829 | 1.25268 | .82727 | 1.20879 | 24 |
| 37 | .74312 | 1.34568 | .77057 | 1.29775 | .79877 | 1.25193 | .82776 | 1.20808 | 23 |
| 38 | .74357 | 1.34487 | .77103 | 1.29696 | .79924 | 1.25118 | .82825 | 1.20736 | 22 |
| 39 | .74402 | 1.34405 | .77149 | 1.29618 | .79972 | 1.25044 | .82874 | 1.20665 | 21 |
| 40 | .74447 | 1.34323 | .77196 | 1.29541 | .80020 | 1.24969 | .82923 | 1.20593 | 20 |
| 41 | .74492 | 1.34242 | .77242 | 1.29463 | .80067 | 1.24895 | .82972 | 1.20522 | 19 |
| 42 | .74538 | 1.34160 | .77289 | 1.29385 | .80115 | 1.24820 | .83022 | 1.20451 | 18 |
| 43 | .74583 | 1.34079 | .77335 | 1.29307 | .80163 | 1.24746 | .83071 | 1.20379 | 17 |
| 44 | .74628 | 1.33998 | .77382 | 1.29229 | .80211 | 1.24672 | .83120 | 1.20308 | 16 |
| 45 | .74674 | 1.33916 | .77428 | 1.29152 | .80258 | 1.24597 | .83169 | 1.20237 | 15 |
| 46 | .74719 | 1.33835 | .77475 | 1.29074 | .80306 | 1.24523 | .83218 | 1.20166 | 14 |
| 47 | .74764 | 1.33754 | .77521 | 1.28997 | .80354 | 1.24449 | .83268 | 1.20095 | 13 |
| 48 | .74810 | 1.33673 | .77568 | 1.28919 | .80402 | 1.24375 | .83317 | 1.20024 | 12 |
| 49 | .74855 | 1.33592 | .77615 | 1.28842 | .80450 | 1.24301 | .83366 | 1.19953 | 11 |
| 50 | .74900 | 1.33511 | .77661 | 1.28764 | .80498 | 1.24227 | .83415 | 1.19882 | 10 |
| 51 | .74946 | 1.33430 | .77708 | 1.28687 | .80546 | 1.24153 | .83465 | 1.19811 | 9 |
| 52 | .74991 | 1.33349 | .77754 | 1.28610 | .80594 | 1.24079 | .83514 | 1.19740 | 8 |
| 53 | .75037 | 1.33268 | .77801 | 1.28533 | .80642 | 1.24005 | .83564 | 1.19669 | 7 |
| 54 | .75082 | 1.33187 | .77848 | 1.28456 | .80690 | 1.23931 | .83613 | 1.19599 | 6 |
| 55 | .75128 | 1.33107 | .77895 | 1.28379 | .80738 | 1.23858 | .83662 | 1.19528 | 5 |
| 56 | .75173 | 1.33026 | .77941 | 1.28302 | .80786 | 1.23784 | .83712 | 1.19457 | 4 |
| 57 | .75219 | 1.32946 | .77988 | 1.28225 | .80834 | 1.23710 | .83761 | 1.19387 | 3 |
| 58 | .75264 | 1.32865 | .78035 | 1.28148 | .80882 | 1.23637 | .83811 | 1.19316 | 2 |
| 59 | .75310 | 1.32785 | .78082 | 1.28071 | .80930 | 1.23563 | .83860 | 1.19246 | 1 |
| 60 | .75355 | 1.32704 | .78129 | 1.27994 | .80978 | 1.23490 | .83910 | 1.19175 | 0 |
| ′ | Cotang | Tang | Cotang | Tang | Cotang | Tang | Cotang | Tang | ′ |
| | 53° | | 52° | | 51° | | 50° | | |

# Table III.—Natural Tangents and Cotangents.

| ′ | 40° | | 41° | | 42° | | 43° | | ′ |
|---|---|---|---|---|---|---|---|---|---|
| | Tang | Cotang | Tang | Cotang | Tang | Cotang | Tang | Cotang | |
| 0 | .83910 | 1.19175 | .86929 | 1.15087 | .90040 | 1.11061 | .93252 | 1.07237 | 60 |
| 1 | .83960 | 1.19105 | .86980 | 1.14969 | .90093 | 1.10996 | .93306 | 1.07174 | 59 |
| 2 | .84009 | 1.19035 | .87031 | 1.14902 | .90146 | 1.10931 | .93360 | 1.07112 | 58 |
| 3 | .84059 | 1.18964 | .87082 | 1.14834 | .90199 | 1.10867 | .93415 | 1.07049 | 57 |
| 4 | .84108 | 1.18894 | .87133 | 1.14767 | .90251 | 1.10802 | .93469 | 1.06987 | 56 |
| 5 | .84158 | 1.18824 | .87184 | 1.14699 | .90304 | 1.10737 | .93524 | 1.06925 | 55 |
| 6 | .84208 | 1.18754 | .87236 | 1.14632 | .90357 | 1.10672 | .93578 | 1.06862 | 54 |
| 7 | .84258 | 1.18684 | .87287 | 1.14565 | .90410 | 1.10607 | .93633 | 1.06800 | 53 |
| 8 | .84307 | 1.18614 | .87338 | 1.14498 | .90463 | 1.10543 | .93688 | 1.06738 | 52 |
| 9 | .84357 | 1.18544 | .87389 | 1.14430 | .90516 | 1.10478 | .93742 | 1.06676 | 51 |
| 10 | .84407 | 1.18474 | .87441 | 1.14363 | .90569 | 1.10414 | .93797 | 1.06613 | 50 |
| 11 | .84457 | 1.18404 | .87492 | 1.14296 | .90621 | 1.10349 | .93852 | 1.06551 | 49 |
| 12 | .84507 | 1.18334 | .87543 | 1.14229 | .90674 | 1.10285 | .93906 | 1.06489 | 48 |
| 13 | .84556 | 1.18264 | .87595 | 1.14162 | .90727 | 1.10220 | .93961 | 1.06427 | 47 |
| 14 | .84606 | 1.18194 | .87646 | 1.14095 | .90781 | 1.10156 | .94016 | 1.06365 | 46 |
| 15 | .84656 | 1.18125 | .87698 | 1.14028 | .90834 | 1.10091 | .94071 | 1.06303 | 45 |
| 16 | .84706 | 1.18055 | .87749 | 1.13961 | .90887 | 1.10027 | .94125 | 1.06241 | 44 |
| 17 | .84756 | 1.17986 | .87801 | 1.13894 | .90940 | 1.09963 | .94180 | 1.06179 | 43 |
| 18 | .84806 | 1.17916 | .87852 | 1.13828 | .90993 | 1.09899 | .94235 | 1.06117 | 42 |
| 19 | .84856 | 1.17846 | .87904 | 1.13761 | .91046 | 1.09834 | .94290 | 1.06056 | 41 |
| 20 | .84906 | 1.17777 | .87955 | 1.13694 | .91099 | 1.09770 | .94345 | 1.05994 | 40 |
| 21 | .84956 | 1.17708 | .88007 | 1.13627 | .91153 | 1.09706 | .94400 | 1.05932 | 39 |
| 22 | .85006 | 1.17638 | .88059 | 1.13561 | .91206 | 1.09642 | .94455 | 1.05870 | 38 |
| 23 | .85057 | 1.17569 | .88110 | 1.13494 | .91259 | 1.09578 | .94510 | 1.05809 | 37 |
| 24 | .85107 | 1.17500 | .88162 | 1.13428 | .91313 | 1.09514 | .94565 | 1.05747 | 36 |
| 25 | .85157 | 1.17430 | .88214 | 1.13361 | .91366 | 1.09450 | .94620 | 1.05685 | 35 |
| 26 | .85207 | 1.17361 | .88265 | 1.13295 | .91419 | 1.09386 | .94676 | 1.05624 | 34 |
| 27 | .85257 | 1.17292 | .88317 | 1.13229 | .91473 | 1.09322 | .94731 | 1.05562 | 33 |
| 28 | .85308 | 1.17223 | .88369 | 1.13162 | .91526 | 1.09258 | .94786 | 1.05501 | 32 |
| 29 | .85358 | 1.17154 | .88421 | 1.13096 | .91580 | 1.09195 | .94841 | 1.05439 | 31 |
| 30 | .85408 | 1.17085 | .88473 | 1.13029 | .91633 | 1.09131 | .94896 | 1.05378 | 30 |
| 31 | .85458 | 1.17016 | .88524 | 1.12963 | .91687 | 1.09067 | .94952 | 1.05317 | 29 |
| 32 | .85509 | 1.16947 | .88576 | 1.12897 | .91740 | 1.09003 | .95007 | 1.05255 | 28 |
| 33 | .85559 | 1.16878 | .88628 | 1.12831 | .91794 | 1.08940 | .95062 | 1.05194 | 27 |
| 34 | .85609 | 1.16809 | .88680 | 1.12765 | .91847 | 1.08876 | .95118 | 1.05133 | 26 |
| 35 | .85660 | 1.16741 | .88732 | 1.12699 | .91901 | 1.08813 | .95173 | 1.05072 | 25 |
| 36 | .85710 | 1.16672 | .88784 | 1.12633 | .91955 | 1.08749 | .95229 | 1.05010 | 24 |
| 37 | .85761 | 1.16603 | .88836 | 1.12567 | .92008 | 1.08686 | .95284 | 1.04949 | 23 |
| 38 | .85811 | 1.16535 | .88888 | 1.12501 | .92062 | 1.08622 | .95340 | 1.04888 | 22 |
| 39 | .85862 | 1.16466 | .88940 | 1.12435 | .92116 | 1.08559 | .95395 | 1.04827 | 21 |
| 40 | .85912 | 1.16398 | .88992 | 1.12369 | .92170 | 1.08496 | .95451 | 1.04766 | 20 |
| 41 | .85963 | 1.16329 | .89045 | 1.12303 | .92224 | 1.08432 | .95506 | 1.04705 | 19 |
| 42 | .86014 | 1.16261 | .89097 | 1.12238 | .92277 | 1.08369 | .95562 | 1.04644 | 18 |
| 43 | .86064 | 1.16192 | .89149 | 1.12172 | .92331 | 1.08306 | .95618 | 1.04583 | 17 |
| 44 | .86115 | 1.16124 | .89201 | 1.12106 | .92385 | 1.08243 | .95673 | 1.04522 | 16 |
| 45 | .86166 | 1.16056 | .89253 | 1.12041 | .92439 | 1.08179 | .95729 | 1.04461 | 15 |
| 46 | .86216 | 1.15987 | .89306 | 1.11975 | .92493 | 1.08116 | .95785 | 1.04401 | 14 |
| 47 | .86267 | 1.15919 | .89358 | 1.11909 | .92547 | 1.08053 | .95841 | 1.04340 | 13 |
| 48 | .86318 | 1.15851 | .89410 | 1.11844 | .92601 | 1.07990 | .95897 | 1.04279 | 12 |
| 49 | .86368 | 1.15783 | .89463 | 1.11778 | .92655 | 1.07927 | .95952 | 1.04218 | 11 |
| 50 | .86419 | 1.15715 | .89515 | 1.11713 | .92709 | 1.07864 | .96008 | 1.04158 | 10 |
| 51 | .86470 | 1.15647 | .89567 | 1.11648 | .92763 | 1.07801 | .96064 | 1.04097 | 9 |
| 52 | .86521 | 1.15579 | .89620 | 1.11582 | .92817 | 1.07738 | .96120 | 1.04036 | 8 |
| 53 | .86572 | 1.15511 | .89672 | 1.11517 | .92872 | 1.07676 | .96176 | 1.03976 | 7 |
| 54 | .86623 | 1.15443 | .89725 | 1.11452 | .92926 | 1.07613 | .96232 | 1.03915 | 6 |
| 55 | .86674 | 1.15375 | .89777 | 1.11387 | .92980 | 1.07550 | .96288 | 1.03855 | 5 |
| 56 | .86725 | 1.15308 | .89830 | 1.11321 | .93034 | 1.07487 | .96344 | 1.03794 | 4 |
| 57 | .86776 | 1.15240 | .89883 | 1.11256 | .93088 | 1.07425 | .96400 | 1.03734 | 3 |
| 58 | .86827 | 1.15172 | .89935 | 1.11191 | .93143 | 1.07362 | .96457 | 1.03674 | 2 |
| 59 | .86878 | 1.15104 | .89988 | 1.11126 | .93197 | 1.07299 | .96513 | 1.03613 | 1 |
| 60 | .86929 | 1.15037 | .90040 | 1.11061 | .93252 | 1.07237 | .96569 | 1.03553 | 0 |
| ′ | Cotang | Tang | Cotang | Tang | Cotang | Tang | Cotang | Tang | |
| | 49° | | 48° | | 47° | | 46° | | |

## TABLE III.—NATURAL TANGENTS AND COTANGENTS.

| ′ | 44° | | ′ | ′ | 44° | | ′ | ′ | 44° | | ′ |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | Tang | Cotang | | | Tang | Cotang | | | Tang | Cotang | |
| 0 | .96569 | 1.03553 | 60 | 20 | .97700 | 1.02355 | 40 | 40 | .98843 | 1.01170 | 20 |
| 1 | .96625 | 1.03498 | 59 | 21 | .97756 | 1.02295 | 39 | 41 | .98901 | 1.01112 | 19 |
| 2 | .96681 | 1.03433 | 58 | 22 | .97813 | 1.02236 | 38 | 42 | .98958 | 1.01053 | 18 |
| 3 | .96738 | 1.03372 | 57 | 23 | .97870 | 1.02176 | 37 | 43 | .99016 | 1.00994 | 17 |
| 4 | .96794 | 1.03312 | 56 | 24 | .97927 | 1.02117 | 36 | 44 | .99073 | 1.00935 | 16 |
| 5 | .96850 | 1.03252 | 55 | 25 | .97984 | 1.02057 | 35 | 45 | .99131 | 1.00876 | 15 |
| 6 | .96907 | 1.03192 | 54 | 26 | .98041 | 1.01998 | 34 | 46 | .99189 | 1.00818 | 14 |
| 7 | .96963 | 1.03132 | 53 | 27 | .98098 | 1.01939 | 33 | 47 | .99247 | 1.00759 | 13 |
| 8 | .97020 | 1.03072 | 52 | 28 | .98155 | 1.01879 | 32 | 48 | .99304 | 1.00701 | 12 |
| 9 | .97076 | 1.03012 | 51 | 29 | .98213 | 1.01820 | 31 | 49 | .99362 | 1.00642 | 11 |
| 10 | .97133 | 1.02952 | 50 | 30 | .98270 | 1.01761 | 30 | 50 | .99420 | 1.00583 | 10 |
| 11 | .97189 | 1.02892 | 49 | 31 | .98327 | 1.01702 | 29 | 51 | .99478 | 1.00525 | 9 |
| 12 | .97246 | 1.02832 | 48 | 32 | .98384 | 1.01642 | 28 | 52 | .99536 | 1.00467 | 8 |
| 13 | .97302 | 1.02772 | 47 | 33 | .98441 | 1.01583 | 27 | 53 | .99594 | 1.00408 | 7 |
| 14 | .97359 | 1.02713 | 46 | 34 | .98499 | 1.01524 | 26 | 54 | .99652 | 1.00350 | 6 |
| 15 | .97416 | 1.02653 | 45 | 35 | .98556 | 1.01465 | 25 | 55 | .99710 | 1.00291 | 5 |
| 16 | .97472 | 1.02593 | 44 | 36 | .98613 | 1.01406 | 24 | 56 | .99768 | 1.00233 | 4 |
| 17 | .97529 | 1.02533 | 43 | 37 | .98671 | 1.01347 | 23 | 57 | .99826 | 1.00175 | 3 |
| 18 | .97586 | 1.02474 | 42 | 38 | .98728 | 1.01288 | 22 | 58 | .99884 | 1.00116 | 2 |
| 19 | .97643 | 1.02414 | 41 | 39 | .98786 | 1.01229 | 21 | 59 | .99942 | 1.00058 | 1 |
| 20 | .97700 | 1.02355 | 40 | 40 | .98843 | 1.01170 | 20 | 60 | 1.00000 | 1.00000 | 0 |
| ′ | Cotang | Tang | ′ | ′ | Cotang | Tang | ′ | ′ | Cotang | Tang | ′ |
| | 45° | | | | 45° | | | | 45° | | |

## Table IV.—Natural Versed Sines and External Secants

| ′ | 0° | | 1° | | 2° | | 3° | | ′ |
|---|---|---|---|---|---|---|---|---|---|
| | Vers. | Ex. sec. | Vers. | Ex. sec. | Vers. | Ex. sec. | Vers. | Ex. sec. | |
| 0 | .00000 | .00000 | .00015 | .00015 | .00061 | .00061 | .00137 | .00137 | 0 |
| 1 | .00000 | .00000 | .00016 | .00016 | .00062 | .00062 | .00139 | .00139 | 1 |
| 2 | .00000 | .00000 | .00016 | .00016 | .00063 | .00063 | .00140 | .00140 | 2 |
| 3 | .00000 | .00000 | .00017 | .00017 | .00064 | .00064 | .00142 | .00142 | 3 |
| 4 | .00000 | .00000 | .00017 | .00017 | .00065 | .00065 | .00143 | .00143 | 4 |
| 5 | .00000 | .00000 | .00018 | .00018 | .00066 | .00066 | .00145 | .00145 | 5 |
| 6 | .00000 | .00000 | .00018 | .00018 | .00067 | .00067 | .00146 | .00147 | 6 |
| 7 | .00000 | .00000 | .00019 | .00019 | .00068 | .00068 | .00148 | .00148 | 7 |
| 8 | .00000 | .00000 | .00020 | .00020 | .00069 | .00069 | .00150 | .00150 | 8 |
| 9 | .00000 | .00000 | .00020 | .00020 | .00070 | .00070 | .00151 | .00151 | 9 |
| 10 | .00000 | .00000 | .00021 | .00021 | .00071 | .00072 | .00153 | .00153 | 10 |
| 11 | .00001 | .00001 | .00021 | .00021 | .00073 | .00073 | .00154 | .00155 | 11 |
| 12 | .00001 | .00001 | .00022 | .00022 | .00074 | .00074 | .00156 | .00156 | 12 |
| 13 | .00001 | .00001 | .00023 | .00023 | .00075 | .00075 | .00158 | .00158 | 13 |
| 14 | .00001 | .00001 | .00023 | .00023 | .00076 | .00076 | .00159 | .00159 | 14 |
| 15 | .00001 | .00001 | .00024 | .00024 | .00077 | .00077 | .00161 | .00161 | 15 |
| 16 | .00001 | .00001 | .00024 | .00024 | .00078 | .00078 | .00162 | .00163 | 16 |
| 17 | .00001 | .00001 | .00025 | .00025 | .00079 | .00079 | .00164 | .00164 | 17 |
| 18 | .00001 | .00001 | .00026 | .00026 | .00081 | .00081 | .00166 | .00166 | 18 |
| 19 | .00002 | .00002 | .00026 | .00026 | .00082 | .00082 | .00168 | .00168 | 19 |
| 20 | .00002 | .00002 | .00027 | .00027 | .00083 | .00083 | .00169 | .00169 | 20 |
| 21 | .00002 | .00002 | .00028 | .00028 | .00084 | .00084 | .00171 | .00171 | 21 |
| 22 | .00002 | .00002 | .00028 | .00028 | .00085 | .00085 | .00173 | .00173 | 22 |
| 23 | .00002 | .00002 | .00029 | .00029 | .00087 | .00087 | .00174 | .00175 | 23 |
| 24 | .00002 | .00002 | .00030 | .00030 | .00088 | .00088 | .00176 | .00176 | 24 |
| 25 | .00003 | .00003 | .00031 | .00031 | .00089 | .00089 | .00178 | .00178 | 25 |
| 26 | .00003 | .00003 | .00031 | .00031 | .00090 | .00090 | .00179 | .00180 | 26 |
| 27 | .00003 | .00003 | .00032 | .00032 | .00091 | .00091 | .00181 | .00182 | 27 |
| 28 | .00003 | .00003 | .00033 | .00033 | .00093 | .00093 | .00183 | .00183 | 28 |
| 29 | .00004 | .00004 | .00034 | .00034 | .00094 | .00094 | .00185 | .00185 | 29 |
| 30 | .00004 | .00004 | .00034 | .00034 | .00095 | .00095 | .00187 | .00187 | 30 |
| 31 | .00004 | .00004 | .00035 | .00035 | .00096 | .00097 | .00188 | .00189 | 31 |
| 32 | .00004 | .00004 | .00036 | .00036 | .00098 | .00098 | .00190 | .00190 | 32 |
| 33 | .00005 | .00005 | .00037 | .00037 | .00099 | .00099 | .00192 | .00192 | 33 |
| 34 | .00005 | .00005 | .00037 | .00037 | .00100 | .00100 | .00194 | .00194 | 34 |
| 35 | .00005 | .00005 | .00038 | .00038 | .00102 | .00102 | .00196 | .00196 | 35 |
| 36 | .00005 | .00005 | .00039 | .00039 | .00103 | .00103 | .00197 | .00198 | 36 |
| 37 | .00006 | .00006 | .00040 | .00040 | .00104 | .00104 | .00199 | .00200 | 37 |
| 38 | .00006 | .00006 | .00041 | .00041 | .00106 | .00106 | .00201 | .00201 | 38 |
| 39 | .00006 | .00006 | .00041 | .00041 | .00107 | .00107 | .00203 | .00203 | 39 |
| 40 | .00007 | .00007 | .00042 | .00042 | .00108 | .00108 | .00205 | .00205 | 40 |
| 41 | .00007 | .00007 | .00043 | .00043 | .00110 | .00110 | .00207 | .00207 | 41 |
| 42 | .00007 | .00007 | .00044 | .00044 | .00111 | .00111 | .00208 | .00209 | 42 |
| 43 | .00008 | .00008 | .00045 | .00045 | .00112 | .00113 | .00210 | .00211 | 43 |
| 44 | .00008 | .00008 | .00046 | .00046 | .00114 | .00114 | .00212 | .00213 | 44 |
| 45 | .00009 | .00009 | .00047 | .00047 | .00115 | .00115 | .00214 | .00215 | 45 |
| 46 | .00009 | .00009 | .00048 | .00048 | .00117 | .00117 | .00216 | .00216 | 46 |
| 47 | .00009 | .00009 | .00048 | .00048 | .00118 | .00118 | .00218 | .00218 | 47 |
| 48 | .00010 | .00010 | .00049 | .00049 | .00119 | .00120 | .00220 | .00220 | 48 |
| 49 | .00010 | .00010 | .00050 | .00050 | .00121 | .00121 | .00222 | .00222 | 49 |
| 50 | .00011 | .00011 | .00051 | .00051 | .00122 | .00122 | .00224 | .00224 | 50 |
| 51 | .00011 | .00011 | .00052 | .00052 | .00124 | .00124 | .00226 | .00226 | 51 |
| 52 | .00011 | .00011 | .00053 | .00053 | .00125 | .00125 | .00228 | .00228 | 52 |
| 53 | .00012 | .00012 | .00054 | .00054 | .00127 | .00127 | .00230 | .00230 | 53 |
| 54 | .00012 | .00012 | .00055 | .00055 | .00128 | .00128 | .00232 | .00232 | 54 |
| 55 | .00013 | .00013 | .00056 | .00056 | .00130 | .00130 | .00234 | .00234 | 55 |
| 56 | .00013 | .00013 | .00057 | .00057 | .00131 | .00131 | .00236 | .00236 | 56 |
| 57 | .00014 | .00014 | .00058 | .00058 | .00133 | .00133 | .00238 | .00238 | 57 |
| 58 | .00014 | .00014 | .00059 | .00059 | .00134 | .00134 | .00240 | .00240 | 58 |
| 59 | .00015 | .00015 | .00060 | .00060 | .00136 | .00136 | .00242 | .00242 | 59 |
| 60 | .00015 | .00015 | .00061 | .00061 | .00137 | .00137 | .00244 | .00244 | 60 |

## Table IV.—Natural Versed Sines and External Secants.

| ′ | 4° | | 5° | | 6° | | 7° | | ′ |
|---|---|---|---|---|---|---|---|---|---|
| | Vers. | Ex. sec. | Vers. | Ex. sec. | Vers. | Ex. sec. | Vers. | Ex. sec. | |
| 0 | .00244 | .00244 | .00381 | .00382 | .00548 | .00551 | .00745 | .00751 | 0 |
| 1 | .00246 | .00246 | .00383 | .00385 | .00551 | .00554 | .00749 | .00755 | 1 |
| 2 | .00248 | .00248 | .00386 | .00387 | .00554 | .00557 | .00752 | .00758 | 2 |
| 3 | .00250 | .00250 | .00388 | .00390 | .00557 | .00560 | .00756 | .00762 | 3 |
| 4 | .00252 | .00252 | .00391 | .00392 | .00560 | .00563 | .00760 | .00765 | 4 |
| 5 | .00254 | .00254 | .00393 | .00395 | .00563 | .00566 | .00763 | .00769 | 5 |
| 6 | .00256 | .00257 | .00396 | .00397 | .00566 | .00569 | .00767 | .00773 | 6 |
| 7 | .00258 | .00259 | .00398 | .00400 | .00569 | .00573 | .00770 | .00776 | 7 |
| 8 | .00260 | .00261 | .00401 | .00403 | .00572 | .00576 | .00774 | .00780 | 8 |
| 9 | .00262 | .00263 | .00404 | .00405 | .00576 | .00579 | .00778 | .00784 | 9 |
| 10 | .00264 | .00265 | .00406 | .00408 | .00579 | .00582 | .00781 | .00787 | 10 |
| 11 | .00266 | .00267 | .00409 | .00411 | .00582 | .00585 | .00785 | .00791 | 11 |
| 12 | .00269 | .00269 | .00412 | .00413 | .00585 | .00588 | .00789 | .00795 | 12 |
| 13 | .00271 | .00271 | .00414 | .00416 | .00588 | .00592 | .00792 | .00799 | 13 |
| 14 | .00273 | .00274 | .00417 | .00419 | .00591 | .00595 | .00796 | .00802 | 14 |
| 15 | .00275 | .00276 | .00420 | .00421 | .00594 | .00598 | .00800 | .00806 | 15 |
| 16 | .00277 | .00278 | .00422 | .00424 | .00598 | .00601 | .00803 | .00810 | 16 |
| 17 | .00279 | .00280 | .00425 | .00427 | .00601 | .00604 | .00807 | .00813 | 17 |
| 18 | .00281 | .00282 | .00428 | .00429 | .00604 | .00608 | .00811 | .00817 | 18 |
| 19 | .00284 | .00284 | .00430 | .00432 | .00607 | .00611 | .00814 | .00821 | 19 |
| 20 | .00286 | .00287 | .00433 | .00435 | .00610 | .00614 | .00818 | .00825 | 20 |
| 21 | .00288 | .00289 | .00436 | .00438 | .00614 | .00617 | .00822 | .00828 | 21 |
| 22 | .00290 | .00291 | .00438 | .00440 | .00617 | .00621 | .00825 | .00832 | 22 |
| 23 | .00293 | .00293 | .00441 | .00443 | .00620 | .00624 | .00829 | .00836 | 23 |
| 24 | .00295 | .00296 | .00444 | .00446 | .00623 | .00627 | .00833 | .00840 | 24 |
| 25 | .00297 | .00298 | .00447 | .00449 | .00626 | .00630 | .00837 | .00844 | 25 |
| 26 | .00299 | .00300 | .00449 | .00451 | .00630 | .00634 | .00840 | .00848 | 26 |
| 27 | .00301 | .00302 | .00452 | .00454 | .00633 | .00637 | .00844 | .00851 | 27 |
| 28 | .00304 | .00305 | .00455 | .00457 | .00636 | .00640 | .00848 | .00855 | 28 |
| 29 | .00306 | .00307 | .00458 | .00460 | .00640 | .00644 | .00852 | .00859 | 29 |
| 30 | .00308 | .00309 | .00460 | .00463 | .00643 | .00647 | .00856 | .00863 | 30 |
| 31 | .00311 | .00312 | .00463 | .00465 | .00646 | .00650 | .00859 | .00867 | 31 |
| 32 | .00313 | .00314 | .00466 | .00468 | .00649 | .00654 | .00863 | .00871 | 32 |
| 33 | .00315 | .00316 | .00469 | .00471 | .00653 | .00657 | .00867 | .00875 | 33 |
| 34 | .00317 | .00318 | .00472 | .00474 | .00656 | .00660 | .00871 | .00878 | 34 |
| 35 | .00320 | .00321 | .00474 | .00477 | .00659 | .00664 | .00875 | .00882 | 35 |
| 36 | .00322 | .00323 | .00477 | .00480 | .00663 | .00667 | .00878 | .00886 | 36 |
| 37 | .00324 | .00326 | .00480 | .00482 | .00666 | .00671 | .00882 | .00890 | 37 |
| 38 | .00327 | .00328 | .00483 | .00485 | .00669 | .00674 | .00886 | .00894 | 38 |
| 39 | .00329 | .00330 | .00486 | .00488 | .00673 | .00677 | .00890 | .00898 | 39 |
| 40 | .00332 | .00333 | .00489 | .00491 | .00676 | .00681 | .00894 | .00902 | 40 |
| 41 | .00334 | .00335 | .00492 | .00494 | .00680 | .00684 | .00898 | .00906 | 41 |
| 42 | .00336 | .00337 | .00494 | .00497 | .00683 | .00688 | .00902 | .00910 | 42 |
| 43 | .00339 | .00340 | .00497 | .00500 | .00686 | .00691 | .00906 | .00914 | 43 |
| 44 | .00341 | .00342 | .00500 | .00503 | .00690 | .00695 | .00909 | .00918 | 44 |
| 45 | .00343 | .00345 | .00503 | .00506 | .00693 | .00698 | .00913 | .00922 | 45 |
| 46 | .00346 | .00347 | .00506 | .00509 | .00697 | .00701 | .00917 | .00926 | 46 |
| 47 | .00348 | .00350 | .00509 | .00512 | .00700 | .00705 | .00921 | .00930 | 47 |
| 48 | .00351 | .00352 | .00512 | .00515 | .00708 | .00708 | .00925 | .00934 | 48 |
| 49 | .00353 | .00354 | .00515 | .00518 | .00707 | .00712 | .00929 | .00938 | 49 |
| 50 | .00356 | .00357 | .00518 | .00521 | .00710 | .00715 | .00933 | .00942 | 50 |
| 51 | .00358 | .00359 | .00521 | .00524 | .00714 | .00719 | .00937 | .00946 | 51 |
| 52 | .00361 | .00362 | .00524 | .00527 | .00717 | .00722 | .00941 | .00950 | 52 |
| 53 | .00363 | .00364 | .00527 | .00530 | .00721 | .00726 | .00945 | .00954 | 53 |
| 54 | .00365 | .00367 | .00530 | .00533 | .00724 | .00730 | .00949 | .00958 | 54 |
| 55 | .00368 | .00369 | .00533 | .00536 | .00728 | .00733 | .00953 | .00962 | 55 |
| 56 | .00370 | .00372 | .00536 | .00539 | .00731 | .00737 | .00957 | .00966 | 56 |
| 57 | .00373 | .00374 | .00539 | .00542 | .00735 | .00740 | .00961 | .00970 | 57 |
| 58 | .00375 | .00377 | .00542 | .00545 | .00738 | .00744 | .00965 | .00975 | 58 |
| 59 | .00378 | .00379 | .00545 | .00548 | .00742 | .00747 | .00969 | .00979 | 59 |
| | .00381 | .00382 | .00548 | .00551 | .00745 | .00751 | .00973 | .00983 | 60 |

# Table IV.—Natural Versed Sines and External Secants

| ′ | 8° | | 9° | | 10° | | 11° | | ′ |
|---|---|---|---|---|---|---|---|---|---|
| | Vers. | Ex. sec. | Vers. | Ex. sec. | Vers. | Ex. sec. | Vers. | Ex. sec. | |
| 0 | .00973 | .00988 | .01231 | .01247 | .01519 | .01543 | .01837 | .01872 | 0 |
| 1 | .00977 | .00987 | .01236 | .01251 | .01524 | .01548 | .01843 | .01877 | 1 |
| 2 | .00981 | .00991 | .01240 | .01256 | .01529 | .01553 | .01848 | .01883 | 2 |
| 3 | .00985 | .00995 | .01245 | .01261 | .01534 | .01558 | .01854 | .01889 | 3 |
| 4 | .00989 | .00999 | .01249 | .01265 | .01540 | .01564 | .01860 | .01895 | 4 |
| 5 | .00994 | .01004 | .01254 | .01270 | .01545 | .01569 | .01865 | .01901 | 5 |
| 6 | .00998 | .01008 | .01259 | .01275 | .01550 | .01574 | .01871 | .01906 | 6 |
| 7 | .01002 | .01012 | .01263 | .01279 | .01555 | .01579 | .01876 | .01912 | 7 |
| 8 | .01006 | .01016 | .01268 | .01284 | .01560 | .01585 | .01882 | .01918 | 8 |
| 9 | .01010 | .01020 | .01272 | .01289 | .01565 | .01590 | .01888 | .01924 | 9 |
| 10 | .01014 | .01024 | .01277 | .01294 | .01570 | .01595 | .01893 | .01930 | 10 |
| 11 | .01018 | .01029 | .01282 | .01298 | .01575 | .01601 | .01899 | .01936 | 11 |
| 12 | .01022 | .01033 | .01286 | .01303 | .01580 | .01606 | .01904 | .01941 | 12 |
| 13 | .01027 | .01037 | .01291 | .01308 | .01586 | .01611 | .01910 | .01947 | 13 |
| 14 | .01031 | .01041 | .01296 | .01313 | .01591 | .01616 | .01916 | .01953 | 14 |
| 15 | .01035 | .01046 | .01300 | .01318 | .01596 | .01622 | .01921 | .01959 | 15 |
| 16 | .01039 | .01050 | .01305 | .01322 | .01601 | .01627 | .01927 | .01965 | 16 |
| 17 | .01043 | .01054 | .01310 | .01327 | .01606 | .01633 | .01933 | .01971 | 17 |
| 18 | .01047 | .01059 | .01314 | .01332 | .01612 | .01638 | .01939 | .01977 | 18 |
| 19 | .01052 | .01063 | .01319 | .01337 | .01617 | .01643 | .01944 | .01983 | 19 |
| 20 | .01056 | .01067 | .01324 | .01342 | .01622 | .01649 | .01950 | .01989 | 20 |
| 21 | .01060 | .01071 | .01329 | .01346 | .01627 | .01654 | .01956 | .01995 | 21 |
| 22 | .01064 | .01076 | .01333 | .01351 | .01632 | .01659 | .01961 | .02001 | 22 |
| 23 | .01069 | .01080 | .01338 | .01356 | .01638 | .01665 | .01967 | .02007 | 23 |
| 24 | .01073 | .01084 | .01343 | .01361 | .01643 | .01670 | .01973 | .02013 | 24 |
| 25 | .01077 | .01089 | .01348 | .01366 | .01648 | .01676 | .01979 | .02019 | 25 |
| 26 | .01081 | .01093 | .01352 | .01371 | .01653 | .01681 | .01984 | .02025 | 26 |
| 27 | .01086 | .01097 | .01357 | .01376 | .01659 | .01687 | .01990 | .02031 | 27 |
| 28 | .01090 | .01102 | .01362 | .01381 | .01664 | .01692 | .01996 | .02037 | 28 |
| 29 | .01094 | .01106 | .01367 | .01386 | .01669 | .01698 | .02002 | .02043 | 29 |
| 30 | .01098 | .01111 | .01371 | .01391 | .01675 | .01703 | .02008 | .02049 | 30 |
| 31 | .01103 | .01115 | .01376 | .01395 | .01680 | .01709 | .02013 | .02055 | 31 |
| 32 | .01107 | .01119 | .01381 | .01400 | .01685 | .01714 | .02019 | .02061 | 32 |
| 33 | .01111 | .01124 | .01386 | .01405 | .01690 | .01720 | .02025 | .02067 | 33 |
| 34 | .01116 | .01128 | .01391 | .01410 | .01696 | .01725 | .02031 | .02073 | 34 |
| 35 | .01120 | .01133 | .01396 | .01415 | .01701 | .01731 | .02037 | .02079 | 35 |
| 36 | .01124 | .01137 | .01400 | .01420 | .01706 | .01736 | .02042 | .02085 | 36 |
| 37 | .01129 | .01142 | .01405 | .01425 | .01712 | .01742 | .02048 | .02091 | 37 |
| 38 | .01133 | .01146 | .01410 | .01430 | .01717 | .01747 | .02054 | .02097 | 38 |
| 39 | .01137 | .01151 | .01415 | .01435 | .01723 | .01753 | .02060 | .02103 | 39 |
| 40 | .01142 | .01155 | .01420 | .01440 | .01728 | .01758 | .02066 | .02110 | 40 |
| 41 | .01146 | .01160 | .01425 | .01445 | .01733 | .01764 | .02072 | .02116 | 41 |
| 42 | .01151 | .01164 | .01430 | .01450 | .01739 | .01769 | .02078 | .02122 | 42 |
| 43 | .01155 | .01169 | .01435 | .01455 | .01744 | .01775 | .02084 | .02128 | 43 |
| 44 | .01159 | .01173 | .01439 | .01461 | .01750 | .01781 | .02090 | .02134 | 44 |
| 45 | .01164 | .01178 | .01444 | .01466 | .01755 | .01786 | .02095 | .02140 | 45 |
| 46 | .01168 | .01182 | .01449 | .01471 | .01760 | .01792 | .02101 | .02146 | 46 |
| 47 | .01173 | .01187 | .01454 | .01476 | .01766 | .01798 | .02107 | .02153 | 47 |
| 48 | .01177 | .01191 | .01459 | .01481 | .01771 | .01803 | .02113 | .02159 | 48 |
| 49 | .01182 | .01196 | .01464 | .01486 | .01777 | .01809 | .02119 | .02165 | 49 |
| 50 | .01186 | .01200 | .01469 | .01491 | .01782 | .01815 | .02125 | .02171 | 50 |
| 51 | .01191 | .01205 | .01474 | .01496 | .01788 | .01820 | .02131 | .02178 | 51 |
| 52 | .01195 | .01209 | .01479 | .01501 | .01793 | .01826 | .02137 | .02184 | 52 |
| 53 | .01200 | .01214 | .01484 | .01506 | .01799 | .01832 | .02143 | .02190 | 53 |
| 54 | .01204 | .01219 | .01489 | .01512 | .01804 | .01837 | .02149 | .02196 | 54 |
| 55 | .01209 | .01223 | .01494 | .01517 | .01810 | .01843 | .02155 | .02203 | 55 |
| 56 | .01213 | .01228 | .01499 | .01522 | .01815 | .01849 | .02161 | .02209 | 56 |
| 57 | .01218 | .01233 | .01504 | .01527 | .01821 | .01854 | .02167 | .02215 | 57 |
| 58 | .01222 | .01237 | .01509 | .01532 | .01826 | .01860 | .02173 | .02221 | 58 |
| 59 | .01227 | .01242 | .01514 | .01537 | .01832 | .01866 | .02179 | .02228 | 59 |
| 60 | .01231 | .01247 | .01519 | .01543 | .01837 | .01872 | .02185 | .02234 | 60 |

## TABLE IV.—NATURAL VERSED SINES AND EXTERNAL SECANTS.

| ′ | 12° | | 13° | | 14° | | 15° | | ′ |
|---|---|---|---|---|---|---|---|---|---|
| | Vers. | Ex. sec. | Vers. | Ex. sec. | Vers. | Ex. sec. | Vers. | Ex. sec. | |
| 0 | .02185 | .02234 | .02563 | .02630 | .02970 | .03061 | .03407 | .03528 | 0 |
| 1 | .02191 | .02240 | .02570 | .02637 | .02977 | .03069 | .03415 | .03536 | 1 |
| 2 | .02197 | .02247 | .02576 | .02644 | .02985 | .03076 | .03422 | .03544 | 2 |
| 3 | .02203 | .02253 | .02583 | .02651 | .02992 | .03084 | .03430 | .03552 | 3 |
| 4 | .02210 | .02259 | .02589 | .02658 | .02999 | .03091 | .03438 | .03560 | 4 |
| 5 | .02216 | .02266 | .02596 | .02665 | .03006 | .03099 | .03445 | .03568 | 5 |
| 6 | .02222 | .02272 | .02602 | .02672 | .03013 | .03106 | .03453 | .03576 | 6 |
| 7 | .02228 | .02279 | .02609 | .02679 | .03020 | .03114 | .03460 | .03584 | 7 |
| 8 | .02234 | .02285 | .02616 | .02686 | .03027 | .03121 | .03468 | .03592 | 8 |
| 9 | .02240 | .02291 | .02622 | .02693 | .03034 | .03129 | .03476 | .03601 | 9 |
| 10 | .02246 | .02298 | .02629 | .02700 | .03041 | .03137 | .03483 | .03609 | 10 |
| 11 | .02252 | .02304 | .02635 | .02707 | .03048 | .03144 | .03491 | .03617 | 11 |
| 12 | .02258 | .02311 | .02642 | .02714 | .03055 | .03152 | .03498 | .03625 | 12 |
| 13 | .02265 | .02317 | .02649 | .02721 | .03063 | .03159 | .03506 | .03633 | 13 |
| 14 | .02271 | .02323 | .02655 | .02728 | .03070 | .03167 | .03514 | .03642 | 14 |
| 15 | .02277 | .02330 | .02662 | .02735 | .03077 | .03175 | .03521 | .03650 | 15 |
| 16 | .02283 | .02336 | .02669 | .02742 | .03084 | .03183 | .03529 | .03658 | 16 |
| 17 | .02289 | .02343 | .02675 | .02749 | .03091 | .03190 | .03537 | .03666 | 17 |
| 18 | .02295 | .02349 | .02682 | .02756 | .03098 | .03198 | .03544 | .03674 | 18 |
| 19 | .02302 | .02356 | .02689 | .02763 | .03106 | .03205 | .03552 | .03683 | 19 |
| 20 | .02308 | .02362 | .02696 | .02770 | .03113 | .03213 | .03560 | .03691 | 20 |
| 21 | .02314 | .02369 | .02702 | .02777 | .03120 | .03221 | .03567 | .03699 | 21 |
| 22 | .02320 | .02375 | .02709 | .02784 | .03127 | .03228 | .03575 | .03708 | 22 |
| 23 | .02327 | .02382 | .02716 | .02791 | .03134 | .03236 | .03583 | .03716 | 23 |
| 24 | .02333 | .02388 | .02722 | .02799 | .03142 | .03244 | .03590 | .03724 | 24 |
| 25 | .02339 | .02395 | .02729 | .02806 | .03149 | .03251 | .03598 | .03732 | 25 |
| 26 | .02345 | .02402 | .02736 | .02813 | .03156 | .03259 | .03606 | .03741 | 26 |
| 27 | .02352 | .02408 | .02743 | .02820 | .03163 | .03267 | .03614 | .03749 | 27 |
| 28 | .02358 | .02415 | .02749 | .02827 | .03171 | .03275 | .03621 | .03758 | 28 |
| 29 | .02364 | .02421 | .02756 | .02834 | .03178 | .03282 | .03629 | .03766 | 29 |
| 30 | .02370 | .02428 | .02763 | .02842 | .03185 | .03290 | .03637 | .03774 | 30 |
| 31 | .02377 | .02435 | .02770 | .02849 | .03193 | .03298 | .03645 | .03783 | 31 |
| 32 | .02383 | .02441 | .02777 | .02856 | .03200 | .03306 | .03653 | .03791 | 32 |
| 33 | .02389 | .02448 | .02783 | .02863 | .03207 | .03313 | .03660 | .03799 | 33 |
| 34 | .02396 | .02454 | .02790 | .02870 | .03214 | .03321 | .03668 | .03808 | 34 |
| 35 | .02402 | .02461 | .02797 | .02878 | .03222 | .03329 | .03676 | .03816 | 35 |
| 36 | .02408 | .02468 | .02804 | .02885 | .03229 | .03337 | .03684 | .03825 | 36 |
| 37 | .02415 | .02474 | .02811 | .02892 | .03236 | .03345 | .03692 | .03833 | 37 |
| 38 | .02421 | .02481 | .02818 | .02899 | .03244 | .03353 | .03699 | .03842 | 38 |
| 39 | .02427 | .02488 | .02824 | .02907 | .03251 | .03360 | .03707 | .03850 | 39 |
| 40 | .02434 | .02494 | .02831 | .02914 | .03258 | .03368 | .03715 | .03858 | 40 |
| 41 | .02440 | .02501 | .02838 | .02921 | .03266 | .03376 | .03723 | .03867 | 41 |
| 42 | .02447 | .02508 | .02845 | .02928 | .03273 | .03384 | .03731 | .03875 | 42 |
| 43 | .02453 | .02515 | .02852 | .02936 | .03281 | .03392 | .03739 | .03884 | 43 |
| 44 | .02459 | .02521 | .02859 | .02943 | .03288 | .03400 | .03747 | .03892 | 44 |
| 45 | .02466 | .02528 | .02866 | .02950 | .03295 | .03408 | .03754 | .03901 | 45 |
| 46 | .02472 | .02535 | .02873 | .02958 | .03303 | .03416 | .03762 | .03909 | 46 |
| 47 | .02479 | .02542 | .02880 | .02965 | .03310 | .03424 | .03770 | .03918 | 47 |
| 48 | .02485 | .02548 | .02887 | .02972 | .03318 | .03432 | .03778 | .03927 | 48 |
| 49 | .02492 | .02555 | .02894 | .02980 | .03325 | .03439 | .03786 | .03935 | 49 |
| 50 | .02498 | .02562 | .02900 | .02987 | .03333 | .03447 | .03794 | .03944 | 50 |
| 51 | .02504 | .02569 | .02907 | .02994 | .03340 | .03455 | .03802 | .03952 | 51 |
| 52 | .02511 | .02576 | .02914 | .03002 | .03347 | .03463 | .03810 | .03961 | 52 |
| 53 | .02517 | .02582 | .02921 | .03009 | .03355 | .03471 | .03818 | .03969 | 53 |
| 54 | .02524 | .02589 | .02928 | .03017 | .03362 | .03479 | .03826 | .03978 | 54 |
| 55 | .02530 | .02596 | .02935 | .03024 | .03370 | .03487 | .03834 | .03987 | 55 |
| 56 | .02537 | .02603 | .02942 | .03032 | .03377 | .03495 | .03842 | .03995 | 56 |
| 57 | .02543 | .02610 | .02949 | .03039 | .03385 | .03503 | .03850 | .04004 | 57 |
| 58 | .02550 | .02617 | .02956 | .03046 | .03392 | .03512 | .03858 | .04013 | 58 |
| 59 | .02556 | .02624 | .02963 | .03054 | .03400 | .03520 | .03866 | .04021 | 59 |
| | .02563 | .02630 | .02970 | .03061 | .03407 | .03528 | .03874 | .04030 | 60 |

## TABLE IV.—NATURAL VERSED SINES AND EXTERNAL SECANTS.

| ′ | 16° | | 17° | | 18° | | 19° | | ′ |
|---|---|---|---|---|---|---|---|---|---|
| | Vers. | Ex. sec. | Vers. | Ex. sec. | Vers. | Ex. sec. | Vers. | Ex. sec. | |
| 0 | .03874 | .04030 | .04370 | .04569 | .04894 | .05146 | .05448 | .05762 | 0 |
| 1 | .03882 | .04039 | .04378 | .04578 | .04903 | .05156 | .05458 | .05773 | 1 |
| 2 | .03890 | .04047 | .04387 | .04588 | .04912 | .05166 | .05467 | .05783 | 2 |
| 3 | .03898 | .04056 | .04395 | .04597 | .04921 | .05176 | .05477 | .05794 | 3 |
| 4 | .03906 | .04065 | .04404 | .04606 | .04930 | .05186 | .05486 | .05805 | 4 |
| 5 | .03914 | .04073 | .04412 | .04616 | .04939 | .05196 | .05496 | .05815 | 5 |
| 6 | .03922 | .04082 | .04421 | .04625 | .04948 | .05206 | .05505 | .05826 | 6 |
| 7 | .03930 | .04091 | .04429 | .04635 | .04957 | .05216 | .05515 | .05836 | 7 |
| 8 | .03938 | .04100 | .04438 | .04644 | .04967 | .05226 | .05524 | .05847 | 8 |
| 9 | .03946 | .04108 | .04446 | .04653 | .04976 | .05236 | .05534 | .05858 | 9 |
| 10 | .03954 | .04117 | .04455 | .04663 | .04985 | .05246 | .05543 | .05869 | 10 |
| 11 | .03963 | .04126 | .04464 | .04672 | .04994 | .05256 | .05553 | .05879 | 11 |
| 12 | .03971 | .04135 | .04472 | .04682 | .05003 | .05266 | .05562 | .05890 | 12 |
| 13 | .03979 | .04144 | .04481 | .04691 | .05012 | .05276 | .05572 | .05901 | 13 |
| 14 | .03987 | .04152 | .04489 | .04700 | .05021 | .05286 | .05582 | .05911 | 14 |
| 15 | .03995 | .04161 | .04498 | .04710 | .05030 | .05297 | .05591 | .05922 | 15 |
| 16 | .04003 | .04170 | .04507 | .04719 | .05039 | .05307 | .05601 | .05933 | 16 |
| 17 | .04011 | .04179 | .04515 | .04729 | .05048 | .05317 | .05610 | .05944 | 17 |
| 18 | .04019 | .04188 | .04524 | .04738 | .05057 | .05327 | .05620 | .05955 | 18 |
| 19 | .04028 | .04197 | .04533 | .04748 | .05067 | .05337 | .05630 | .05965 | 19 |
| 20 | .04036 | .04206 | .04541 | .04757 | .05076 | .05347 | .05639 | .05976 | 20 |
| 21 | .04044 | .04214 | .04550 | .04767 | .05085 | .05357 | .05649 | .05987 | 21 |
| 22 | .04052 | .04223 | .04559 | .04776 | .05094 | .05367 | .05658 | .05998 | 22 |
| 23 | .04060 | .04232 | .04567 | .04786 | .05103 | .05378 | .05668 | .06009 | 23 |
| 24 | .04069 | .04241 | .04576 | .04795 | .05112 | .05388 | .05678 | .06020 | 24 |
| 25 | .04077 | .04250 | .04585 | .04805 | .05122 | .05398 | .05687 | .06030 | 25 |
| 26 | .04085 | .04259 | .04593 | .04815 | .05131 | .05408 | .05697 | .06041 | 26 |
| 27 | .04093 | .04268 | .04602 | .04824 | .05140 | .05418 | .05707 | .06052 | 27 |
| 28 | .04102 | .04277 | .04611 | .04834 | .05149 | .05429 | .05716 | .06063 | 28 |
| 29 | .04110 | .04286 | .04620 | .04843 | .05158 | .05439 | .05726 | .06074 | 29 |
| 30 | .04118 | .04295 | .04628 | .04853 | .05168 | .05449 | .05736 | .06085 | 30 |
| 31 | .04126 | .04304 | .04637 | .04863 | .05177 | .05460 | .05746 | .06096 | 31 |
| 32 | .04135 | .04313 | .04646 | .04872 | .05186 | .05470 | .05755 | .06107 | 32 |
| 33 | .04143 | .04322 | .04655 | .04882 | .05195 | .05480 | .05765 | .06118 | 33 |
| 34 | .04151 | .04331 | .04663 | .04891 | .05205 | .05490 | .05775 | .06129 | 34 |
| 35 | .04159 | .04340 | .04672 | .04901 | .05214 | .05501 | .05785 | .06140 | 35 |
| 36 | .04168 | .04349 | .04681 | .04911 | .05223 | .05511 | .05794 | .06151 | 36 |
| 37 | .04176 | .04358 | .04690 | .04920 | .05232 | .05521 | .05804 | .06162 | 37 |
| 38 | .04184 | .04367 | .04699 | .04930 | .05242 | .05532 | .05814 | .06173 | 38 |
| 39 | .04193 | .04376 | .04707 | .04940 | .05251 | .05542 | .05824 | .06184 | 39 |
| 40 | .04201 | .04385 | .04716 | .04950 | .05260 | .05552 | .05833 | .06195 | 40 |
| 41 | .04209 | .04394 | .04725 | .04959 | .05270 | .05563 | .05843 | .06206 | 41 |
| 42 | .04218 | .04403 | .04734 | .04969 | .05279 | .05573 | .05853 | .06217 | 42 |
| 43 | .04226 | .04413 | .04743 | .04979 | .05288 | .05584 | .05863 | .06228 | 43 |
| 44 | .04234 | .04422 | .04752 | .04989 | .05298 | .05594 | .05873 | .06239 | 44 |
| 45 | .04243 | .04431 | .04760 | .04998 | .05307 | .05604 | .05882 | .06250 | 45 |
| 46 | .04251 | .04440 | .04769 | .05008 | .05316 | .05615 | .05892 | .06261 | 46 |
| 47 | .04260 | .04449 | .04778 | .05018 | .05326 | .05625 | .05902 | .06272 | 47 |
| 48 | .04268 | .04458 | .04787 | .05028 | .05335 | .05636 | .05912 | .06283 | 48 |
| 49 | .04276 | .04468 | .04796 | .05038 | .05344 | .05646 | .05922 | .06295 | 49 |
| 50 | .04285 | .04477 | .04805 | .05047 | .05354 | .05657 | .05932 | .06306 | 50 |
| 51 | .04293 | .04486 | .04814 | .05057 | .05363 | .05667 | .05942 | .06317 | 51 |
| 52 | .04302 | .04495 | .04823 | .05067 | .05373 | .05678 | .05951 | .06328 | 52 |
| 53 | .04310 | .04504 | .04832 | .05077 | .05382 | .05688 | .05961 | .06339 | 53 |
| 54 | .04319 | .04514 | .04841 | .05087 | .05391 | .05699 | .05971 | .06350 | 54 |
| 55 | .04327 | .04523 | .04850 | .05097 | .05401 | .05709 | .05981 | .06362 | 55 |
| 56 | .04336 | .04532 | .04858 | .05107 | .05410 | .05720 | .05991 | .06373 | 56 |
| 57 | .04344 | .04541 | .04867 | .05116 | .05420 | .05730 | .06001 | .06384 | 57 |
| 58 | .04353 | .04551 | .04876 | .05126 | .05429 | .05741 | .06011 | .06395 | 58 |
| 59 | .04361 | .04560 | .04885 | .05136 | .05439 | .05751 | .06021 | .06407 | 59 |
| 60 | .04370 | .04569 | .04894 | .05146 | .05448 | .05762 | .06031 | .06418 | 60 |

# Table IV.—Natural Versed Sines and External Secants.

| ′ | 20° | | 21° | | 22° | | 23° | | ′ |
|---|---|---|---|---|---|---|---|---|---|
| | Vers. | Ex. sec. | Vers. | Ex. sec. | Vers. | Ex. sec. | Vers. | Ex. sec. | |
| 0 | .06031 | .06418 | .06642 | .07115 | .07282 | .07853 | .07950 | .08636 | 0 |
| 1 | .06041 | .06429 | .06652 | .07126 | .07293 | .07866 | .07961 | .08649 | 1 |
| 2 | .06051 | .06440 | .06663 | .07138 | .07303 | .07879 | .07972 | .08663 | 2 |
| 3 | .06061 | .06452 | .06673 | .07150 | .07314 | .07892 | .07984 | .08676 | 3 |
| 4 | .06071 | .06463 | .06684 | .07162 | .07325 | .07904 | .07995 | .08690 | 4 |
| 5 | .06081 | .06474 | .06694 | .07174 | .07336 | .07917 | .08006 | .08703 | 5 |
| 6 | .06091 | .06486 | .06705 | .07186 | .07347 | .07930 | .08018 | .08717 | 6 |
| 7 | .06101 | .06497 | .06715 | .07199 | .07358 | .07943 | .08029 | .08730 | 7 |
| 8 | .06111 | .06508 | .06726 | .07211 | .07369 | .07955 | .08041 | .08744 | 8 |
| 9 | .06121 | .06520 | .06736 | .07223 | .07380 | .07968 | .08052 | .08757 | 9 |
| 10 | .06131 | .06531 | .06747 | .07235 | .07391 | .07981 | .08064 | .08771 | 10 |
| 11 | .06141 | .06542 | .06757 | .07247 | .07402 | .07994 | .08075 | .08784 | 11 |
| 12 | .06151 | .06554 | .06768 | .07259 | .07413 | .08006 | .08086 | .08798 | 12 |
| 13 | .06161 | .06565 | .06778 | .07271 | .07424 | .08019 | .08098 | .08811 | 13 |
| 14 | .06171 | .06577 | .06789 | .07283 | .07435 | .08032 | .08109 | .08825 | 14 |
| 15 | .06181 | .06588 | .06799 | .07295 | .07446 | .08045 | .08121 | .08839 | 15 |
| 16 | .06191 | .06600 | .06810 | .07307 | .07457 | .08058 | .08132 | .08852 | 16 |
| 17 | .06201 | .06611 | .06820 | .07320 | .07468 | .08071 | .08144 | .08866 | 17 |
| 18 | .06211 | .06622 | .06831 | .07332 | .07479 | .08084 | .08155 | .08880 | 18 |
| 19 | .06221 | .06634 | .06841 | .07344 | .07490 | .08097 | .08167 | .08893 | 19 |
| 20 | .06231 | .06645 | .06852 | .07356 | .07501 | .08109 | .08178 | .08907 | 20 |
| 21 | .06241 | .06657 | .06863 | .07368 | .07512 | .08122 | .08190 | .08921 | 21 |
| 22 | .06252 | .06668 | .06873 | .07380 | .07523 | .08135 | .08201 | .08934 | 22 |
| 23 | .06262 | .06680 | .06884 | .07393 | .07534 | .08148 | .08213 | .08948 | 23 |
| 24 | .06272 | .06691 | .06894 | .07405 | .07545 | .08161 | .08225 | .08962 | 24 |
| 25 | .06282 | .06703 | .06905 | .07417 | .07556 | .08174 | .08236 | .08975 | 25 |
| 26 | .06292 | .06715 | .06916 | .07429 | .07568 | .08187 | .08248 | .08989 | 26 |
| 27 | .06303 | .06726 | .06926 | .07442 | .07579 | .08200 | .08259 | .09003 | 27 |
| 28 | .06312 | .06738 | .06937 | .07454 | .07590 | .08213 | .08271 | .09017 | 28 |
| 29 | .06323 | .06749 | .06948 | .07466 | .07601 | .08226 | .08282 | .09030 | 29 |
| 30 | .06333 | .06761 | .06958 | .07479 | .07612 | .08239 | .08294 | .09044 | 30 |
| 31 | .06343 | .06773 | .06969 | .07491 | .07623 | .08252 | .08306 | .09058 | 31 |
| 32 | .06353 | .06784 | .06980 | .07503 | .07634 | .08265 | .08317 | .09072 | 32 |
| 33 | .06363 | .06796 | .06990 | .07516 | .07645 | .08278 | .08329 | .09086 | 33 |
| 34 | .06374 | .06807 | .07001 | .07528 | .07657 | .08291 | .08340 | .09099 | 34 |
| 35 | .06384 | .06819 | .07012 | .07540 | .07668 | .08305 | .08352 | .09113 | 35 |
| 36 | .06394 | .06831 | .07022 | .07553 | .07679 | .08318 | .08364 | .09127 | 36 |
| 37 | .06404 | .06843 | .07033 | .07565 | .07690 | .08331 | .08375 | .09141 | 37 |
| 38 | .06415 | .06854 | .07044 | .07578 | .07701 | .08344 | .08387 | .09155 | 38 |
| 39 | .06425 | .06866 | .07055 | .07590 | .07713 | .08357 | .08399 | .09169 | 39 |
| 40 | .06435 | .06878 | .07065 | .07602 | .07724 | .08370 | .08410 | .09183 | 40 |
| 41 | .06445 | .06889 | .07076 | .07615 | .07735 | .08383 | .08422 | .09197 | 41 |
| 42 | .06456 | .06901 | .07087 | .07627 | .07746 | .08397 | .08434 | .09211 | 42 |
| 43 | .06466 | .06913 | .07098 | .07640 | .07757 | .08410 | .08445 | .09224 | 43 |
| 44 | .06476 | .06925 | .07108 | .07652 | .07769 | .08423 | .08457 | .09238 | 44 |
| 45 | .06486 | .06936 | .07119 | .07665 | .07780 | .08436 | .08469 | .09252 | 45 |
| 46 | .06497 | .06948 | .07130 | .07677 | .07791 | .08449 | .08481 | .09266 | 46 |
| 47 | .06507 | .06960 | .07141 | .07690 | .07802 | .08463 | .08492 | .09280 | 47 |
| 48 | .06517 | .06972 | .07151 | .07702 | .07814 | .08476 | .08504 | .09294 | 48 |
| 49 | .06528 | .06984 | .07162 | .07715 | .07825 | .08489 | .08516 | .09308 | 49 |
| 50 | .06538 | .06995 | .07173 | .07727 | .07836 | .08503 | .08528 | .09323 | 50 |
| 51 | .06548 | .07007 | .07184 | .07740 | .07848 | .08516 | .08539 | .09337 | 51 |
| 52 | .06559 | .07019 | .07195 | .07752 | .07859 | .08529 | .08551 | .09351 | 52 |
| 53 | .06569 | .07031 | .07206 | .07765 | .07870 | .08542 | .08563 | .09365 | 53 |
| 54 | .06580 | .07043 | .07216 | .07778 | .07881 | .08556 | .08575 | .09379 | 54 |
| 55 | .06590 | .07055 | .07227 | .07790 | .07893 | .08569 | .08586 | .09393 | 55 |
| 56 | .06600 | .07067 | .07238 | .07803 | .07904 | .08582 | .08598 | .09407 | 56 |
| 57 | .06611 | .07079 | .07249 | .07816 | .07915 | .08596 | .08610 | .09421 | 57 |
| | .06621 | .07091 | .07260 | .07828 | .07927 | .08609 | .08622 | .09435 | 58 |
| | .06632 | .07103 | .07271 | .07841 | .07938 | .08623 | .08634 | .09449 | 59 |
| | .06642 | .07115 | .07282 | .07853 | .07950 | .08636 | .08645 | .09464 | 60 |

| ′ | 24° | | 25° | | 26° | | 27° | |
|---|---|---|---|---|---|---|---|---|
| | Vers. | Ex. sec. | Vers. | Ex. sec. | Vers. | Ex. sec. | Vers. | Ex. sec. |
| 0 | .08645 | .09464 | .09369 | .10338 | .10121 | .11260 | .10899 | .12233 |
| 1 | .08657 | .09478 | .09382 | .10353 | .10133 | .11276 | .10913 | .12249 |
| 2 | .08669 | .09492 | .09394 | .10368 | .10146 | .11292 | .10926 | .12266 |
| 3 | .08681 | .09506 | .09406 | .10383 | .10159 | .11308 | .10939 | .12283 |
| 4 | .08693 | .09520 | .09418 | .10398 | .10172 | .11323 | .10952 | .12299 |
| 5 | .08705 | .09535 | .09431 | .10413 | .10184 | .11339 | .10965 | .12316 |
| 6 | .08717 | .09549 | .09443 | .10428 | .10197 | .11355 | .10979 | .12333 |
| 7 | .08728 | .09563 | .09455 | .10443 | .10210 | .11371 | .10992 | .12349 |
| 8 | .08740 | .09577 | .09468 | .10458 | .10223 | .11387 | .11005 | .12366 |
| 9 | .08752 | .09592 | .09480 | .10473 | .10236 | .11403 | .11019 | .12383 |
| 10 | .08764 | .09606 | .09493 | .10488 | .10248 | .11419 | .11032 | .12400 |
| 11 | .08776 | .09620 | .09505 | .10503 | .10261 | .11435 | .11045 | .12416 |
| 12 | .08788 | .09635 | .09517 | .10518 | .10274 | .11451 | .11058 | .12433 |
| 13 | .08800 | .09649 | .09530 | .10533 | .10287 | .11467 | .11072 | .12450 |
| 14 | .08812 | .09663 | .09542 | .10549 | .10300 | .11483 | .11085 | .12467 |
| 15 | .08824 | .09678 | .09554 | .10564 | .10313 | .11499 | .11098 | .12484 |
| 16 | .08836 | .09692 | .09567 | .10579 | .10326 | .11515 | .11112 | .12501 |
| 17 | .08848 | .09707 | .09579 | .10594 | .10338 | .11531 | .11125 | .12518 |
| 18 | .08860 | .09721 | .09592 | .10609 | .10351 | .11547 | .11138 | .12534 |
| 19 | .08872 | .09735 | .09604 | .10625 | .10364 | .11563 | .11152 | .12551 |
| 20 | .08884 | .09750 | .09617 | .10640 | .10377 | .11579 | .11165 | .12568 |
| 21 | .08896 | .09764 | .09629 | .10655 | .10390 | .11595 | .11178 | .12585 |
| 22 | .08908 | .09779 | .09642 | .10670 | .10403 | .11611 | .11192 | .12602 |
| 23 | .08920 | .09793 | .09654 | .10686 | .10416 | .11627 | .11205 | .12619 |
| 24 | .08933 | .09808 | .09666 | .10701 | .10429 | .11643 | .11218 | .12636 |
| 25 | .08944 | .09822 | .09679 | .10716 | .10442 | .11659 | .11232 | .12653 |
| 26 | .08956 | .09837 | .09691 | .10731 | .10455 | .11675 | .11245 | .12670 |
| 27 | .08968 | .09851 | .09704 | .10747 | .10468 | .11691 | .11259 | .12687 |
| 28 | .08980 | .09866 | .09716 | .10762 | .10481 | .11708 | .11272 | .12704 |
| 29 | .08992 | .09880 | .09729 | .10777 | .10494 | .11724 | .11285 | .12721 |
| 30 | .09004 | .09895 | .09741 | .10793 | .10507 | .11740 | .11299 | .12738 |
| 31 | .09016 | .09909 | .09754 | .10808 | .10520 | .11756 | .11312 | .12755 |
| 32 | .09028 | .09924 | .09767 | .10824 | .10533 | .11772 | .11326 | .12772 |
| 33 | .09040 | .09939 | .09779 | .10839 | .10546 | .11789 | .11339 | .12789 |
| 34 | .09052 | .09953 | .09792 | .10854 | .10559 | .11805 | .11353 | .12807 |
| 35 | .09064 | .09968 | .09804 | .10870 | .10572 | .11821 | .11366 | .12824 |
| 36 | .09076 | .09983 | .09817 | .10885 | .10585 | .11838 | .11380 | .12841 |
| 37 | .09089 | .09997 | .09829 | .10901 | .10598 | .11854 | .11393 | .12858 |
| 38 | .09101 | .10012 | .09842 | .10916 | .10611 | .11870 | .11407 | .12875 |
| 39 | .09113 | .10026 | .09854 | .10932 | .10624 | .11886 | .11420 | .12892 |
| 40 | .09125 | .10041 | .09867 | .10947 | .10637 | .11903 | .11434 | .12910 |
| 41 | .09137 | .10055 | .09880 | .10963 | .10650 | .11919 | .11447 | .12927 |
| 42 | .09149 | .10071 | .09892 | .10978 | .10663 | .11936 | .11461 | .12944 |
| 43 | .09161 | .10085 | .09905 | .10994 | .10676 | .11952 | .11474 | .12961 |
| 44 | .09174 | .10100 | .09918 | .11009 | .10689 | .11968 | .11488 | .12979 |
| 45 | .09186 | .10115 | .09930 | .11025 | .10702 | .11985 | .11501 | .12996 |
| 46 | .09198 | .10130 | .09943 | .11041 | .10715 | .12001 | .11515 | .13013 |
| 47 | .09210 | .10144 | .09955 | .11056 | .10728 | .12018 | .11528 | .13031 |
| 48 | .09222 | .10159 | .09968 | .11072 | .10741 | .12034 | .11542 | .13048 |
| 49 | .09234 | .10174 | .09981 | .11087 | .10755 | .12051 | .11555 | .13065 |
| 50 | .09247 | .10189 | .09993 | .11103 | .10768 | .12067 | .11569 | .13083 |
| 51 | .09259 | .10204 | .10006 | .11119 | .10781 | .12084 | .11583 | .13100 |
| 52 | .09271 | .10218 | .10019 | .11134 | .10794 | .12100 | .11596 | .13117 |
| 53 | .09283 | .10233 | .10032 | .11150 | .10807 | .12117 | .11610 | .13135 |
| 54 | .09296 | .10248 | .10044 | .11166 | .10820 | .12133 | .11623 | .13152 |
| 55 | .09308 | .10263 | .10057 | .11181 | .10833 | .12150 | .11637 | .13170 |
| 56 | .09320 | .10278 | .10070 | .11197 | .10847 | .12166 | .11651 | .13187 |
| 57 | .09332 | .10293 | .10082 | .11213 | .10860 | .12183 | .11664 | .13205 |
| 58 | .09345 | .10308 | .10095 | .11229 | .10873 | .12199 | .11678 | .13222 |
| 59 | .09357 | .10323 | .10108 | .11244 | .10886 | .12216 | .11692 | .13240 |
| 60 | .09369 | .10338 | .10121 | .11260 | .10899 | .12233 | .11705 | .13257 |

| 28° | | 29° | | 30° | | 31° | | ′ |
|---|---|---|---|---|---|---|---|---|
| Vers. | Ex. sec. | Vers. | Ex. sec. | Vers. | Ex. sec. | Vers. | Ex. sec. | |
| .11705 | .13257 | .12538 | .14335 | .13397 | .15470 | .14283 | .16663 | 0 |
| .11719 | .13275 | .12552 | .14354 | .13412 | .15489 | .14298 | .16684 | 1 |
| .11733 | .13292 | .12566 | .14372 | .13427 | .15509 | .14313 | .16704 | 2 |
| .11746 | .13310 | .12580 | .14391 | .13441 | .15528 | .14328 | .16725 | 3 |
| .11760 | .13327 | .12595 | .14409 | .13456 | .15548 | .14343 | .16745 | 4 |
| .11774 | .13345 | .12609 | .14428 | .13470 | .15567 | .14358 | .16766 | 5 |
| .11787 | .13362 | .12623 | .14446 | .13485 | .15587 | .14373 | .16786 | 6 |
| .11801 | .13380 | .12637 | .14465 | .13499 | .15606 | .14388 | .16806 | 7 |
| .11815 | .13398 | .12651 | .14483 | .13514 | .15626 | .14403 | .16827 | 8 |
| .11828 | .13415 | .12665 | .14502 | .13529 | .15645 | .14418 | .16848 | 9 |
| .11842 | .13433 | .12679 | .14521 | .13543 | .15665 | .14433 | .16868 | 10 |
| .11856 | .13451 | .12694 | .14539 | .13558 | .15684 | .14449 | .16889 | 11 |
| .11870 | .13468 | .12708 | .14558 | .13573 | .15704 | .14464 | .16909 | 12 |
| .11883 | .13486 | .12722 | .14576 | .13587 | .15724 | .14479 | .16930 | 13 |
| .11897 | .13504 | .12736 | .14595 | .13602 | .15743 | .14494 | .16950 | 14 |
| .11911 | .13521 | .12750 | .14614 | .13616 | .15763 | .14509 | .16971 | 15 |
| .11925 | .13539 | .12765 | .14632 | .13631 | .15782 | .14524 | .16992 | 16 |
| .11938 | .13557 | .12779 | .14651 | .13646 | .15802 | .14539 | .17012 | 17 |
| .11952 | .13575 | .12793 | .14670 | .13660 | .15822 | .14554 | .17033 | 18 |
| .11966 | .13593 | .12807 | .14689 | .13675 | .15841 | .14569 | .17054 | 19 |
| .11980 | .13610 | .12822 | .14707 | .13690 | .15861 | .14584 | .17075 | 20 |
| .11994 | .13628 | .12836 | .14726 | .13705 | .15881 | .14599 | .17095 | 21 |
| .12007 | .13646 | .12850 | .14745 | .13719 | .15901 | .14615 | .17116 | 22 |
| .12021 | .13664 | .12864 | .14764 | .13734 | .15920 | .14630 | .17137 | 23 |
| .12035 | .13682 | .12879 | .14782 | .13749 | .15940 | .14645 | .17158 | 24 |
| .12049 | .13700 | .12893 | .14801 | .13763 | .15960 | .14660 | .17178 | 25 |
| .12063 | .13718 | .12907 | .14820 | .13778 | .15980 | .14675 | .17199 | 26 |
| .12077 | .13735 | .12921 | .14839 | .13793 | .16000 | .14690 | .17220 | 27 |
| .12091 | .13753 | .12936 | .14858 | .13808 | .16019 | .14706 | .17241 | 28 |
| .12104 | .13771 | .12950 | .14877 | .13822 | .16039 | .14721 | .17262 | 29 |
| .12118 | .13789 | .12964 | .14896 | .13837 | .16059 | .14736 | .17283 | 30 |
| .12132 | .13807 | .12979 | .14914 | .13852 | .16079 | .14751 | .17304 | 31 |
| .12146 | .13825 | .12993 | .14933 | .13867 | .16099 | .14766 | .17325 | 32 |
| .12160 | .13843 | .13007 | .14952 | .13881 | .16119 | .14782 | .17346 | 33 |
| .12174 | .13861 | .13022 | .14971 | .13896 | .16139 | .14797 | .17367 | 34 |
| .12188 | .13879 | .13036 | .14990 | .13911 | .16159 | .14812 | .17388 | 35 |
| .12202 | .13897 | .13051 | .15009 | .13926 | .16179 | .14827 | .17409 | 36 |
| .12216 | .13916 | .13065 | .15028 | .13941 | .16199 | .14843 | .17430 | 37 |
| .12230 | .13934 | .13079 | .15047 | .13955 | .16219 | .14858 | .17451 | 38 |
| .12244 | .13952 | .13094 | .15066 | .13970 | .16239 | .14873 | .17472 | 39 |
| .12257 | .13970 | .13108 | .15085 | .13985 | .16259 | .14888 | .17493 | 40 |
| .12271 | .13988 | .13122 | .15105 | .14000 | .16279 | .14904 | .17514 | 41 |
| .12285 | .14006 | .13137 | .15124 | .14015 | .16299 | .14919 | .17535 | 42 |
| .12299 | .14024 | .13151 | .15143 | .14030 | .16319 | .14934 | .17556 | 43 |
| .12313 | .14042 | .13166 | .15162 | .14044 | .16339 | .14949 | .17577 | 44 |
| .12327 | .14061 | .13180 | .15181 | .14059 | .16359 | .14965 | .17598 | 45 |
| .12341 | .14079 | .13195 | .15200 | .14074 | .16380 | .14980 | .17620 | 46 |
| .12355 | .14097 | .13209 | .15219 | .14089 | .16400 | .14995 | .17641 | 47 |
| .12369 | .14115 | .13223 | .15239 | .14104 | .16420 | .15011 | .17662 | 48 |
| .12383 | .14134 | .13238 | .15258 | .14119 | .16440 | .15026 | .17683 | 49 |
| .12397 | .14152 | .13252 | .15277 | .14134 | .16460 | .15041 | .17704 | 50 |
| .12411 | .14170 | .13267 | .15296 | .14149 | .16481 | .15057 | .17726 | 51 |
| .12425 | .14188 | .13281 | .15315 | .14164 | .16501 | .15072 | .17747 | 52 |
| .12439 | .14207 | .13296 | .15335 | .14179 | .16521 | .15087 | .17768 | 53 |
| .12454 | .14225 | .13310 | .15354 | .14194 | .16541 | .15103 | .17790 | 54 |
| .12468 | .14243 | .13325 | .15373 | .14208 | .16562 | .15118 | .17811 | 55 |
| .12482 | .14262 | .13339 | .15393 | .14223 | .16582 | .15134 | .17832 | 56 |
| .12496 | .14280 | .13354 | .15412 | .14238 | .16602 | .15149 | .17854 | 57 |
| .12510 | .14299 | .13368 | .15431 | .14253 | .16623 | .15164 | .17875 | 58 |
| .12524 | .14317 | .13383 | .15451 | .14268 | .16643 | .15180 | .17896 | 59 |
| .12538 | .14335 | .13397 | .15470 | .14283 | .16663 | .15195 | .17918 | 60 |

| ′ | 32° | | 33° | | 34° | | 35° | |
|---|---|---|---|---|---|---|---|---|
| | Vers. | Ex. sec. | Vers. | Ex. sec. | Vers. | Ex. sec. | Vers. | Ex. sec. |
| 0 | .15195 | .17918 | .16133 | .19236 | .17096 | .20622 | .18085 | .22077 |
| 1 | .15211 | .17939 | .16149 | .19259 | .17113 | .20645 | .18101 | .22102 |
| 2 | .15226 | .17961 | .16165 | .19281 | .17129 | .20669 | .18118 | .22127 |
| 3 | .15241 | .17982 | .16181 | .19304 | .17145 | .20693 | .18135 | .22152 |
| 4 | .15257 | .18004 | .16196 | .19327 | .17161 | .20717 | .18152 | .22177 |
| 5 | .15272 | .18025 | .16212 | .19349 | .17178 | .20740 | .18168 | .22202 |
| 6 | .15288 | .18047 | .16228 | .19372 | .17194 | .20764 | .18185 | .22227 |
| 7 | .15303 | .18068 | .16244 | .19394 | .17210 | .20788 | .18202 | .22252 |
| 8 | .15319 | .18090 | .16260 | .19417 | .17227 | .20812 | .18218 | .22277 |
| 9 | .15334 | .18111 | .16276 | .19440 | .17243 | .20836 | .18235 | .22302 |
| 10 | .15350 | .18133 | .16292 | .19463 | .17259 | .20859 | .18252 | .22327 |
| 11 | .15365 | .18155 | .16308 | .19485 | .17276 | .20883 | .18269 | .22352 |
| 12 | .15381 | .18176 | .16324 | .19508 | .17292 | .20907 | .18286 | .22377 |
| 13 | .15396 | .18198 | .16340 | .19531 | .17308 | .20931 | .18302 | .22402 |
| 14 | .15412 | .18220 | .16355 | .19554 | .17325 | .20955 | .18319 | .22428 |
| 15 | .15427 | .18241 | .16371 | .19576 | .17341 | .20979 | .18336 | .22453 |
| 16 | .15443 | .18263 | .16387 | .19599 | .17357 | .21003 | .18353 | .22478 |
| 17 | .15458 | .18285 | .16403 | .19622 | .17374 | .21027 | .18369 | .22503 |
| 18 | .15474 | .18307 | .16419 | .19645 | .17390 | .21051 | .18386 | .22528 |
| 19 | .15489 | .18328 | .16435 | .19668 | .17407 | .21075 | .18403 | .22554 |
| 20 | .15505 | .18350 | .16451 | .19691 | .17423 | .21099 | .18420 | .22579 |
| 21 | .15520 | .18372 | .16467 | .19713 | .17439 | .21123 | .18437 | .22604 |
| 22 | .15536 | .18394 | .16483 | .19736 | .17456 | .21147 | .18454 | .22629 |
| 23 | .15552 | .18416 | .16499 | .19759 | .17472 | .21171 | .18470 | .22655 |
| 24 | .15567 | .18437 | .16515 | .19782 | .17489 | .21195 | .18487 | .22680 |
| 25 | .15583 | .18459 | .16531 | .19805 | .17505 | .21220 | .18504 | .22706 |
| 26 | .15598 | .18481 | .16547 | .19828 | .17522 | .21244 | .18521 | .22731 |
| 27 | .15614 | .18503 | .16563 | .19851 | .17538 | .21268 | .18538 | .22756 |
| 28 | .15630 | .18525 | .16579 | .19874 | .17554 | .21292 | .18555 | .22782 |
| 29 | .15645 | .18547 | .16595 | .19897 | .17571 | .21316 | .18572 | .22807 |
| 30 | .15661 | .18569 | .16611 | .19920 | .17587 | .21341 | .18588 | .22833 |
| 31 | .15676 | .18591 | .16627 | .19944 | .17604 | .21365 | .18605 | .22858 |
| 32 | .15692 | .18613 | .16644 | .19967 | .17620 | .21389 | .18622 | .22884 |
| 33 | .15708 | .18635 | .16660 | .19990 | .17637 | .21414 | .18639 | .22909 |
| 34 | .15723 | .18657 | .16676 | .20013 | .17653 | .21438 | .18656 | .22935 |
| 35 | .15739 | .18679 | .16692 | .20036 | .17670 | .21462 | .18673 | .22960 |
| 36 | .15755 | .18701 | .16708 | .20059 | .17686 | .21487 | .18690 | .22986 |
| 37 | .15770 | .18723 | .16724 | .20083 | .17703 | .21511 | .18707 | .23012 |
| 38 | .15786 | .18745 | .16740 | .20106 | .17719 | .21535 | .18724 | .23037 |
| 39 | .15802 | .18767 | .16756 | .20129 | .17736 | .21560 | .18741 | .23063 |
| 40 | .15818 | .18790 | .16772 | .20152 | .17752 | .21584 | .18758 | .23089 |
| 41 | .15833 | .18812 | .16788 | .20176 | .17769 | .21609 | .18775 | .23114 |
| 42 | .15849 | .18834 | .16805 | .20199 | .17786 | .21633 | .18792 | .23140 |
| 43 | .15865 | .18856 | .16821 | .20222 | .17802 | .21658 | .18809 | .23166 |
| 44 | .15880 | .18878 | .16837 | .20246 | .17819 | .21682 | .18826 | .23192 |
| 45 | .15896 | .18901 | .16853 | .20269 | .17835 | .21707 | .18843 | .23217 |
| 46 | .15912 | .18923 | .16869 | .20292 | .17852 | .21731 | .18860 | .23243 |
| 47 | .15928 | .18945 | .16885 | .20316 | .17868 | .21756 | .18877 | .23269 |
| 48 | .15943 | .18967 | .16902 | .20339 | .17885 | .21781 | .18894 | .23295 |
| 49 | .15959 | .18990 | .16918 | .20363 | .17902 | .21805 | .18911 | .23321 |
| 50 | .15975 | .19012 | .16934 | .20386 | .17918 | .21830 | .18928 | .23347 |
| 51 | .15991 | .19034 | .16950 | .20410 | .17935 | .21855 | .18945 | .23373 |
| 52 | .16006 | .19057 | .16966 | .20433 | .17952 | .21879 | .18962 | .23399 |
| 53 | .16022 | .19079 | .16983 | .20457 | .17968 | .21904 | .18979 | .23424 |
| 54 | .16038 | .19102 | .16999 | .20480 | .17985 | .21929 | .18996 | .23450 |
| 55 | .16054 | .19124 | .17015 | .20504 | .18001 | .21953 | .19013 | .23476 |
| 56 | .16070 | .19146 | .17031 | .20527 | .18018 | .21978 | .19030 | .23502 |
| 57 | .16085 | .19169 | .17047 | .20551 | .18035 | .22003 | .19047 | .23529 |
| 58 | .16101 | .19191 | .17064 | .20575 | .18051 | .22028 | .19064 | .23555 |
| 59 | .16117 | .19214 | .17080 | .20598 | .18068 | .22053 | .19081 | .23581 |
| 60 | .16133 | .19236 | .17096 | .20622 | .18085 | .22077 | .19098 | .23607 |

## Table IV.—Natural Versed Sines and External Secants.

| ′ | 36° | | 37° | | 38° | | 39° | | ′ |
|---|---|---|---|---|---|---|---|---|---|
| | Vers. | Ex. sec. | Vers. | Ex. sec. | Vers. | Ex. sec. | Vers. | Ex. sec. | |
| 0 | .19098 | .23607 | .20136 | .25214 | .21199 | .26902 | .22285 | .28676 | 0 |
| 1 | .19115 | .23638 | .20154 | .25241 | .21217 | .26931 | .22304 | .28706 | 1 |
| 2 | .19133 | .23659 | .20171 | .25269 | .21235 | .26960 | .22322 | .28737 | 2 |
| 3 | .19150 | .23685 | .20189 | .25296 | .21253 | .26988 | .22340 | .28767 | 3 |
| 4 | .19167 | .23711 | .20207 | .25324 | .21271 | .27017 | .22359 | .28797 | 4 |
| 5 | .19184 | .23738 | .20224 | .25351 | .21289 | .27046 | .22377 | .28828 | 5 |
| 6 | .19201 | .23764 | .20242 | .25379 | .21307 | .27075 | .22395 | .28858 | 6 |
| 7 | .19218 | .23790 | .20259 | .25406 | .21324 | .27104 | .22414 | .28889 | 7 |
| 8 | .19235 | .23816 | .20277 | .25434 | .21342 | .27133 | .22432 | .28919 | 8 |
| 9 | .19252 | .23843 | .20294 | .25462 | .21360 | .27162 | .22450 | .28950 | 9 |
| 10 | .19270 | .23869 | .20313 | .25489 | .21378 | .27191 | .22469 | .28980 | 10 |
| 11 | .19287 | .23895 | .20329 | .25517 | .21396 | .27221 | .22487 | .29011 | 11 |
| 12 | .19304 | .23922 | .20347 | .25545 | .21414 | .27250 | .22506 | .29042 | 12 |
| 13 | .19321 | .23948 | .20365 | .25572 | .21433 | .27279 | .22524 | .29072 | 13 |
| 14 | .19338 | .23975 | .20382 | .25600 | .21450 | .27308 | .22542 | .29103 | 14 |
| 15 | .19356 | .24001 | .20400 | .25628 | .21468 | .27337 | .22561 | .29133 | 15 |
| 16 | .19373 | .24028 | .20417 | .25656 | .21486 | .27366 | .22579 | .29164 | 16 |
| 17 | .19390 | .24054 | .20435 | .25683 | .21504 | .27396 | .22598 | .29195 | 17 |
| 18 | .19407 | .24081 | .20453 | .25711 | .21522 | .27425 | .22616 | .29226 | 18 |
| 19 | .19424 | .24107 | .20470 | .25739 | .21540 | .27454 | .22634 | .29256 | 19 |
| 20 | .19442 | .24134 | .20488 | .25767 | .21558 | .27483 | .22653 | .29287 | 20 |
| 21 | .19459 | .24160 | .20506 | .25795 | .21576 | .27513 | .22671 | .29318 | 21 |
| 22 | .19476 | .24187 | .20523 | .25823 | .21595 | .27542 | .22690 | .29349 | 22 |
| 23 | .19493 | .24213 | .20541 | .25851 | .21613 | .27572 | .22708 | .29380 | 23 |
| 24 | .19511 | .24240 | .20559 | .25879 | .21631 | .27601 | .22727 | .29411 | 24 |
| 25 | .19528 | .24267 | .20576 | .25907 | .21649 | .27630 | .22745 | .29442 | 25 |
| 26 | .19545 | .24293 | .20594 | .25935 | .21667 | .27660 | .22764 | .29473 | 26 |
| 27 | .19562 | .24320 | .20612 | .25963 | .21685 | .27689 | .22782 | .29504 | 27 |
| 28 | .19580 | .24347 | .20629 | .25991 | .21703 | .27719 | .22801 | .29535 | 28 |
| 29 | .19597 | .24373 | .20647 | .26019 | .21721 | .27748 | .22819 | .29566 | 29 |
| 30 | .19614 | .24400 | .20665 | .26047 | .21739 | .27778 | .22838 | .29597 | 30 |
| 31 | .19632 | .24427 | .20682 | .26075 | .21757 | .27807 | .22856 | .29628 | 31 |
| 32 | .19649 | .24454 | .20700 | .26104 | .21775 | .27837 | .22875 | .29659 | 32 |
| 33 | .19666 | .24481 | .20718 | .26132 | .21794 | .27867 | .22893 | .29690 | 33 |
| 34 | .19684 | .24508 | .20736 | .26160 | .21812 | .27896 | .22912 | .29721 | 34 |
| 35 | .19701 | .24534 | .20753 | .26188 | .21830 | .27926 | .22930 | .29752 | 35 |
| 36 | .19718 | .24561 | .20771 | .26216 | .21848 | .27956 | .22949 | .29784 | 36 |
| 37 | .19736 | .24588 | .20789 | .26245 | .21866 | .27985 | .22967 | .29815 | 37 |
| 38 | .19753 | .24615 | .20807 | .26273 | .21884 | .28015 | .22986 | .29846 | 38 |
| 39 | .19770 | .24642 | .20824 | .26301 | .21902 | .28045 | .23004 | .29877 | 39 |
| 40 | .19788 | .24669 | .20842 | .26330 | .21921 | .28075 | .23023 | .29909 | 40 |
| 41 | .19805 | .24696 | .20860 | .26358 | .21939 | .28105 | .23041 | .29940 | 41 |
| 42 | .19822 | .24723 | .20878 | .26387 | .21957 | .28134 | .23060 | .29971 | 42 |
| 43 | .19840 | .24750 | .20895 | .26415 | .21975 | .28164 | .23079 | .30003 | 43 |
| 44 | .19857 | .24777 | .20913 | .26443 | .21993 | .28194 | .23097 | .30034 | 44 |
| 45 | .19875 | .24804 | .20931 | .26472 | .22012 | .28224 | .23116 | .30066 | 45 |
| 46 | .19892 | .24832 | .20949 | .26500 | .22030 | .28254 | .23134 | .30097 | 46 |
| 47 | .19909 | .24859 | .20967 | .26529 | .22048 | .28284 | .23153 | .30129 | 47 |
| 48 | .19927 | .24886 | .20985 | .26557 | .22066 | .28314 | .23172 | .30160 | 48 |
| 49 | .19944 | .24913 | .21002 | .26586 | .22084 | .28344 | .23190 | .30192 | 49 |
| 50 | .19962 | .24940 | .21020 | .26615 | .22103 | .28374 | .23209 | .30223 | 50 |
| 51 | .19979 | .24967 | .21038 | .26643 | .22121 | .28404 | .23228 | .30255 | 51 |
| 52 | .19997 | .24995 | .21056 | .26672 | .22139 | .28434 | .23246 | .30287 | 52 |
| 53 | .20014 | .25022 | .21074 | .26701 | .22157 | .28464 | .23265 | .30318 | 53 |
| 54 | .20032 | .25049 | .21092 | .26729 | .22176 | .28495 | .23283 | .30350 | 54 |
| 55 | .20049 | .25077 | .21109 | .26758 | .22194 | .28525 | .23302 | .30382 | 55 |
| 56 | .20066 | .25104 | .21127 | .26787 | .22212 | .28555 | .23321 | .30413 | 56 |
| 57 | .20084 | .25131 | .21145 | .26815 | .22231 | .28585 | .23339 | .30445 | 57 |
| 58 | .20101 | .25159 | .21163 | .26844 | .22249 | .28615 | .23358 | .30477 | 58 |
| 59 | .20119 | .25186 | .21181 | .26873 | .22267 | .28646 | .23377 | .30509 | 59 |
| 60 | .20136 | .25214 | .21199 | .26902 | .22285 | .28676 | .23396 | .30541 | 60 |

## TABLE IV.—NATURAL VERSED SINES AND EXTERNAL SECANTS

| ′ | 40° | | 41° | | 42° | | 43° | | ′ |
|---|---|---|---|---|---|---|---|---|---|
| | Vers. | Ex. sec. | Vers. | Ex. sec. | Vers. | Ex. sec. | Vers. | Ex. sec. | |
| 0 | .23396 | .30541 | .24529 | .32501 | .25686 | .34563 | .26865 | .36733 | 0 |
| 1 | .23414 | .30573 | .24548 | .32535 | .25705 | .34599 | .26884 | .36770 | 1 |
| 2 | .23433 | .30605 | .24567 | .32568 | .25724 | .34634 | .26904 | .36807 | 2 |
| 3 | .23452 | .30636 | .24586 | .32602 | .25744 | .34669 | .26924 | .36844 | 3 |
| 4 | .23470 | .30668 | .24605 | .32636 | .25763 | .34704 | .26944 | .36881 | 4 |
| 5 | .23489 | .30700 | .24625 | .32669 | .25783 | .34740 | .26964 | .36919 | 5 |
| 6 | .23508 | .30732 | .24644 | .32703 | .25802 | .34775 | .26984 | .36956 | 6 |
| 7 | .23527 | .30764 | .24663 | .32737 | .25822 | .34811 | .27004 | .36993 | 7 |
| 8 | .23545 | .30796 | .24682 | .32770 | .25841 | .34846 | .27024 | .37030 | 8 |
| 9 | .23564 | .30829 | .24701 | .32804 | .25861 | .34882 | .27043 | .37068 | 9 |
| 10 | .23583 | .30861 | .24720 | .32838 | .25880 | .34917 | .27063 | .37105 | 10 |
| 11 | .23602 | .30893 | .24739 | .32872 | .25900 | .34953 | .27083 | .37143 | 11 |
| 12 | .23620 | .30925 | .24759 | .32905 | .25920 | .34988 | .27103 | .37180 | 12 |
| 13 | .23639 | .30957 | .24778 | .32939 | .25939 | .35024 | .27123 | .37218 | 13 |
| 14 | .23658 | .30989 | .24797 | .32973 | .25959 | .35060 | .27143 | .37255 | 14 |
| 15 | .23677 | .31022 | .24816 | .33007 | .25978 | .35095 | .27163 | .37293 | 15 |
| 16 | .23696 | .31054 | .24835 | .33041 | .25998 | .35131 | .27183 | .37330 | 16 |
| 17 | .23714 | .31086 | .24854 | .33075 | .26017 | .35167 | .27203 | .37368 | 17 |
| 18 | .23733 | .31119 | .24874 | .33109 | .26037 | .35203 | .27223 | .37406 | 18 |
| 19 | .23752 | .31151 | .24893 | .33143 | .26056 | .35238 | .27243 | .37443 | 19 |
| 20 | .23771 | .31183 | .24912 | .33177 | .26076 | .35274 | .27263 | .37481 | 20 |
| 21 | .23790 | .31216 | .24931 | .33211 | .26096 | .35310 | .27283 | .37519 | 21 |
| 22 | .23808 | .31248 | .24950 | .33245 | .26115 | .35346 | .27303 | .37556 | 22 |
| 23 | .23827 | .31281 | .24970 | .33279 | .26135 | .35382 | .27323 | .37594 | 23 |
| 24 | .23846 | .31313 | .24989 | .33314 | .26154 | .35418 | .27343 | .37632 | 24 |
| 25 | .23865 | .31346 | .25008 | .33348 | .26174 | .35454 | .27363 | .37670 | 25 |
| 26 | .23884 | .31378 | .25027 | .33382 | .26194 | .35490 | .27383 | .37708 | 26 |
| 27 | .23903 | .31411 | .25047 | .33416 | .26213 | .35526 | .27403 | .37746 | 27 |
| 28 | .23922 | .31443 | .25066 | .33451 | .26233 | .35562 | .27423 | .37784 | 28 |
| 29 | .23941 | .31476 | .25085 | .33485 | .26253 | .35598 | .27443 | .37822 | 29 |
| 30 | .23959 | .31509 | .25104 | .33519 | .26272 | .35634 | .27463 | .37860 | 30 |
| 31 | .23978 | .31541 | .25124 | .33554 | .26292 | .35670 | .27483 | .37898 | 31 |
| 32 | .23997 | .31574 | .25143 | .33588 | .26312 | .35707 | .27503 | .37936 | 32 |
| 33 | .24016 | .31607 | .25162 | .33622 | .26331 | .35743 | .27523 | .37974 | 33 |
| 34 | .24035 | .31640 | .25182 | .33657 | .26351 | .35779 | .27543 | .38012 | 34 |
| 35 | .24054 | .31672 | .25201 | .33691 | .26371 | .35815 | .27563 | .38051 | 35 |
| 36 | .24073 | .31705 | .25220 | .33726 | .26390 | .35852 | .27583 | .38089 | 36 |
| 37 | .24092 | .31738 | .25240 | .33760 | .26410 | .35888 | .27603 | .38127 | 37 |
| 38 | .24111 | .31771 | .25259 | .33795 | .26430 | .35924 | .27623 | .38165 | 38 |
| 39 | .24130 | .31804 | .25278 | .33830 | .26449 | .35961 | .27643 | .38204 | 39 |
| 40 | .24149 | .31837 | .25297 | .33864 | .26469 | .35997 | .27663 | .38242 | 40 |
| 41 | .24168 | .31870 | .25317 | .33899 | .26489 | .36034 | .27683 | .38280 | 41 |
| 42 | .24187 | .31903 | .25336 | .33934 | .26509 | .36070 | .27703 | .38319 | 42 |
| 43 | .24206 | .31936 | .25356 | .33968 | .26528 | .36107 | .27723 | .38357 | 43 |
| 44 | .24225 | .31969 | .25375 | .34003 | .26548 | .36143 | .27743 | .38396 | 44 |
| 45 | .24244 | .32002 | .25394 | .34038 | .26568 | .36180 | .27764 | .38434 | 45 |
| 46 | .24262 | .32035 | .25414 | .34073 | .26588 | .36217 | .27784 | .38473 | 46 |
| 47 | .24281 | .32068 | .25433 | .34108 | .26607 | .36253 | .27804 | .38512 | 47 |
| 48 | .24300 | .32101 | .25452 | .34142 | .26627 | .36290 | .27824 | .38550 | 48 |
| 49 | .24320 | .32134 | .25472 | .34177 | .26647 | .36327 | .27844 | .38589 | 49 |
| 50 | .24339 | .32168 | .25491 | .34212 | .26667 | .36363 | .27864 | .38628 | 50 |
| 51 | .24358 | .32201 | .25511 | .34247 | .26686 | .36400 | .27884 | .38666 | 51 |
| 52 | .24377 | .32234 | .25530 | .34282 | .26706 | .36437 | .27905 | .38705 | 52 |
| 53 | .24396 | .32267 | .25549 | .34317 | .26726 | .36474 | .27925 | .38744 | 53 |
| 54 | .24415 | .32301 | .25569 | .34352 | .26746 | .36511 | .27945 | .38783 | 54 |
| 55 | .24434 | .32334 | .25588 | .34387 | .26766 | .36548 | .27965 | .38822 | 55 |
| 56 | .24453 | .32368 | .25608 | .34423 | .26785 | .36585 | .27985 | .38860 | 56 |
| 57 | .24472 | .32401 | .25627 | .34458 | .26805 | .36622 | .28005 | .38899 | 57 |
| 58 | .24491 | .32434 | .25647 | .34493 | .26825 | .36659 | .28026 | .38938 | 58 |
| 59 | .24510 | .32468 | .25666 | .34528 | .26845 | .36696 | .28046 | .38977 | 59 |
| 60 | .24529 | .32501 | .25686 | .34563 | .26865 | .36733 | .28066 | .39016 | 60 |

## Table IV.—Natural Versed Sines and External Secants.

| ′ | 44° | | 45° | | 46° | | 47° | | ′ |
|---|---|---|---|---|---|---|---|---|---|
| | Vers. | Ex. sec. | Vers. | Ex. sec. | Vers. | Ex. sec. | Vers. | Ex. sec. | |
| 0 | .28066 | .39016 | .29289 | .41421 | .30534 | .43956 | .31800 | .46628 | 0 |
| 1 | .28086 | .39055 | .29310 | .41463 | .30555 | .43999 | .31821 | .46674 | 1 |
| 2 | .28106 | .39095 | .29330 | .41504 | .30576 | .44042 | .31843 | .46719 | 2 |
| 3 | .28127 | .39134 | .29351 | .41545 | .30597 | .44086 | .31864 | .46765 | 3 |
| 4 | .28147 | .39173 | .29372 | .41586 | .30618 | .44129 | .31885 | .46811 | 4 |
| 5 | .28167 | .39212 | .29392 | .41627 | .30639 | .44173 | .31907 | .46857 | 5 |
| 6 | .28187 | .39251 | .29413 | .41669 | .30660 | .44217 | .31928 | .46903 | 6 |
| 7 | .28208 | .39291 | .29433 | .41710 | .30681 | .44260 | .31949 | .46949 | 7 |
| 8 | .28228 | .39330 | .29454 | .41752 | .30702 | .44304 | .31971 | .46995 | 8 |
| 9 | .28248 | .39369 | .29475 | .41793 | .30723 | .44347 | .31992 | .47041 | 9 |
| 10 | .28268 | .39409 | .29495 | .41835 | .30744 | .44391 | .32013 | .47087 | 10 |
| 11 | .28289 | .39448 | .29516 | .41876 | .30765 | .44435 | .32035 | .47134 | 11 |
| 12 | .28309 | .39487 | .29537 | .41918 | .30786 | .44479 | .32056 | .47180 | 12 |
| 13 | .28329 | .39527 | .29557 | .41959 | .30807 | .44523 | .32077 | .47226 | 13 |
| 14 | .28350 | .39566 | .29578 | .42001 | .30828 | .44567 | .32099 | .47272 | 14 |
| 15 | .28370 | .39606 | .29599 | .42042 | .30849 | .44610 | .32120 | .47319 | 15 |
| 16 | .28390 | .39646 | .29619 | .42084 | .30870 | .44654 | .32141 | .47365 | 16 |
| 17 | .28410 | .39685 | .29640 | .42126 | .30891 | .44698 | .32163 | .47411 | 17 |
| 18 | .28431 | .39725 | .29661 | .42168 | .30912 | .44742 | .32184 | .47458 | 18 |
| 19 | .28451 | .39764 | .29681 | .42210 | .30933 | .44787 | .32205 | .47504 | 19 |
| 20 | .28471 | .39804 | .29702 | .42251 | .30954 | .44831 | .32227 | .47551 | 20 |
| 21 | .28492 | .39844 | .29723 | .42293 | .30975 | .44875 | .32248 | .47598 | 21 |
| 22 | .28512 | .39884 | .29743 | .42335 | .30996 | .44919 | .32270 | .47644 | 22 |
| 23 | .28532 | .39924 | .29764 | .42377 | .31017 | .44963 | .32291 | .47691 | 23 |
| 24 | .28553 | .39963 | .29785 | .42419 | .31038 | .45007 | .32312 | .47738 | 24 |
| 25 | .28573 | .40003 | .29805 | .42461 | .31059 | .45052 | .32334 | .47784 | 25 |
| 26 | .28593 | .40043 | .29826 | .42503 | .31080 | .45096 | .32355 | .47831 | 26 |
| 27 | .28614 | .40083 | .29847 | .42545 | .31101 | .45141 | .32377 | .47878 | 27 |
| 28 | .28634 | .40123 | .29868 | .42587 | .31122 | .45185 | .32398 | .47925 | 28 |
| 29 | .28655 | .40163 | .29888 | .42630 | .31143 | .45229 | .32420 | .47972 | 29 |
| 30 | .28675 | .40203 | .29909 | .42672 | .31165 | .45274 | .32441 | .48019 | 30 |
| 31 | .28695 | .40243 | .29930 | .42714 | .31186 | .45319 | .32462 | .48066 | 31 |
| 32 | .28716 | .40283 | .29951 | .42756 | .31207 | .45363 | .32484 | .48113 | 32 |
| 33 | .28736 | .40324 | .29971 | .42799 | .31228 | .45408 | .32505 | .48160 | 33 |
| 34 | .28757 | .40364 | .29992 | .42841 | .31249 | .45452 | .32527 | .48207 | 34 |
| 35 | .28777 | .40404 | .30013 | .42883 | .31270 | .45497 | .32548 | .48254 | 35 |
| 36 | .28797 | .40444 | .30034 | .42926 | .31291 | .45542 | .32570 | .48301 | 36 |
| 37 | .28818 | .40485 | .30054 | .42968 | .31312 | .45587 | .32591 | .48349 | 37 |
| 38 | .28838 | .40525 | .30075 | .43011 | .31334 | .45631 | .32613 | .48396 | 38 |
| 39 | .28859 | .40565 | .30096 | .43053 | .31355 | .45676 | .32634 | .48443 | 39 |
| 40 | .28879 | .40606 | .30117 | .43096 | .31376 | .45721 | .32656 | .48491 | 40 |
| 41 | .28900 | .40646 | .30138 | .43139 | .31397 | .45766 | .32677 | .48538 | 41 |
| 42 | .28920 | .40687 | .30158 | .43181 | .31418 | .45811 | .32699 | .48586 | 42 |
| 43 | .28941 | .40727 | .30179 | .43224 | .31439 | .45856 | .32720 | .48633 | 43 |
| 44 | .28961 | .40768 | .30200 | .43267 | .31461 | .45901 | .32742 | .48681 | 44 |
| 45 | .28981 | .40808 | .30221 | .43310 | .31482 | .45946 | .32763 | .48728 | 45 |
| 46 | .29002 | .40849 | .30242 | .43353 | .31503 | .45992 | .32785 | .48776 | 46 |
| 47 | .29022 | .40890 | .30263 | .43395 | .31524 | .46037 | .32806 | .48824 | 47 |
| 48 | .29043 | .40930 | .30283 | .43438 | .31545 | .46082 | .32828 | .48871 | 48 |
| 49 | .29063 | .40971 | .30304 | .43481 | .31567 | .46127 | .32849 | .48919 | 49 |
| 50 | .29084 | .41012 | .30325 | .43524 | .31588 | .46173 | .32871 | .48967 | 50 |
| 51 | .29104 | .41053 | .30346 | .43567 | .31609 | .46218 | .32893 | .49015 | 51 |
| 52 | .29125 | .41093 | .30367 | .43610 | .31630 | .46263 | .32914 | .49063 | 52 |
| 53 | .29145 | .41134 | .30388 | .43653 | .31651 | .46309 | .32936 | .49111 | 53 |
| 54 | .29166 | .41175 | .30409 | .43696 | .31673 | .46354 | .32957 | .49159 | 54 |
| 55 | .29187 | .41216 | .30430 | .43739 | .31694 | .46400 | .32979 | .49207 | 55 |
| 56 | .29207 | .41257 | .30451 | .43783 | .31715 | .46445 | .33001 | .49255 | 56 |
| 57 | .29228 | .41298 | .30471 | .43826 | .31736 | .46491 | .33022 | .49303 | 57 |
| 58 | .29248 | .41339 | .30492 | .43869 | .31758 | .46537 | .33044 | .49351 | 58 |
| 59 | .29269 | .41380 | .30513 | .43912 | .31779 | .46582 | .33065 | .49399 | 59 |
| 60 | .29289 | .41421 | .30534 | .43956 | .31800 | .46628 | .33087 | .49448 | 60 |

## TABLE IV.—NATURAL VERSED SINES AND EXTERNAL SECANTS.

| ′ | 48° | | 49° | | 50° | | 51° | | ′ |
|---|---|---|---|---|---|---|---|---|---|
| | Vers. | Ex. sec. | Vers. | Ex. sec. | Vers. | Ex. sec. | Vers. | Ex. sec. | |
| 0 | .33067 | .49448 | .34394 | .52425 | .35721 | .55572 | .37068 | .58902 | 0 |
| 1 | .33109 | .49496 | .34416 | .52476 | .35744 | .55626 | .37091 | .58959 | 1 |
| 2 | .33130 | .49544 | .34438 | .52527 | .35766 | .55680 | .37113 | .59016 | 2 |
| 3 | .33152 | .49593 | .34460 | .52579 | .35788 | .55734 | .37136 | .59073 | 3 |
| 4 | .33173 | .49641 | .34482 | .52630 | .35810 | .55789 | .37158 | .59130 | 4 |
| 5 | .33195 | .49690 | .34504 | .52681 | .35833 | .55843 | .37181 | .59188 | 5 |
| 6 | .33217 | .49738 | .34526 | .52732 | .35855 | .55897 | .37204 | .59245 | 6 |
| 7 | .33238 | .49787 | .34548 | .52784 | .35877 | .55951 | .37226 | .59302 | 7 |
| 8 | .33260 | .49835 | .34570 | .52835 | .35900 | .56005 | .37249 | .59360 | 8 |
| 9 | .33282 | .49884 | .34592 | .52886 | .35922 | .56060 | .37272 | .59418 | 9 |
| 10 | .33303 | .49933 | .34614 | .52938 | .35944 | .56114 | .37294 | .59475 | 10 |
| 11 | .33325 | .49981 | .34636 | .52989 | .35967 | .56169 | .37317 | .59533 | 11 |
| 12 | .33347 | .50030 | .34658 | .53041 | .35989 | .56223 | .37340 | .59590 | 12 |
| 13 | .33368 | .50079 | .34680 | .53092 | .36011 | .56278 | .37362 | .59648 | 13 |
| 14 | .33390 | .50128 | .34702 | .53144 | .36034 | .56332 | .37385 | .59706 | 14 |
| 15 | .33412 | .50177 | .34724 | .53196 | .36056 | .56387 | .37408 | .59764 | 15 |
| 16 | .33434 | .50226 | .34746 | .53247 | .36078 | .56442 | .37430 | .59822 | 16 |
| 17 | .33455 | .50275 | .34768 | .53299 | .36101 | .56497 | .37453 | .59880 | 17 |
| 18 | .33477 | .50324 | .34790 | .53351 | .36123 | .56551 | .37476 | .59938 | 18 |
| 19 | .33499 | .50373 | .34812 | .53403 | .36146 | .56606 | .37498 | .59996 | 19 |
| 20 | .33520 | .50422 | .34834 | .53455 | .36168 | .56661 | .37521 | .60054 | 20 |
| 21 | .33542 | .50471 | .34856 | .53507 | .36190 | .56716 | .37544 | .60112 | 21 |
| 22 | .33564 | .50521 | .34878 | .53559 | .36213 | .56771 | .37567 | .60171 | 22 |
| 23 | .33586 | .50570 | .34900 | .53611 | .36235 | .56826 | .37589 | .60229 | 23 |
| 24 | .33607 | .50619 | .34923 | .53663 | .36258 | .56881 | .37612 | .60287 | 24 |
| 25 | .33629 | .50669 | .34945 | .53715 | .36280 | .56937 | .37635 | .60346 | 25 |
| 26 | .33651 | .50718 | .34967 | .53768 | .36302 | .56992 | .37658 | .60404 | 26 |
| 27 | .33673 | .50767 | .34989 | .53820 | .36325 | .57047 | .37680 | .60463 | 27 |
| 28 | .33694 | .50817 | .35011 | .53872 | .36347 | .57103 | .37703 | .60521 | 28 |
| 29 | .33716 | .50866 | .35033 | .53924 | .36370 | .57158 | .37726 | .60580 | 29 |
| 30 | .33738 | .50916 | .35055 | .53977 | .36392 | .57213 | .37749 | .60639 | 30 |
| 31 | .33760 | .50966 | .35077 | .54029 | .36415 | .57269 | .37771 | .60698 | 31 |
| 32 | .33782 | .51015 | .35099 | .54082 | .36437 | .57324 | .37794 | .60756 | 32 |
| 33 | .33803 | .51065 | .35122 | .54134 | .36460 | .57380 | .37817 | .60815 | 33 |
| 34 | .33825 | .51115 | .35144 | .54187 | .36482 | .57436 | .37840 | .60874 | 34 |
| 35 | .33847 | .51165 | .35166 | .54240 | .36504 | .57491 | .37862 | .60933 | 35 |
| 36 | .33869 | .51215 | .35188 | .54292 | .36527 | .57547 | .37885 | .60992 | 36 |
| 37 | .33891 | .51265 | .35210 | .54345 | .36549 | .57603 | .37908 | .61051 | 37 |
| 38 | .33912 | .51314 | .35232 | .54398 | .36572 | .57659 | .37931 | .61111 | 38 |
| 39 | .33934 | .51364 | .35254 | .54451 | .36594 | .57715 | .37954 | .61170 | 39 |
| 40 | .33956 | .51415 | .35277 | .54504 | .36617 | .57771 | .37976 | .61229 | 40 |
| 41 | .33978 | .51465 | .35299 | .54557 | .36639 | .57827 | .37999 | .61288 | 41 |
| 42 | .34000 | .51515 | .35321 | .54610 | .36662 | .57883 | .38022 | .61348 | 42 |
| 43 | .34022 | .51565 | .35343 | .54663 | .36684 | .57939 | .38045 | .61407 | 43 |
| 44 | .34044 | .51615 | .35365 | .54716 | .36707 | .57995 | .38068 | .61467 | 44 |
| 45 | .34065 | .51665 | .35388 | .54769 | .36729 | .58051 | .38091 | .61526 | 45 |
| 46 | .34087 | .51716 | .35410 | .54822 | .36752 | .58108 | .38113 | .61586 | 46 |
| 47 | .34109 | .51766 | .35432 | .54876 | .36775 | .58164 | .38136 | .61646 | 47 |
| 48 | .34131 | .51817 | .35454 | .54929 | .36797 | .58221 | .38159 | .61705 | 48 |
| 49 | .34153 | .51867 | .35476 | .54982 | .36820 | .58277 | .38182 | .61765 | 49 |
| 50 | .34175 | .51918 | .35499 | .55036 | .36842 | .58333 | .38205 | .61825 | 50 |
| 51 | .34197 | .51968 | .35521 | .55089 | .36865 | .58390 | .38228 | .61885 | 51 |
| 52 | .34219 | .52019 | .35543 | .55143 | .36887 | .58447 | .38251 | .61945 | 52 |
| 53 | .34241 | .52069 | .35565 | .55196 | .36910 | .58503 | .38274 | .62005 | 53 |
| 54 | .34262 | .52120 | .35588 | .55250 | .36932 | .58560 | .38296 | .62065 | 54 |
| 55 | .34284 | .52171 | .35610 | .55303 | .36955 | .58617 | .38319 | .62125 | 55 |
| 56 | .34306 | .52222 | .35632 | .55357 | .36978 | .58674 | .38342 | .62185 | 56 |
| 57 | .34328 | .52273 | .35654 | .55411 | .37000 | .58731 | .38365 | .62246 | 57 |
| 58 | .34350 | .52323 | .35677 | .55465 | .37023 | .58788 | .38388 | .62306 | 58 |
| 59 | .34372 | .52374 | .35699 | .55518 | .37045 | .58845 | .38411 | .62266 | 59 |
| 60 | .34394 | .52425 | .35721 | .55572 | .37068 | .58902 | .38434 | .62427 | 60 |

## TABLE IV.—NATURAL VERSED SINES AND EXTERNAL SECANTS.

| ′ | 52° | | 53° | | 54° | | 55° | | ′ |
|---|---|---|---|---|---|---|---|---|---|
| | Vers. | Ex. sec. | Vers. | Ex. sec. | Vers. | Ex. sec. | Vers. | Ex. sec. | |
| 0 | .38434 | .62427 | .39819 | .66164 | .41221 | .70130 | .42642 | .74345 | 0 |
| 1 | .38457 | .62487 | .39842 | .66226 | .41245 | .70196 | .42666 | .74417 | 1 |
| 2 | .38480 | .62548 | .39865 | .66292 | .41269 | .70267 | .42690 | .74490 | 2 |
| 3 | .38503 | .62609 | .39888 | .66357 | .41292 | .70335 | .42714 | .74562 | 3 |
| 4 | .38526 | .62669 | .39911 | .66421 | .41316 | .70403 | .42738 | .74635 | 4 |
| 5 | .38549 | .62730 | .39935 | .66486 | .41339 | .70472 | .42762 | .74708 | 5 |
| 6 | .38571 | .62791 | .39958 | .66550 | .41363 | .70540 | .42785 | .74781 | 6 |
| 7 | .38594 | .62852 | .39981 | .66615 | .41386 | .70609 | .42809 | .74854 | 7 |
| 8 | .38617 | .62913 | .40005 | .66679 | .41410 | .70677 | .42833 | .74927 | 8 |
| 9 | .38640 | .62974 | .40028 | .66744 | .41433 | .70746 | .42857 | .75000 | 9 |
| 10 | .38663 | .63035 | .40051 | .66809 | .41457 | .70815 | .42881 | .75073 | 10 |
| 11 | .38686 | .63096 | .40074 | .66873 | .41481 | .70884 | .42905 | .75146 | 11 |
| 12 | .38709 | .63157 | .40098 | .66938 | .41504 | .70953 | .42929 | .75219 | 12 |
| 13 | .38732 | .63218 | .40121 | .67003 | .41528 | .71022 | .42953 | .75293 | 13 |
| 14 | .38755 | .63279 | .40144 | .67068 | .41551 | .71091 | .42976 | .75366 | 14 |
| 15 | .38778 | .63341 | .40168 | .67133 | .41575 | .71160 | .43000 | .75440 | 15 |
| 16 | .38801 | .63402 | .40191 | .67199 | .41599 | .71229 | .43024 | .75513 | 16 |
| 17 | .38824 | .63464 | .40214 | .67264 | .41622 | .71298 | .43048 | .75587 | 17 |
| 18 | .38847 | .63525 | .40237 | .67329 | .41646 | .71368 | .43072 | .75661 | 18 |
| 19 | .38870 | .63587 | .40261 | .67394 | .41670 | .71437 | .43096 | .75734 | 19 |
| 20 | .38893 | .63648 | .40284 | .67460 | .41693 | .71506 | .43120 | .75808 | 20 |
| 21 | .38916 | .63710 | .40307 | .67525 | .41717 | .71576 | .43144 | .75882 | 21 |
| 22 | .38939 | .63772 | .40331 | .67591 | .41740 | .71646 | .43168 | .75956 | 22 |
| 23 | .38962 | .63834 | .40354 | .67656 | .41764 | .71715 | .43192 | .76031 | 23 |
| 24 | .38985 | .63895 | .40378 | .67722 | .41788 | .71785 | .43216 | .76105 | 24 |
| 25 | .39009 | .63957 | .40401 | .67788 | .41811 | .71855 | .43240 | .76179 | 25 |
| 26 | .39032 | .64019 | .40424 | .67853 | .41835 | .71925 | .43264 | .76253 | 26 |
| 27 | .39055 | .64081 | .40448 | .67919 | .41859 | .71995 | .43287 | .76328 | 27 |
| 28 | .39078 | .64144 | .40471 | .67985 | .41882 | .72065 | .43311 | .76402 | 28 |
| 29 | .39101 | .64206 | .40494 | .68051 | .41906 | .72135 | .43335 | .76477 | 29 |
| 30 | .39124 | .64268 | .40518 | .68117 | .41930 | .72205 | .43359 | .76552 | 30 |
| 31 | .39147 | .64330 | .40541 | .68183 | .41953 | .72275 | .43383 | .76626 | 31 |
| 32 | .39170 | .64393 | .40565 | .68250 | .41977 | .72346 | .43407 | .76701 | 32 |
| 33 | .39193 | .64455 | .40588 | .68316 | .42001 | .72416 | .43431 | .76776 | 33 |
| 34 | .39216 | .64518 | .40611 | .68382 | .42024 | .72487 | .43455 | .76851 | 34 |
| 35 | .39239 | .64580 | .40635 | .68449 | .42048 | .72557 | .43479 | .76926 | 35 |
| 36 | .39262 | .64643 | .40658 | .68515 | .42072 | .72628 | .43503 | .77001 | 36 |
| 37 | .39286 | .64705 | .40682 | .68582 | .42096 | .72698 | .43527 | .77077 | 37 |
| 38 | .39309 | .64768 | .40705 | .68648 | .42119 | .72769 | .43551 | .77152 | 38 |
| 39 | .39332 | .64831 | .40728 | .68715 | .42143 | .72840 | .43575 | .77227 | 39 |
| 40 | .39355 | .64894 | .40752 | .68782 | .42167 | .72911 | .43599 | .77303 | 40 |
| 41 | .39378 | .64957 | .40775 | .68848 | .42191 | .72982 | .43623 | .77378 | 41 |
| 42 | .39401 | .65020 | .40799 | .68915 | .42214 | .73053 | .43647 | .77454 | 42 |
| 43 | .39424 | .65083 | .40822 | .68982 | .42238 | .73124 | .43671 | .77530 | 43 |
| 44 | .39447 | .65146 | .40846 | .69049 | .42262 | .73195 | .43695 | .77606 | 44 |
| 45 | .39471 | .65209 | .40869 | .69116 | .42285 | .73267 | .43720 | .77681 | 45 |
| 46 | .39494 | .65272 | .40893 | .69183 | .42309 | .73338 | .43744 | .77757 | 46 |
| 47 | .39517 | .65336 | .40916 | .69250 | .42333 | .73409 | .43768 | .77833 | 47 |
| 48 | .39540 | .65399 | .40939 | .69318 | .42357 | .73481 | .43792 | .77910 | 48 |
| 49 | .39563 | .65462 | .40963 | .69385 | .42381 | .73552 | .43816 | .77986 | 49 |
| 50 | .39586 | .65526 | .40986 | .69452 | .42404 | .73624 | .43840 | .78062 | 50 |
| 51 | .39610 | .65589 | .41010 | .69520 | .42428 | .73696 | .43864 | .78138 | 51 |
| 52 | .39633 | .65653 | .41033 | .69587 | .42452 | .73768 | .43888 | .78215 | 52 |
| 53 | .39656 | .65717 | .41057 | .69655 | .42476 | .73840 | .43912 | .78291 | 53 |
| 54 | .39679 | .65780 | .41080 | .69723 | .42499 | .73911 | .43936 | .78368 | 54 |
| 55 | .39703 | .65844 | .41104 | .69790 | .42523 | .73983 | .43960 | .78445 | 55 |
| 56 | .39726 | .65908 | .41127 | .69858 | .42547 | .74056 | .43984 | .78521 | 56 |
| 57 | .39749 | .65972 | .41151 | .69926 | .42571 | .74128 | .44008 | .78598 | 57 |
| 58 | .39772 | .66036 | .41174 | .69994 | .42595 | .74200 | .44032 | .78675 | 58 |
| 59 | .39795 | .66100 | .41198 | .70062 | .42619 | .74272 | .44057 | .78752 | 59 |
| 60 | .39819 | .66164 | .41221 | .70130 | .42642 | .74345 | .44081 | .78829 | 60 |

| ′ | 56° | | 57° | | 58° | | 59° | | ′ |
|---|---|---|---|---|---|---|---|---|---|
| | Vers. | Ex. sec. | Vers. | Ex. sec. | Vers. | Ex. sec. | Vers. | Ex. sec. | |
| 0 | .44081 | .78829 | .45536 | .83608 | .47008 | .88708 | .48496 | .94160 | 0 |
| 1 | .44105 | .78906 | .45560 | .83690 | .47033 | .88796 | .48521 | .94254 | 1 |
| 2 | .44129 | .78984 | .45585 | .83773 | .47057 | .88884 | .48546 | .94349 | 2 |
| 3 | .44153 | .79061 | .45609 | .83855 | .47082 | .88972 | .48571 | .94443 | 3 |
| 4 | .44177 | .79138 | .45634 | .83938 | .47107 | .89060 | .48596 | .94537 | 4 |
| 5 | .44201 | .79216 | .45658 | .84020 | .47131 | .89148 | .48621 | .94632 | 5 |
| 6 | .44225 | .79293 | .45683 | .84103 | .47156 | .89237 | .48646 | .94726 | 6 |
| 7 | .44250 | .79371 | .45707 | .84186 | .47181 | .89325 | .48671 | .94821 | 7 |
| 8 | .44274 | .79449 | .45731 | .84269 | .47206 | .89414 | .48696 | .94916 | 8 |
| 9 | .44298 | .79527 | .45756 | .84352 | .47230 | .89503 | .48721 | .95011 | 9 |
| 10 | .44322 | .79604 | .45780 | .84435 | .47255 | .89591 | .48746 | .95106 | 10 |
| 11 | .44346 | .79682 | .45805 | .84518 | .47280 | .89680 | .48771 | .95201 | 11 |
| 12 | .44370 | .79761 | .45829 | .84601 | .47304 | .89769 | .48796 | .95296 | 12 |
| 13 | .44395 | .79839 | .45854 | .84685 | .47329 | .89858 | .48821 | .95392 | 13 |
| 14 | .44419 | .79917 | .45878 | .84768 | .47354 | .89948 | .48846 | .95487 | 14 |
| 15 | .44443 | .79995 | .45903 | .84852 | .47379 | .90037 | .48871 | .95583 | 15 |
| 16 | .44467 | .80074 | .45927 | .84935 | .47403 | .90126 | .48896 | .95678 | 16 |
| 17 | .44491 | .80153 | .45951 | .85019 | .47428 | .90216 | .48921 | .95774 | 17 |
| 18 | .44516 | .80231 | .45976 | .85103 | .47453 | .90305 | .48946 | .95870 | 18 |
| 19 | .44540 | .80309 | .46000 | .85187 | .47478 | .90395 | .48971 | .95966 | 19 |
| 20 | .44564 | .80388 | .46025 | .85271 | .47502 | .90485 | .48996 | .96062 | 20 |
| 21 | .44588 | .80467 | .46049 | .85355 | .47527 | .90575 | .49021 | .96158 | 21 |
| 22 | .44612 | .80546 | .46074 | .85439 | .47552 | .90665 | .49046 | .96255 | 22 |
| 23 | .44637 | .80625 | .46098 | .85523 | .47577 | .90755 | .49071 | .96351 | 23 |
| 24 | .44661 | .80704 | .46123 | .85608 | .47601 | .90845 | .49096 | .96448 | 24 |
| 25 | .44685 | .80783 | .46147 | .85692 | .47626 | .90935 | .49121 | .96544 | 25 |
| 26 | .44709 | .80862 | .46172 | .85777 | .47651 | .91026 | .49146 | .96641 | 26 |
| 27 | .44734 | .80942 | .46196 | .85861 | .47676 | .91116 | .49171 | .96738 | 27 |
| 28 | .44758 | .81021 | .46221 | .85946 | .47701 | .91207 | .49196 | .96835 | 28 |
| 29 | .44782 | .81101 | .46246 | .86031 | .47725 | .91297 | .49221 | .96932 | 29 |
| 30 | .44806 | .81180 | .46270 | .86116 | .47750 | .91388 | .49246 | .97029 | 30 |
| 31 | .44831 | .81260 | .46295 | .86201 | .47775 | .91479 | .49271 | .97127 | 31 |
| 32 | .44855 | .81340 | .46319 | .86286 | .47800 | .91570 | .49296 | .97224 | 32 |
| 33 | .44879 | .81419 | .46344 | .86371 | .47825 | .91661 | .49321 | .97322 | 33 |
| 34 | .44903 | .81499 | .46368 | .86457 | .47849 | .91752 | .49346 | .97420 | 34 |
| 35 | .44928 | .81579 | .46393 | .86542 | .47874 | .91844 | .49372 | .97517 | 35 |
| 36 | .44952 | .81659 | .46417 | .86627 | .47899 | .91935 | .49397 | .97615 | 36 |
| 37 | .44976 | .81740 | .46442 | .86713 | .47924 | .92027 | .49422 | .97713 | 37 |
| 38 | .45001 | .81820 | .46466 | .86799 | .47949 | .92118 | .49447 | .97811 | 38 |
| 39 | .45025 | .81900 | .46491 | .86885 | .47974 | .92210 | .49472 | .97910 | 39 |
| 40 | .45049 | .81981 | .46516 | .86990 | .47998 | .92302 | .49497 | .98008 | 40 |
| 41 | .45073 | .82061 | .46540 | .87056 | .48023 | .92394 | .49522 | .98107 | 41 |
| 42 | .45098 | .82142 | .46565 | .87142 | .48048 | .92486 | .49547 | .98205 | 42 |
| 43 | .45122 | .82222 | .46589 | .87229 | .48073 | .92578 | .49572 | .98304 | 43 |
| 44 | .45146 | .82303 | .46614 | .87315 | .48098 | .92670 | .49597 | .98403 | 44 |
| 45 | .45171 | .82384 | .46639 | .87401 | .48123 | .92762 | .49623 | .98502 | 45 |
| 46 | .45195 | .82465 | .46663 | .87488 | .48148 | .92855 | .49648 | .98601 | 46 |
| 47 | .45219 | .82546 | .46688 | .87574 | .48172 | .92947 | .49673 | .98700 | 47 |
| 48 | .45244 | .82627 | .46712 | .87661 | .48197 | .93040 | .49698 | .98799 | 48 |
| 49 | .45268 | .82709 | .46737 | .87748 | .48222 | .93133 | .49723 | .98899 | 49 |
| 50 | .45292 | .82790 | .46762 | .87834 | .48247 | .93226 | .49748 | .98998 | 50 |
| 51 | .45317 | .82871 | .46786 | .87921 | .48272 | .93319 | .49773 | .99098 | 51 |
| 52 | .45341 | .82953 | .46811 | .88008 | .48297 | .93412 | .49799 | .99198 | 52 |
| 53 | .45365 | .83034 | .46836 | .88095 | .48322 | .93505 | .49824 | .99298 | 53 |
| 54 | .45390 | .83116 | .46860 | .88183 | .48347 | .93598 | .49849 | .99398 | 54 |
| 55 | .45414 | .83198 | .46885 | .88270 | .48372 | .93692 | .49874 | .99498 | 55 |
| 56 | .45439 | .83280 | .46909 | .88357 | .48396 | .93785 | .49899 | .99598 | 56 |
| 57 | .45463 | .83362 | .46934 | .88445 | .48421 | .93879 | .49924 | .99698 | 57 |
| 58 | .45487 | .83444 | .46959 | .88532 | .48446 | .93973 | .49950 | .99799 | 58 |
| 59 | .45512 | .83526 | .46983 | .88620 | .48471 | .94066 | .49975 | .99899 | 59 |
| 60 | .45536 | .83608 | .47008 | .88708 | .48496 | .94160 | .50000 | 1.00000 | 60 |

## Table IV.—Natural Versed Sines and External Secants.

| ′ | 60° | | 61° | | 62° | | 63° | | ′ |
|---|---|---|---|---|---|---|---|---|---|
| | Vers. | Ex .se | Vers. | Ex. sec. | Vers. | Ex. sec. | Vers. | Ex. sec. | |
| 0 | .50000 | 1.00000 | .51519 | 1.06267 | .53053 | 1.13005 | .54601 | 1.20269 | 0 |
| 1 | .50025 | 1.00101 | .51544 | 1.06375 | .53079 | 1.13122 | .54627 | 1.20395 | 1 |
| 2 | .50050 | 1.00202 | .51570 | 1.06483 | .53104 | 1.13239 | .54653 | 1.20521 | 2 |
| 3 | .50076 | 1.00303 | .51595 | 1.06592 | .53130 | 1.13356 | .54679 | 1.20647 | 3 |
| 4 | .50101 | 1.00404 | .51621 | 1.06701 | .53156 | 1.13473 | .54705 | 1.20773 | 4 |
| 5 | .50126 | 1.00505 | .51646 | 1.06809 | .53181 | 1.13590 | .54731 | 1.20900 | 5 |
| 6 | .50151 | 1.00607 | .51672 | 1.06918 | .53207 | 1.13707 | .54757 | 1.21026 | 6 |
| 7 | .50176 | 1.00708 | .51697 | 1.07027 | .53233 | 1.13825 | .54782 | 1.21153 | 7 |
| 8 | .50202 | 1.00810 | .51723 | 1.07137 | .53258 | 1.13942 | .54808 | 1.21280 | 8 |
| 9 | .50227 | 1.00912 | .51748 | 1.07246 | .53284 | 1.14060 | .54834 | 1.21407 | 9 |
| 10 | .50252 | 1.01014 | .51774 | 1.07356 | .53310 | 1.14178 | .54860 | 1.21585 | 10 |
| 11 | .50277 | 1.01116 | .51799 | 1.07465 | .53336 | 1.14296 | .54886 | 1.21662 | 11 |
| 12 | .50303 | 1.01218 | .51825 | 1.07575 | .53361 | 1.14414 | .54912 | 1.21790 | 12 |
| 13 | .50328 | 1.01320 | .51850 | 1.07685 | .53387 | 1.14533 | .54938 | 1.21918 | 13 |
| 14 | .50353 | 1.01422 | .51876 | 1.07795 | .53413 | 1.14651 | .54964 | 1.22045 | 14 |
| 15 | .50378 | 1.01525 | .51901 | 1.07905 | .53439 | 1.14770 | .54990 | 1.22174 | 15 |
| 16 | .50404 | 1.01628 | .51927 | 1.08015 | .53464 | 1.14889 | .55016 | 1.22302 | 16 |
| 17 | .50429 | 1.01730 | .51952 | 1.08126 | .53490 | 1.15008 | .55042 | 1.22430 | 17 |
| 18 | .50454 | 1.01833 | .51978 | 1.08236 | .53516 | 1.15127 | .55068 | 1.22559 | 18 |
| 19 | .50479 | 1.01936 | .52003 | 1.08347 | .53542 | 1.15246 | .55094 | 1.22688 | 19 |
| 20 | .50505 | 1.02039 | .52029 | 1.08458 | .53567 | 1.15366 | .55120 | 1.22817 | 20 |
| 21 | .50530 | 1.02143 | .52054 | 1.08569 | .53593 | 1.15485 | .55146 | 1.22946 | 21 |
| 22 | .50555 | 1.02246 | .52080 | 1.08680 | .53619 | 1.15605 | .55172 | 1.23075 | 22 |
| 23 | .50581 | 1.02349 | .52105 | 1.08791 | .53645 | 1.15725 | .55198 | 1.23205 | 23 |
| 24 | .50606 | 1.02453 | .52131 | 1.08903 | .53670 | 1.15845 | .55224 | 1.23334 | 24 |
| 25 | .50631 | 1.02557 | .52156 | 1.09014 | .53696 | 1.15965 | .55250 | 1.23464 | 25 |
| 26 | .50656 | 1.02661 | .52182 | 1.09126 | .53722 | 1.16085 | .55276 | 1.23594 | 26 |
| 27 | .50682 | 1.02765 | .52207 | 1.09238 | .53748 | 1.16206 | .55302 | 1.23724 | 27 |
| 28 | .50707 | 1.02869 | .52233 | 1.09350 | .53774 | 1.16326 | .55328 | 1.23855 | 28 |
| 29 | .50732 | 1.02973 | .52259 | 1.09462 | .53799 | 1.16447 | .55354 | 1.23985 | 29 |
| 30 | .50758 | 1.03077 | .52284 | 1.09574 | .53825 | 1.16568 | .55380 | 1.24116 | 30 |
| 31 | .50783 | 1.03182 | .52310 | 1.09686 | .53851 | 1.16689 | .55406 | 1.24247 | 31 |
| 32 | .50808 | 1.03286 | .52335 | 1.09799 | .53877 | 1.16810 | .55432 | 1.24378 | 32 |
| 33 | .50834 | 1.03391 | .52361 | 1.09911 | .53903 | 1.16932 | .55458 | 1.24509 | 33 |
| 34 | .50859 | 1.03496 | .52386 | 1.10024 | .53928 | 1.17053 | .55484 | 1.24640 | 34 |
| 35 | .50884 | 1.03601 | .52412 | 1.10137 | .53954 | 1.17175 | .55510 | 1.24772 | 35 |
| 36 | .50910 | 1.03706 | .52438 | 1.10250 | .53980 | 1.17297 | .55536 | 1.24903 | 36 |
| 37 | .50935 | 1.03811 | .52463 | 1.10363 | .54006 | 1.17419 | .55563 | 1.25035 | 37 |
| 38 | .50960 | 1.03916 | .52489 | 1.10477 | .54032 | 1.17541 | .55589 | 1.25167 | 38 |
| 39 | .50986 | 1.04022 | .52514 | 1.10590 | .54058 | 1.17663 | .55615 | 1.25300 | 39 |
| 40 | .51011 | 1.04128 | .52540 | 1.10704 | .54083 | 1.17786 | .55641 | 1.25432 | 40 |
| 41 | .51036 | 1.04233 | .52566 | 1.10817 | .54109 | 1.17909 | .55667 | 1.25565 | 41 |
| 42 | .51062 | 1.04339 | .52591 | 1.10931 | .54135 | 1.18031 | .55693 | 1.25697 | 42 |
| 43 | .51087 | 1.04445 | .52617 | 1.11045 | .54161 | 1.18154 | .55719 | 1.25830 | 43 |
| 44 | .51113 | 1.04551 | .52642 | 1.11159 | .54187 | 1.18277 | .55745 | 1.25963 | 44 |
| 45 | .51138 | 1.04658 | .52668 | 1.11274 | .54213 | 1.18401 | .55771 | 1.26097 | 45 |
| 46 | .51163 | 1.04764 | .52694 | 1.11388 | .54288 | 1.18524 | .55797 | 1.26230 | 46 |
| 47 | .51189 | 1.04870 | .52719 | 1.11503 | .54264 | 1.18648 | .55823 | 1.26364 | 47 |
| 48 | .51214 | 1.04977 | .52745 | 1.11617 | .54290 | 1.18772 | .55849 | 1.26498 | 48 |
| 49 | .51239 | 1.05084 | .52771 | 1.11732 | .54316 | 1.18895 | .55876 | 1.26632 | 49 |
| 50 | .51265 | 1.05191 | .52796 | 1.11847 | .54342 | 1.19019 | .55902 | 1.26766 | 50 |
| 51 | .51290 | 1.05298 | .52822 | 1.11963 | .54368 | 1.19144 | .55928 | 1.26900 | 51 |
| 52 | .51316 | 1.05405 | .52848 | 1.12078 | .54394 | 1.19268 | .55954 | 1.27035 | 52 |
| 53 | .51341 | 1.05512 | .52873 | 1.12193 | .54420 | 1.19393 | .55980 | 1.27169 | 53 |
| 54 | .51366 | 1.05619 | .52899 | 1.12309 | .54446 | 1.19517 | .56006 | 1.27304 | 54 |
| 55 | .51392 | 1.05727 | .52924 | 1.12425 | .54471 | 1.19642 | .56032 | 1.27439 | 55 |
| 56 | .51417 | 1.05835 | .52950 | 1.12540 | .54497 | 1.19767 | .56058 | 1.27574 | 56 |
| 57 | .51443 | 1.05942 | .52976 | 1.12657 | .54523 | 1.19892 | .56084 | 1.27710 | 57 |
| 58 | .51468 | 1.06050 | .53001 | 1.12773 | .54549 | 1.20018 | .56111 | 1.27845 | 58 |
| 59 | .51494 | 1.06158 | .53027 | 1.12889 | .54575 | 1.20143 | .56137 | 1.27981 | 59 |
| 60 | .51519 | 1.06267 | .53053 | 1.13005 | .54601 | 1.20269 | .56163 | 1.28117 | 60 |

## Table IV.—Natural Versed Sines and External Secants.

| ′ | 64° | | 65° | | 66° | | 67° | | ′ |
|---|---|---|---|---|---|---|---|---|---|
| | Vers. | Ex. sec. | Vers. | Ex. sec. | Vers. | Ex. sec. | Vers. | Ex. sec. | |
| 0 | .56163 | 1.28117 | .57738 | 1.36620 | .59326 | 1.45859 | .60927 | 1.55930 | 0 |
| 1 | .56189 | 1.28253 | .57765 | 1.36768 | .59353 | 1.46020 | .60954 | 1.56106 | 1 |
| 2 | .56215 | 1.28390 | .57791 | 1.36916 | .59379 | 1.46181 | .60980 | 1.56282 | 2 |
| 3 | .56241 | 1.28526 | .57817 | 1.37064 | .59406 | 1.46342 | .61007 | 1.56458 | 3 |
| 4 | .56267 | 1.28663 | .57844 | 1.37212 | .59433 | 1.46504 | .61034 | 1.56634 | 4 |
| 5 | .56294 | 1.28800 | .57870 | 1.37361 | .59459 | 1.46665 | .61061 | 1.56811 | 5 |
| 6 | .56320 | 1.28937 | .57896 | 1.37509 | .59486 | 1.46827 | .61088 | 1.56988 | 6 |
| 7 | .56346 | 1.29074 | .57923 | 1.37658 | .59512 | 1.46989 | .61114 | 1.57165 | 7 |
| 8 | .56372 | 1.29211 | .57949 | 1.37808 | .59539 | 1.47152 | .61141 | 1.57342 | 8 |
| 9 | .56398 | 1.29349 | .57976 | 1.37957 | .59566 | 1.47314 | .61168 | 1.57520 | 9 |
| 10 | .56425 | 1.29487 | .58002 | 1.38107 | .59592 | 1.47477 | .61195 | 1.57698 | 10 |
| 11 | .56451 | 1.29625 | .58028 | 1.38256 | .59619 | 1.47640 | .61222 | 1.57876 | 11 |
| 12 | .56477 | 1.29763 | .58055 | 1.38406 | .59645 | 1.47804 | .61248 | 1.58054 | 12 |
| 13 | .56503 | 1.29901 | .58081 | 1.38556 | .59672 | 1.47967 | .61275 | 1.58233 | 13 |
| 14 | .56529 | 1.30040 | .58108 | 1.38707 | .59699 | 1.48131 | .61302 | 1.58412 | 14 |
| 15 | .56555 | 1.30179 | .58134 | 1.38857 | .59725 | 1.48295 | .61329 | 1.58591 | 15 |
| 16 | .56582 | 1.30318 | .58160 | 1.39008 | .59752 | 1.48459 | .61356 | 1.58771 | 16 |
| 17 | .56608 | 1.30457 | .58187 | 1.39159 | .59779 | 1.48624 | .61383 | 1.58950 | 17 |
| 18 | .56634 | 1.30596 | .58213 | 1.39311 | .59805 | 1.48789 | .61409 | 1.59130 | 18 |
| 19 | .56660 | 1.30735 | .58240 | 1.39462 | .59832 | 1.48954 | .61436 | 1.59311 | 19 |
| 20 | .56687 | 1.30875 | .58266 | 1.39614 | .59859 | 1.49119 | .61463 | 1.59491 | 20 |
| 21 | .56713 | 1.31015 | .58293 | 1.39766 | .59885 | 1.49284 | .61490 | 1.59672 | 21 |
| 22 | .56739 | 1.31155 | .58319 | 1.39918 | .59912 | 1.49450 | .61517 | 1.59853 | 22 |
| 23 | .56765 | 1.31295 | .58345 | 1.40070 | .59938 | 1.49616 | .61544 | 1.60035 | 23 |
| 24 | .56791 | 1.31436 | .58372 | 1.40222 | .59965 | 1.49782 | .61570 | 1.60217 | 24 |
| 25 | .56818 | 1.31576 | .58398 | 1.40375 | .59992 | 1.49948 | .61597 | 1.60399 | 25 |
| 26 | .56844 | 1.31717 | .58425 | 1.40528 | .60018 | 1.50115 | .61624 | 1.60581 | 26 |
| 27 | .56870 | 1.31858 | .58451 | 1.40681 | .60045 | 1.50282 | .61651 | 1.60763 | 27 |
| 28 | .56896 | 1.31999 | .58478 | 1.40835 | .60072 | 1.50449 | .61678 | 1.60946 | 28 |
| 29 | .56923 | 1.32140 | .58504 | 1.40988 | .60098 | 1.50617 | .61705 | 1.61129 | 29 |
| 30 | .56949 | 1.32282 | .58531 | 1.41142 | .60125 | 1.50784 | .61733 | 1.61313 | 30 |
| 31 | .56975 | 1.32424 | .58557 | 1.41296 | .60152 | 1.50952 | .61759 | 1.61496 | 31 |
| 32 | .57001 | 1.32566 | .58584 | 1.41450 | .60178 | 1.51120 | .61785 | 1.61680 | 32 |
| 33 | .57028 | 1.32708 | .58610 | 1.41605 | .60205 | 1.51289 | .61812 | 1.61864 | 33 |
| 34 | .57054 | 1.32850 | .58637 | 1.41760 | .60232 | 1.51457 | .61839 | 1.62049 | 34 |
| 35 | .57080 | 1.32993 | .58663 | 1.41914 | .60259 | 1.51626 | .61866 | 1.62234 | 35 |
| 36 | .57106 | 1.33135 | .58690 | 1.42070 | .60285 | 1.51795 | .61893 | 1.62419 | 36 |
| 37 | .57133 | 1.33278 | .58716 | 1.42225 | .60312 | 1.51965 | .61920 | 1.62604 | 37 |
| 38 | .57159 | 1.33422 | .58743 | 1.42380 | .60339 | 1.52134 | .61947 | 1.62790 | 38 |
| 39 | .57185 | 1.33565 | .58769 | 1.42536 | .60365 | 1.52304 | .61974 | 1.62976 | 39 |
| 40 | .57212 | 1.33708 | .58796 | 1.42692 | .60392 | 1.52474 | .62001 | 1.63162 | 40 |
| 41 | .57238 | 1.33852 | .58822 | 1.42848 | .60419 | 1.52645 | .62027 | 1.63348 | 41 |
| 42 | .57264 | 1.33996 | .58849 | 1.43005 | .60445 | 1.52815 | .62054 | 1.63535 | 42 |
| 43 | .57291 | 1.34140 | .58875 | 1.43162 | .60472 | 1.52986 | .62081 | 1.63722 | 43 |
| 44 | .57317 | 1.34284 | .58902 | 1.43318 | .60499 | 1.53157 | .62108 | 1.63909 | 44 |
| 45 | .57343 | 1.34429 | .58928 | 1.43476 | .60526 | 1.53329 | .62135 | 1.64097 | 45 |
| 46 | .57369 | 1.34573 | .58955 | 1.43633 | .60552 | 1.53500 | .62162 | 1.64285 | 46 |
| 47 | .57396 | 1.34718 | .58981 | 1.43790 | .60579 | 1.53672 | .62189 | 1.64478 | 47 |
| 48 | .57422 | 1.34863 | .59008 | 1.43948 | .60606 | 1.53845 | .62216 | 1.64662 | 48 |
| 49 | .57448 | 1.35009 | .59034 | 1.44106 | .60633 | 1.54017 | .62243 | 1.64851 | 49 |
| 50 | .57475 | 1.35154 | .59061 | 1.44264 | .60659 | 1.54190 | .62270 | 1.65040 | 50 |
| 51 | .57501 | 1.35300 | .59087 | 1.44423 | .60686 | 1.54363 | .62297 | 1.65229 | 51 |
| 52 | .57527 | 1.35446 | .59114 | 1.44582 | .60713 | 1.54536 | .62324 | 1.65419 | 52 |
| 53 | .57554 | 1.35592 | .59140 | 1.44741 | .60740 | 1.54709 | .62351 | 1.65609 | 53 |
| 54 | .57580 | 1.35738 | .59167 | 1.44900 | .60766 | 1.54883 | .62378 | 1.65799 | 54 |
| 55 | .57606 | 1.35885 | .59194 | 1.45059 | .60793 | 1.55057 | .62405 | 1.65989 | 55 |
| 56 | .57633 | 1.36031 | .59220 | 1.45219 | .60820 | 1.55231 | .62431 | 1.66180 | 56 |
| 57 | .57659 | 1.36178 | .59247 | 1.45378 | .60847 | 1.55405 | .62458 | 1.66371 | 57 |
| 58 | .57685 | 1.36325 | .59273 | 1.45539 | .60873 | 1.55580 | .62485 | 1.66563 | 58 |
| 59 | .57712 | 1.36473 | .59300 | 1.45699 | .60900 | 1.55755 | .62512 | 1.66755 | 59 |
| 60 | .57738 | 1.36620 | .59326 | 1.45859 | .60927 | 1.55930 | .62539 | 1.66947 | 60 |

# Table IV.—Natural Versed Sines and External Secants.

| ′ | 68° | | 69° | | 70° | | 71° | | ′ |
|---|---|---|---|---|---|---|---|---|---|
| | Vers. | Ex. sec. | Vers. | Ex. sec. | Vers. | Ex. sec. | Vers. | Ex. sec. | |
| 0 | .62539 | 1.66947 | .64163 | 1.79043 | .65798 | 1.92380 | .67443 | 2.07155 | 0 |
| 1 | .62566 | 1.67139 | .64190 | 1.79254 | .65825 | 1.92614 | .67471 | 2.07415 | 1 |
| 2 | .62593 | 1.67332 | .64218 | 1.79466 | .65853 | 1.92849 | .67498 | 2.07675 | 2 |
| 3 | .62620 | 1.67525 | .64245 | 1.79679 | .65880 | 1.93083 | .67526 | 2.07936 | 3 |
| 4 | .62647 | 1.67718 | .64272 | 1.79891 | .65907 | 1.93318 | .67553 | 2.08197 | 4 |
| 5 | .62674 | 1.67911 | .64299 | 1.80104 | .65935 | 1.93554 | .67581 | 2.08459 | 5 |
| 6 | .62701 | 1.68105 | .64326 | 1.80318 | .65962 | 1.93790 | .67608 | 2.08721 | 6 |
| 7 | .62728 | 1.68299 | .64353 | 1.80531 | .65989 | 1.94026 | .67636 | 2.08983 | 7 |
| 8 | .62755 | 1.68494 | .64381 | 1.80746 | .66017 | 1.94263 | .67663 | 2.09246 | 8 |
| 9 | .62782 | 1.68689 | .64408 | 1.80960 | .66044 | 1.94500 | .67691 | 2.09510 | 9 |
| 10 | .62809 | 1.68884 | .64435 | 1.81175 | .66071 | 1.94737 | .67718 | 2.09774 | 10 |
| 11 | .62836 | 1.69079 | .64462 | 1.81390 | .66099 | 1.94975 | .67746 | 2.10038 | 11 |
| 12 | .62863 | 1.69275 | .64489 | 1.81605 | .66126 | 1.95213 | .67773 | 2.10303 | 12 |
| 13 | .62890 | 1.69471 | .64517 | 1.81821 | .66154 | 1.95452 | .67801 | 2.10568 | 13 |
| 14 | .62917 | 1.69667 | .64544 | 1.82037 | .66181 | 1.95691 | .67829 | 2.10834 | 14 |
| 15 | .62944 | 1.69864 | .64571 | 1.82254 | .66208 | 1.95931 | .67856 | 2.11101 | 15 |
| 16 | .62971 | 1.70061 | .64598 | 1.82471 | .66236 | 1.96171 | .67884 | 2.11367 | 16 |
| 17 | .62998 | 1.70258 | .64625 | 1.82688 | .66263 | 1.96411 | .67911 | 2.11635 | 17 |
| 18 | .63025 | 1.70455 | .64653 | 1.82906 | .66290 | 1.96652 | .67939 | 2.11903 | 18 |
| 19 | .63052 | 1.70653 | .64680 | 1.83124 | .66318 | 1.96893 | .67966 | 2.12171 | 19 |
| 20 | .63079 | 1.70851 | .64707 | 1.83342 | .66345 | 1.97135 | .67994 | 2.12440 | 20 |
| 21 | .63106 | 1.71050 | .64734 | 1.83561 | .66373 | 1.97377 | .68021 | 2.12709 | 21 |
| 22 | .63133 | 1.71249 | .64761 | 1.83780 | .66400 | 1.97619 | .68049 | 2.12979 | 22 |
| 23 | .63161 | 1.71448 | .64789 | 1.83999 | .66427 | 1.97862 | .68077 | 2.13249 | 23 |
| 24 | .63188 | 1.71647 | .64816 | 1.84219 | .66455 | 1.98106 | .68104 | 2.13520 | 24 |
| 25 | .63215 | 1.71847 | .64843 | 1.84439 | .66482 | 1.98349 | .68132 | 2.13791 | 25 |
| 26 | .63242 | 1.72047 | .64870 | 1.84659 | .66510 | 1.98594 | .68159 | 2.14063 | 26 |
| 27 | .63269 | 1.72247 | .64898 | 1.84880 | .66537 | 1.98838 | .68187 | 2.14335 | 27 |
| 28 | .63296 | 1.72448 | .64925 | 1.85102 | .66564 | 1.99083 | .68214 | 2.14606 | 28 |
| 29 | .63323 | 1.72649 | .64952 | 1.85323 | .66592 | 1.99329 | .68242 | 2.14881 | 29 |
| 30 | .63350 | 1.72850 | .64979 | 1.85545 | .66619 | 1.99574 | .68270 | 2.15155 | 30 |
| 31 | .63377 | 1.73052 | .65007 | 1.85767 | .66647 | 1.99821 | .68297 | 2.15429 | 31 |
| 32 | .63404 | 1.73254 | .65034 | 1.85990 | .66674 | 2.00067 | .68325 | 2.15704 | 32 |
| 33 | .63431 | 1.73456 | .65061 | 1.86213 | .66702 | 2.00315 | .68352 | 2.15979 | 33 |
| 34 | .63458 | 1.73659 | .65088 | 1.86437 | .66729 | 2.00562 | .68380 | 2.16255 | 34 |
| 35 | .63485 | 1.73862 | .65116 | 1.86661 | .66756 | 2.00810 | .68408 | 2.16531 | 35 |
| 36 | .63512 | 1.74065 | .65143 | 1.86885 | .66784 | 2.01059 | .68435 | 2.16808 | 36 |
| 37 | .63539 | 1.74269 | .65170 | 1.87109 | .66811 | 2.01308 | .68463 | 2.17085 | 37 |
| 38 | .63566 | 1.74473 | .65197 | 1.87334 | .66839 | 2.01557 | .68490 | 2.17363 | 38 |
| 39 | .63594 | 1.74677 | .65225 | 1.87560 | .66866 | 2.01807 | .68518 | 2.17641 | 39 |
| 40 | .63621 | 1.74881 | .65252 | 1.87785 | .66894 | 2.02057 | .68546 | 2.17920 | 40 |
| 41 | .63648 | 1.75086 | .65279 | 1.88011 | .66921 | 2.02308 | .68573 | 2.18199 | 41 |
| 42 | .63675 | 1.75292 | .65306 | 1.88238 | .66949 | 2.02559 | .68601 | 2.18479 | 42 |
| 43 | .63702 | 1.75497 | .65334 | 1.88465 | .66976 | 2.02810 | .68628 | 2.18759 | 43 |
| 44 | .63729 | 1.75703 | .65361 | 1.88692 | .67003 | 2.03062 | .68656 | 2.19040 | 44 |
| 45 | .63756 | 1.75909 | .65388 | 1.88920 | .67031 | 2.03315 | .68684 | 2.19322 | 45 |
| 46 | .63783 | 1.76116 | .65416 | 1.89148 | .67058 | 2.03568 | .68711 | 2.19604 | 46 |
| 47 | .63810 | 1.76323 | .65443 | 1.89376 | .67086 | 2.03821 | .68739 | 2.19886 | 47 |
| 48 | .63838 | 1.76530 | .65470 | 1.89605 | .67113 | 2.04075 | .68767 | 2.20169 | 48 |
| 49 | .63865 | 1.76737 | .65497 | 1.89834 | .67141 | 2.04329 | .68794 | 2.20453 | 49 |
| 50 | .63892 | 1.76945 | .65525 | 1.90063 | .67168 | 2.04584 | .68822 | 2.20737 | 50 |
| 51 | .63919 | 1.77154 | .65552 | 1.90293 | .67196 | 2.04839 | .68849 | 2.21021 | 51 |
| 52 | .63946 | 1.77362 | .65579 | 1.90524 | .67223 | 2.05094 | .68877 | 2.21306 | 52 |
| 53 | .63973 | 1.77571 | .65607 | 1.90754 | .67251 | 2.05350 | .68905 | 2.21592 | 53 |
| 54 | .64000 | 1.77780 | .65634 | 1.90986 | .67278 | 2.05607 | .68933 | 2.21878 | 54 |
| 55 | .64027 | 1.77990 | .65661 | 1.91217 | .67306 | 2.05864 | .68960 | 2.22165 | 55 |
| 56 | .64055 | 1.78200 | .65689 | 1.91449 | .67333 | 2.06121 | .68988 | 2.22452 | 56 |
| 57 | .64082 | 1.78410 | .65716 | 1.91681 | .67361 | 2.06379 | .69015 | 2.22740 | 57 |
| 58 | .64109 | 1.78621 | .65743 | 1.91914 | .67388 | 2.06637 | .69043 | 2.23028 | 58 |
| 59 | .64136 | 1.78832 | .65771 | 1.92147 | .67416 | 2.06896 | .69071 | 2.23317 | 59 |
| 60 | .64163 | 1.79043 | .65798 | 1.92380 | .67443 | 2.07155 | .69098 | 2.23607 | 60 |

| ′ | 72° | | 73° | | 74° | | 75° | | ′ |
|---|---|---|---|---|---|---|---|---|---|
| | Vers. | Ex. sec. | Vers. | Ex. sec. | Vers. | Ex. sec. | Vers. | Ex. sec. | |
| 0 | .69098 | 2.23607 | .70763 | 2.42030 | .72436 | 2.62796 | .74118 | 2.86370 | 0 |
| 1 | .69126 | 2.23697 | .70791 | 2.42356 | .72464 | 2.63164 | .74146 | 2.86790 | 1 |
| 2 | .69154 | 2.24187 | .70818 | 2.42683 | .72492 | 2.63533 | .74174 | 2.87211 | 2 |
| 3 | .69181 | 2.24478 | .70846 | 2.43010 | .72520 | 2.63903 | .74202 | 2.87633 | 3 |
| 4 | .69209 | 2.24770 | .70874 | 2.43337 | .72548 | 2.64274 | .74231 | 2.88056 | 4 |
| 5 | .69237 | 2.25062 | .70902 | 2.43666 | .72576 | 2.64645 | .74259 | 2.88479 | 5 |
| 6 | .69264 | 2.25355 | .70930 | 2.43995 | .72604 | 2.65018 | .74287 | 2.88904 | 6 |
| 7 | .69292 | 2.25648 | .70958 | 2.44324 | .72632 | 2.65391 | .74315 | 2.89330 | 7 |
| 8 | .69320 | 2.25942 | .70985 | 2.44655 | .72660 | 2.65765 | .74343 | 2.89756 | 8 |
| 9 | .69347 | 2.26237 | .71013 | 2.44986 | .72688 | 2.66140 | .74371 | 2.90184 | 9 |
| 10 | .69375 | 2.26531 | .71041 | 2.45317 | .72716 | 2.66515 | .74399 | 2.90613 | 10 |
| 11 | .69403 | 2.26827 | .71069 | 2.45650 | .72744 | 2.66892 | .74427 | 2.91042 | 11 |
| 12 | .69430 | 2.27123 | .71097 | 2.45983 | .72772 | 2.67269 | .74455 | 2.91473 | 12 |
| 13 | .69458 | 2.27420 | .71125 | 2.46316 | .72800 | 2.67647 | .74484 | 2.91904 | 13 |
| 14 | .69486 | 2.27717 | .71153 | 2.46651 | .72828 | 2.68025 | .74512 | 2.92337 | 14 |
| 15 | .69514 | 2.28015 | .71180 | 2.46986 | .72856 | 2.68405 | .74540 | 2.92770 | 15 |
| 16 | .69541 | 2.28313 | .71208 | 2.47321 | .72884 | 2.68785 | .74568 | 2.93204 | 16 |
| 17 | .69569 | 2.28612 | .71236 | 2.47658 | .72912 | 2.69167 | .74596 | 2.93640 | 17 |
| 18 | .69597 | 2.28912 | .71264 | 2.47995 | .72940 | 2.69549 | .74624 | 2.94076 | 18 |
| 19 | .69624 | 2.29212 | .71292 | 2.48333 | .72968 | 2.69931 | .74652 | 2.94514 | 19 |
| 20 | .69652 | 2.29512 | .71320 | 2.48671 | .72996 | 2.70315 | .74680 | 2.94952 | 20 |
| 21 | .69680 | 2.29814 | .71348 | 2.49010 | .73024 | 2.70700 | .74709 | 2.95392 | 21 |
| 22 | .69708 | 2.30115 | .71375 | 2.49350 | .73052 | 2.71085 | .74737 | 2.95832 | 22 |
| 23 | .69735 | 2.30418 | .71403 | 2.49691 | .73080 | 2.71471 | .74765 | 2.96274 | 23 |
| 24 | .69763 | 2.30721 | .71431 | 2.50032 | .73108 | 2.71858 | .74793 | 2.96716 | 24 |
| 25 | .69791 | 2.31024 | .71459 | 2.50374 | .73136 | 2.72246 | .74821 | 2.97160 | 25 |
| 26 | .69818 | 2.31328 | .71487 | 2.50716 | .73164 | 2.72635 | .74849 | 2.97604 | 26 |
| 27 | .69846 | 2.31633 | .71515 | 2.51060 | .73192 | 2.73024 | .74878 | 2.98050 | 27 |
| 28 | .69874 | 2.31939 | .71543 | 2.51404 | .73220 | 2.73414 | .74906 | 2.98497 | 28 |
| 29 | .69902 | 2.32244 | .71571 | 2.51748 | .73248 | 2.73806 | .74934 | 2.98944 | 29 |
| 30 | .69929 | 2.32551 | .71598 | 2.52094 | .73276 | 2.74198 | .74962 | 2.99393 | 30 |
| 31 | .69957 | 2.32858 | .71626 | 2.52440 | .73304 | 2.74591 | .74990 | 2.99843 | 31 |
| 32 | .69985 | 2.33166 | .71654 | 2.52787 | .73332 | 2.74984 | .75018 | 3.00293 | 32 |
| 33 | .70013 | 2.33474 | .71682 | 2.53134 | .73360 | 2.75379 | .75047 | 3.00745 | 33 |
| 34 | .70040 | 2.33783 | .71710 | 2.53482 | .73388 | 2.75775 | .75075 | 3.01198 | 34 |
| 35 | .70068 | 2.34092 | .71738 | 2.53831 | .73416 | 2.76171 | .75103 | 3.01652 | 35 |
| 36 | .70096 | 2.34403 | .71766 | 2.54181 | .73444 | 2.76568 | .75131 | 3.02107 | 36 |
| 37 | .70124 | 2.34713 | .71794 | 2.54531 | .73472 | 2.76966 | .75159 | 3.02563 | 37 |
| 38 | .70151 | 2.35025 | .71822 | 2.54883 | .73500 | 2.77365 | .75187 | 3.03020 | 38 |
| 39 | .70179 | 2.35336 | .71850 | 2.55235 | .73529 | 2.77765 | .75216 | 3.03479 | 39 |
| 40 | .70207 | 2.35649 | .71877 | 2.55587 | .73557 | 2.78166 | .75244 | 3.03938 | 40 |
| 41 | .70235 | 2.35962 | .71905 | 2.55940 | .73585 | 2.78568 | .75272 | 3.04398 | 41 |
| 42 | .70263 | 2.36276 | .71933 | 2.56294 | .73613 | 2.78970 | .75300 | 3.04860 | 42 |
| 43 | .70290 | 2.36590 | .71961 | 2.56649 | .73641 | 2.79374 | .75328 | 3.05322 | 43 |
| 44 | .70318 | 2.36905 | .71989 | 2.57005 | .73669 | 2.79778 | .75356 | 3.05786 | 44 |
| 45 | .70346 | 2.37221 | .72017 | 2.57361 | .73697 | 2.80183 | .75385 | 3.06251 | 45 |
| 46 | .70374 | 2.37537 | .72045 | 2.57718 | .73725 | 2.80589 | .75413 | 3.06717 | 46 |
| 47 | .70401 | 2.37854 | .72073 | 2.58076 | .73753 | 2.80996 | .75441 | 3.07184 | 47 |
| 48 | .70429 | 2.38171 | .72101 | 2.58434 | .73781 | 2.81404 | .75469 | 3.07652 | 48 |
| 49 | .70457 | 2.38489 | .72129 | 2.58794 | .73809 | 2.81813 | .75497 | 3.08121 | 49 |
| 50 | .70485 | 2.38808 | .72157 | 2.59154 | .73837 | 2.82223 | .75526 | 3.08591 | 50 |
| 51 | .70513 | 2.39128 | .72185 | 2.59514 | .73865 | 2.82633 | .75554 | 3.09063 | 51 |
| 52 | .70540 | 2.39448 | .72213 | 2.59876 | .73893 | 2.83045 | .75582 | 3.09535 | 52 |
| 53 | .70568 | 2.39768 | .72241 | 2.60238 | .73921 | 2.83457 | .75610 | 3.10009 | 53 |
| 54 | .70596 | 2.40089 | .72269 | 2.60601 | .73950 | 2.83871 | .75639 | 3.10484 | 54 |
| 55 | .70624 | 2.40411 | .72296 | 2.60965 | .73978 | 2.84285 | .75667 | 3.10960 | 55 |
| 56 | .70652 | 2.40734 | .72324 | 2.61330 | .74006 | 2.84700 | .75695 | 3.11437 | 56 |
| 57 | .70679 | 2.41057 | .72352 | 2.61695 | .74034 | 2.85116 | .75723 | 3.11915 | 57 |
| 58 | .70707 | 2.41381 | .72380 | 2.62061 | .74062 | 2.85533 | .75751 | 3.12394 | 58 |
| 59 | .70735 | 2.41705 | .72408 | 2.62428 | .74090 | 2.85951 | .75780 | 3.12875 | 59 |
| 60 | .70763 | 2.42030 | .72436 | 2.62796 | .74118 | 2.86370 | .75808 | 3.13357 | 60 |

# Table IV.—Natural Versed Sines and External Secants.

| ′ | 76° | | 77° | | 78° | | 79° | | ′ |
|---|---|---|---|---|---|---|---|---|---|
| | Vers. | Ex. sec. | Vers. | Ex. sec. | Vers. | Ex. sec. | Vers. | Ex. sec. | |
| 0 | .75808 | 3.13357 | .77505 | 3.44541 | .79209 | 3.80973 | .80919 | 4.24084 | 0 |
| 1 | .75836 | 3.13389 | .77533 | 3.45102 | .79237 | 3.81633 | .80948 | 4.24870 | 1 |
| 2 | .75864 | 3.14323 | .77562 | 3.45664 | .79266 | 3.82294 | .80976 | 4.25658 | 2 |
| 3 | .75892 | 3.14809 | .77590 | 3.46228 | .79294 | 3.82956 | .81005 | 4.26448 | 3 |
| 4 | .75921 | 3.15295 | .77618 | 3.46793 | .79322 | 3.83621 | .81033 | 4.27241 | 4 |
| 5 | .75949 | 3.15782 | .77647 | 3.47360 | .79351 | 3.84288 | .81062 | 4.28036 | 5 |
| 6 | .75977 | 3.16271 | .77675 | 3.47928 | .79380 | 3.84956 | .81090 | 4.28833 | 6 |
| 7 | .76005 | 3.16761 | .77703 | 3.48498 | .79408 | 3.85627 | .81119 | 4.29634 | 7 |
| 8 | .76034 | 3.17252 | .77732 | 3.49069 | .79437 | 3.86299 | .81148 | 4.30436 | 8 |
| 9 | .76062 | 3.17744 | .77760 | 3.49642 | .79465 | 3.86973 | .81176 | 4.31241 | 9 |
| 10 | .76090 | 3.18238 | .77788 | 3.50216 | .79493 | 3.87649 | .81205 | 4.32049 | 10 |
| 11 | .76118 | 3.18733 | .77817 | 3.50791 | .79522 | 3.88327 | .81233 | 4.32859 | 11 |
| 12 | .76147 | 3.19228 | .77845 | 3.51368 | .79550 | 3.89007 | .81262 | 4.33671 | 12 |
| 13 | .76175 | 3.19725 | .77874 | 3.51947 | .79579 | 3.89689 | .81290 | 4.34486 | 13 |
| 14 | .76203 | 3.20224 | .77902 | 3.52527 | .79607 | 3.90373 | .81319 | 4.35304 | 14 |
| 15 | .76231 | 3.20723 | .77930 | 3.53109 | .79636 | 3.91058 | .81348 | 4.36124 | 15 |
| 16 | .76260 | 3.21224 | .77959 | 3.53692 | .79664 | 3.91746 | .81376 | 4.36947 | 16 |
| 17 | .76288 | 3.21726 | .77987 | 3.54277 | .79693 | 3.92436 | .81405 | 4.37772 | 17 |
| 18 | .76316 | 3.22229 | .78015 | 3.54863 | .79721 | 3.93128 | .81433 | 4.38600 | 18 |
| 19 | .76344 | 3.22734 | .78044 | 3.55451 | .79750 | 3.93821 | .81462 | 4.39430 | 19 |
| 20 | .76373 | 3.23239 | .78072 | 3.56041 | .79778 | 3.94517 | .81491 | 4.40263 | 20 |
| 21 | .76401 | 3.23746 | .78101 | 3.56632 | .79807 | 3.95215 | .81519 | 4.41099 | 21 |
| 22 | .76429 | 3.24255 | .78129 | 3.57224 | .79835 | 3.95914 | .81548 | 4.41937 | 22 |
| 23 | .76458 | 3.24764 | .78157 | 3.57819 | .79864 | 3.96616 | .81576 | 4.42778 | 23 |
| 24 | .76486 | 3.25275 | .78186 | 3.58414 | .79892 | 3.97320 | .81605 | 4.43622 | 24 |
| 25 | .76514 | 3.25787 | .78214 | 3.59012 | .79921 | 3.98025 | .81633 | 4.44468 | 25 |
| 26 | .76542 | 3.26300 | .78242 | 3.59611 | .79949 | 3.98733 | .81662 | 4.45317 | 26 |
| 27 | .76571 | 3.26814 | .78271 | 3.60211 | .79978 | 3.99443 | .81691 | 4.46169 | 27 |
| 28 | .76599 | 3.27330 | .78299 | 3.60813 | .80006 | 4.00155 | .81719 | 4.47023 | 28 |
| 29 | .76627 | 3.27847 | .78328 | 3.61417 | .80035 | 4.00869 | .81748 | 4.47881 | 29 |
| 30 | .76655 | 3.28366 | .78356 | 3.62023 | .80063 | 4.01585 | .81776 | 4.48740 | 30 |
| 31 | .76684 | 3.28885 | .78384 | 3.62630 | .80092 | 4.02303 | .81805 | 4.49603 | 31 |
| 32 | .76712 | 3.29406 | .78413 | 3.63238 | .80120 | 4.03024 | .81834 | 4.50468 | 32 |
| 33 | .76740 | 3.29929 | .78441 | 3.63849 | .80149 | 4.03746 | .81862 | 4.51337 | 33 |
| 34 | .76769 | 3.30452 | .78470 | 3.64461 | .80177 | 4.04471 | .81891 | 4.52208 | 34 |
| 35 | .76797 | 3.30977 | .78498 | 3.65074 | .80206 | 4.05197 | .81919 | 4.53081 | 35 |
| 36 | .76825 | 3.31503 | .78526 | 3.65690 | .80234 | 4.05926 | .81948 | 4.53958 | 36 |
| 37 | .76854 | 3.32031 | .78555 | 3.66307 | .80263 | 4.06657 | .81977 | 4.54837 | 37 |
| 38 | .76882 | 3.32560 | .78583 | 3.66925 | .80291 | 4.07390 | .82005 | 4.55720 | 38 |
| 39 | .76910 | 3.33090 | .78612 | 3.67545 | .80320 | 4.08125 | .82034 | 4.56605 | 39 |
| 40 | .76938 | 3.33622 | .78640 | 3.68167 | .80348 | 4.08863 | .82063 | 4.57493 | 40 |
| 41 | .76967 | 3.34154 | .78669 | 3.68791 | .80377 | 4.09602 | .82091 | 4.58383 | 41 |
| 42 | .76995 | 3.34689 | .78697 | 3.69417 | .80405 | 4.10344 | .82120 | 4.59277 | 42 |
| 43 | .77023 | 3.35224 | .78725 | 3.70044 | .80434 | 4.11088 | .82148 | 4.60174 | 43 |
| 44 | .77052 | 3.35761 | .78754 | 3.70673 | .80462 | 4.11835 | .82177 | 4.61073 | 44 |
| 45 | .77080 | 3.36299 | .78782 | 3.71303 | .80491 | 4.12583 | .82206 | 4.61976 | 45 |
| 46 | .77108 | 3.36839 | .78811 | 3.71935 | .80520 | 4.13334 | .82234 | 4.62881 | 46 |
| 47 | .77137 | 3.37380 | .78839 | 3.72569 | .80548 | 4.14087 | .82263 | 4.63790 | 47 |
| 48 | .77165 | 3.37923 | .78868 | 3.73205 | .80577 | 4.14842 | .82292 | 4.64701 | 48 |
| 49 | .77193 | 3.38466 | .78896 | 3.73843 | .80605 | 4.15599 | .82320 | 4.65616 | 49 |
| 50 | .77222 | 3.39012 | .78924 | 3.74482 | .80634 | 4.16359 | .82349 | 4.66533 | 50 |
| 51 | .77250 | 3.39558 | .78953 | 3.75123 | .80662 | 4.17121 | .82377 | 4.67454 | 51 |
| 52 | .77278 | 3.40106 | .78981 | 3.75766 | .80691 | 4.17886 | .82406 | 4.68377 | 52 |
| 53 | .77307 | 3.40656 | .79010 | 3.76411 | .80719 | 4.18652 | .82435 | 4.69304 | 53 |
| 54 | .77335 | 3.41206 | .79038 | 3.77057 | .80748 | 4.19421 | .82463 | 4.70234 | 54 |
| 55 | .77363 | 3.41759 | .79067 | 3.77705 | .80776 | 4.20193 | .82492 | 4.71166 | 55 |
| 56 | .77392 | 3.42312 | .79095 | 3.78355 | .80805 | 4.20966 | .82521 | 4.72102 | 56 |
| 57 | .77420 | 3.42867 | .79123 | 3.79007 | .80833 | 4.21742 | .82549 | 4.73041 | 57 |
| 58 | .77448 | 3.43424 | .79152 | 3.79661 | .80862 | 4.22521 | .82578 | 4.73983 | 58 |
| 59 | .77477 | 3.43982 | .79180 | 3.80316 | .80891 | 4.23301 | .82607 | 4.74929 | 59 |
| 60 | .77505 | 3.44541 | .79209 | 3.80973 | .80919 | 4.24084 | .82635 | 4.75877 | 60 |

# Table IV.—Natural Versed Sines and External Secants.

| ′ | 80° | | 81° | | 82° | | 83° | | ′ |
|---|---|---|---|---|---|---|---|---|---|
| | Vers. | Ex. sec. | Vers. | Ex. sec. | Vers. | Ex. sec. | Vers. | Ex. sec. | |
| 0 | .82635 | 4.75877 | .84357 | 5.39245 | .86083 | 6.18530 | .87813 | 7.20551 | 0 |
| 1 | .82664 | 4.76829 | .84385 | 5.40122 | .86112 | 6.20020 | .87842 | 7.22500 | 1 |
| 2 | .82692 | 4.77784 | .84414 | 5.41002 | .86140 | 6.21517 | .87871 | 7.24457 | 2 |
| 3 | .82721 | 4.78742 | .84443 | 5.42787 | .86169 | 6.23019 | .87900 | 7.26425 | 3 |
| 4 | .82750 | 4.79703 | .84471 | 5.43977 | .86198 | 6.24529 | .87929 | 7.28402 | 4 |
| 5 | .82778 | 4.80667 | .84500 | 5.45171 | .86227 | 6.26044 | .87957 | 7.30388 | 5 |
| 6 | .82807 | 4.81635 | .84529 | 5.46369 | .86256 | 6.27566 | .87986 | 7.32384 | 6 |
| 7 | .82836 | 4.82606 | .84558 | 5.47572 | .86284 | 6.29095 | .88015 | 7.34390 | 7 |
| 8 | .82864 | 4.83581 | .84586 | 5.48779 | .86313 | 6.30630 | .88044 | 7.36405 | 8 |
| 9 | .82893 | 4.84558 | .84615 | 5.49991 | .86342 | 6.32171 | .88073 | 7.38431 | 9 |
| 10 | .82922 | 4.85539 | .84644 | 5.51208 | .86371 | 6.33719 | .88102 | 7.40466 | 10 |
| 11 | .82950 | 4.86524 | .84673 | 5.52429 | .86400 | 6.35274 | .88131 | 7.42511 | 11 |
| 12 | .82979 | 4.87511 | .84701 | 5.53655 | .86428 | 6.36835 | .88160 | 7.44566 | 12 |
| 13 | .83008 | 4.88502 | .84730 | 5.54886 | .86457 | 6.38403 | .88188 | 7.46632 | 13 |
| 14 | .83036 | 4.89497 | .84759 | 5.56121 | .86486 | 6.39978 | .88217 | 7.48707 | 14 |
| 15 | .83065 | 4.90495 | .84788 | 5.57361 | .86515 | 6.41560 | .88246 | 7.50793 | 15 |
| 16 | .83094 | 4.91496 | .84816 | 5.58606 | .86544 | 6.43148 | .88275 | 7.52889 | 16 |
| 17 | .83122 | 4.92501 | .84845 | 5.59855 | .86573 | 6.44743 | .88304 | 7.54996 | 17 |
| 18 | .83151 | 4.93509 | .84874 | 5.61110 | .86601 | 6.46346 | .88333 | 7.57113 | 18 |
| 19 | .83180 | 4.94521 | .84903 | 5.62369 | .86630 | 6.47955 | .88362 | 7.59241 | 19 |
| 20 | .83208 | 4.95536 | .84931 | 5.63633 | .86659 | 6.49571 | .88391 | 7.61379 | 20 |
| 21 | .83237 | 4.96555 | .84960 | 5.64902 | .86688 | 6.51194 | .88420 | 7.63528 | 21 |
| 22 | .83266 | 4.97577 | .84989 | 5.66176 | .86717 | 6.52825 | .88448 | 7.65688 | 22 |
| 23 | .83294 | 4.98603 | .85018 | 5.67454 | .86746 | 6.54462 | .88477 | 7.67859 | 23 |
| 24 | .83323 | 4.99633 | .85046 | 5.68738 | .86774 | 6.56107 | .88506 | 7.70041 | 24 |
| 25 | .83352 | 5.00666 | .85075 | 5.70027 | .86803 | 6.57759 | .88535 | 7.72234 | 25 |
| 26 | .83380 | 5.01703 | .85104 | 5.71321 | .86832 | 6.59418 | .88564 | 7.74438 | 26 |
| 27 | .83409 | 5.02743 | .85133 | 5.72620 | .86861 | 6.61085 | .88593 | 7.76653 | 27 |
| 28 | .83438 | 5.03787 | .85162 | 5.73924 | .86890 | 6.62759 | .88622 | 7.78880 | 28 |
| 29 | .83467 | 5.04834 | .85190 | 5.75233 | .86919 | 6.64441 | .88651 | 7.81118 | 29 |
| 30 | .83495 | 5.05886 | .85219 | 5.76547 | .86947 | 6.66130 | .88680 | 7.83367 | 30 |
| 31 | .83524 | 5.06941 | .85248 | 5.77866 | .86976 | 6.67826 | .88709 | 7.85628 | 31 |
| 32 | .83553 | 5.08000 | .85277 | 5.79191 | .87005 | 6.69530 | .88737 | 7.87901 | 32 |
| 33 | .83581 | 5.09062 | .85305 | 5.80521 | .87034 | 6.71242 | .88766 | 7.90186 | 33 |
| 34 | .83610 | 5.10129 | .85334 | 5.81856 | .87063 | 6.72962 | .88795 | 7.92482 | 34 |
| 35 | .83639 | 5.11199 | .85363 | 5.83196 | .87092 | 6.74689 | .88824 | 7.94791 | 35 |
| 36 | .83667 | 5.12273 | .85392 | 5.84542 | .87120 | 6.76424 | .88853 | 7.97111 | 36 |
| 37 | .83696 | 5.13350 | .85420 | 5.85893 | .87149 | 6.78167 | .88882 | 7.99444 | 37 |
| 38 | .83725 | 5.14432 | .85449 | 5.87250 | .87178 | 6.79918 | .88911 | 8.01788 | 38 |
| 39 | .83754 | 5.15517 | .85478 | 5.88612 | .87207 | 6.81677 | .88940 | 8.04146 | 39 |
| 40 | .83782 | 5.16607 | .85507 | 5.89979 | .87236 | 6.83443 | .88969 | 8.06515 | 40 |
| 41 | .83811 | 5.17700 | .85536 | 5.91352 | .87265 | 6.85218 | .88998 | 8.08897 | 41 |
| 42 | .83840 | 5.18797 | .85564 | 5.92731 | .87294 | 6.87001 | .89027 | 8.11292 | 42 |
| 43 | .83868 | 5.19898 | .85593 | 5.94115 | .87322 | 6.88792 | .89055 | 8.13699 | 43 |
| 44 | .83897 | 5.21004 | .85622 | 5.95505 | .87351 | 6.90592 | .89084 | 8.16120 | 44 |
| 45 | .83926 | 5.22113 | .85651 | 5.96900 | .87380 | 6.92400 | .89113 | 8.18553 | 45 |
| 46 | .83954 | 5.23226 | .85680 | 5.98301 | .87409 | 6.94216 | .89142 | 8.20999 | 46 |
| 47 | .83983 | 5.24343 | .85708 | 5.99708 | .87438 | 6.96040 | .89171 | 8.23459 | 47 |
| 48 | .84012 | 5.25464 | .85737 | 6.01120 | .87467 | 6.97873 | .89200 | 8.25931 | 48 |
| 49 | .84041 | 5.26590 | .85766 | 6.02538 | .87496 | 6.99714 | .89229 | 8.28417 | 49 |
| 50 | .84069 | 5.27719 | .85795 | 6.03962 | .87524 | 7.01565 | .89258 | 8.30917 | 50 |
| 51 | .84098 | 5.28853 | .85823 | 6.05392 | .87553 | 7.03423 | .89287 | 8.33430 | 51 |
| 52 | .84127 | 5.29991 | .85852 | 6.06828 | .87582 | 7.05291 | .89316 | 8.35957 | 52 |
| 53 | .84155 | 5.31133 | .85881 | 6.08269 | .87611 | 7.07167 | .89345 | 8.38497 | 53 |
| 54 | .84184 | 5.32279 | .85910 | 6.09717 | .87640 | 7.09052 | .89374 | 8.41052 | 54 |
| 55 | .84213 | 5.33429 | .85939 | 6.11171 | .87669 | 7.10946 | .89403 | 8.43620 | 55 |
| 56 | .84242 | 5.34584 | .85967 | 6.12630 | .87698 | 7.12849 | .89431 | 8.46203 | 56 |
| 57 | .84270 | 5.35743 | .85996 | 6.14096 | .87726 | 7.14760 | .89460 | 8.48800 | 57 |
| 58 | .84299 | 5.36906 | .86025 | 6.15568 | .87755 | 7.16681 | .89489 | 8.51411 | 58 |
| 59 | .84328 | 5.38073 | .86054 | 6.17046 | .87784 | 7.18612 | .89518 | 8.54037 | ′ |
| 60 | .84357 | 5.39245 | .86083 | 6.18530 | .87813 | 7.20551 | .89547 | 8.56677 | |

## Table IV.—Natural Versed Sines and External Secants.

| ′ | 84° | | 85° | | 86° | | ′ |
|---|---|---|---|---|---|---|---|
| | Vers. | Ex. sec. | Vers. | Ex. sec. | Vers. | Ex. sec. | |
| 0 | .89547 | 8.50677 | .91284 | 10.47371 | .93024 | 13.33559 | 0 |
| 1 | .89576 | 8.50832 | .91313 | 10.51199 | .93053 | 13.39547 | 1 |
| 2 | .89605 | 8.62002 | .91342 | 10.55052 | .93082 | 13.45586 | 2 |
| 3 | .89634 | 8.64657 | .91371 | 10.58932 | .93111 | 13.51676 | 3 |
| 4 | .89663 | 8.67357 | .91400 | 10.62837 | .93140 | 13.57817 | 4 |
| 5 | .89692 | 8.70103 | .91429 | 10.66769 | .93169 | 13.64011 | 5 |
| 6 | .89721 | 8.72893 | .91458 | 10.70728 | .93198 | 13.70258 | 6 |
| 7 | .89750 | 8.75579 | .91487 | 10.74714 | .93227 | 13.76558 | 7 |
| 8 | .89779 | 8.78341 | .91516 | 10.78727 | .93257 | 13.82913 | 8 |
| 9 | .89808 | 8.81119 | .91545 | 10.82768 | .93286 | 13.89323 | 9 |
| 10 | .89836 | 8.83913 | .91574 | 10.86837 | .93315 | 13.95788 | 10 |
| 11 | .89865 | 8.86722 | .91603 | 10.90934 | .93344 | 14.02310 | 11 |
| 12 | .89894 | 8.89547 | .91632 | 10.95060 | .93373 | 14.08890 | 12 |
| 13 | .89923 | 8.92389 | .91661 | 10.99214 | .93402 | 14.15527 | 13 |
| 14 | .89952 | 8.95248 | .91690 | 11.03397 | .93431 | 14.22223 | 14 |
| 15 | .89981 | 8.98123 | .91719 | 11.07610 | .93460 | 14.28979 | 15 |
| 16 | .90010 | 9.01015 | .91748 | 11.11852 | .93489 | 14.35795 | 16 |
| 17 | .90039 | 9.03923 | .91777 | 11.16125 | .93518 | 14.42672 | 17 |
| 18 | .90068 | 9.06849 | .91806 | 11.20427 | .93547 | 14.49611 | 18 |
| 19 | .90097 | 9.09792 | .91835 | 11.24761 | .93576 | 14.56614 | 19 |
| 20 | .90126 | 9.12752 | .91864 | 11.29125 | .93605 | 14.63679 | 20 |
| 21 | .90155 | 9.15730 | .91893 | 11.33521 | .93634 | 14.70810 | 21 |
| 22 | .90184 | 9.18725 | .91922 | 11.37948 | .93663 | 14.78005 | 22 |
| 23 | .90213 | 9.21739 | .91951 | 11.42408 | .93692 | 14.85268 | 23 |
| 24 | .90242 | 9.24770 | .91980 | 11.46900 | .93721 | 14.92597 | 24 |
| 25 | .90271 | 9.27819 | .92009 | 11.51424 | .93750 | 14.99995 | 25 |
| 26 | .90300 | 9.30887 | .92038 | 11.55982 | .93779 | 15.07462 | 26 |
| 27 | .90329 | 9.33973 | .92067 | 11.60572 | .93808 | 15.14999 | 27 |
| 28 | .90358 | 9.37077 | .92096 | 11.65197 | .93837 | 15.22607 | 28 |
| 29 | .90386 | 9.40201 | .92125 | 11.69856 | .93866 | 15.30287 | 29 |
| 30 | .90415 | 9.43343 | .92154 | 11.74550 | .93895 | 15.38041 | 30 |
| 31 | .90444 | 9.46505 | .92183 | 11.79278 | .93924 | 15.45869 | 31 |
| 32 | .90473 | 9.49685 | .92212 | 11.84042 | .93953 | 15.53773 | 32 |
| 33 | .90502 | 9.52886 | .92241 | 11.88841 | .93982 | 15.61751 | 33 |
| 34 | .90531 | 9.56106 | .92270 | 11.93677 | .94011 | 15.69806 | 34 |
| 35 | .90560 | 9.59346 | .92299 | 11.98549 | .94040 | 15.77944 | 35 |
| 36 | .90589 | 9.62605 | .92328 | 12.03458 | .94069 | 15.86159 | 36 |
| 37 | .90618 | 9.65885 | .92357 | 12.08040 | .94098 | 15.94456 | 37 |
| 38 | .90647 | 9.69186 | .92386 | 12.13388 | .94127 | 16.02885 | 38 |
| 39 | .90676 | 9.72507 | .92415 | 12.18411 | .94156 | 16.11297 | 39 |
| 40 | .90705 | 9.75849 | .92444 | 12.23472 | .94186 | 16.19843 | 40 |
| 41 | .90734 | 9.79212 | .92473 | 12.28573 | .94215 | 16.28476 | 41 |
| 42 | .90763 | 9.82596 | .92502 | 12.33712 | .94244 | 16.37196 | 42 |
| 43 | .90792 | 9.86001 | .92531 | 12.38891 | .94273 | 16.46005 | 43 |
| 44 | .90821 | 9.89428 | .92560 | 12.44112 | .94302 | 16.54903 | 44 |
| 45 | .90850 | 9.92877 | .92589 | 12.49373 | .94331 | 16.63893 | 45 |
| 46 | .90879 | 9.96348 | .92618 | 12.54676 | .94360 | 16.72975 | 46 |
| 47 | .90908 | 9.99841 | .92647 | 12.60021 | .94389 | 16.82152 | 47 |
| 48 | .90937 | 10.03356 | .92676 | 12.65408 | .94418 | 16.91424 | 48 |
| 49 | .90966 | 10.06894 | .92705 | 12.70838 | .94447 | 17.00794 | 49 |
| 50 | .90995 | 10.10455 | .92734 | 12.76312 | .94476 | 17.10263 | 50 |
| 51 | .91024 | 10.14039 | .92763 | 12.81829 | .94505 | 17.19830 | 51 |
| 52 | .91053 | 10.17646 | .92792 | 12.87391 | .94534 | 17.29501 | 52 |
| 53 | .91082 | 10.21277 | .92821 | 12.92999 | .94563 | 17.39274 | 53 |
| 54 | .91111 | 10.24932 | .92850 | 12.98651 | .94592 | 17.49153 | 54 |
| 55 | .91140 | 10.28610 | .92879 | 13 04350 | .94621 | 17.59139 | 55 |
| 56 | .91169 | 10.32313 | .92908 | 13.10096 | .94650 | 17.69233 | 56 |
| 57 | .91197 | 10.36040 | .92937 | 13.15889 | .94679 | 17.79438 | 57 |
| 58 | .91226 | 10.39792 | .92966 | 13.21730 | .94708 | 17.89755 | 58 |
| 59 | .91255 | 10.43569 | .92995 | 13.27620 | .94737 | 18.00185 | 59 |
| 60 | .91284 | 10.47371 | .93024 | 13.33559 | .94766 | 18.10732 | 60 |

## Table IV.—Natural Versed Sines and External Secants

| ′ | 87° | | 88° | | 89° | | ′ |
|---|---|---|---|---|---|---|---|
| | Vers. | Ex. sec. | Vers. | Ex. sec. | Vers. | Ex. sec. | |
| 0 | .94766 | 18.10732 | .96510 | 27.65871 | .98255 | 56.29869 | 0 |
| 1 | .94795 | 18.21397 | .96539 | 27.89440 | .98284 | 57.26976 | 1 |
| 2 | .94825 | 18.32182 | .96568 | 28.13917 | .98313 | 58.27431 | 2 |
| 3 | .94854 | 18.43088 | .96597 | 28.38812 | .98342 | 59.31411 | 3 |
| 4 | .94883 | 18.54119 | .96626 | 28.64137 | .98371 | 60.39105 | 4 |
| 5 | .94912 | 18.65275 | .96655 | 28.89903 | .98400 | 61.50715 | 5 |
| 6 | .94941 | 18.76560 | .96684 | 29.16120 | .98429 | 62.66460 | 6 |
| 7 | .94970 | 18.87976 | .96714 | 29.42802 | .98458 | 63.86572 | 7 |
| 8 | .94999 | 18.99524 | .96743 | 29.69960 | .98487 | 65.11304 | 8 |
| 9 | .95028 | 19.11208 | .96772 | 29.97607 | .98517 | 66.40927 | 9 |
| 10 | .95057 | 19.23028 | .96801 | 30.25758 | .98546 | 67.75736 | 10 |
| 11 | .95086 | 19.34989 | .96830 | 30.54425 | .98575 | 69.16047 | 11 |
| 12 | .95115 | 19.47093 | .96859 | 30.83623 | .98604 | 70.62285 | 12 |
| 13 | .95144 | 19.59341 | .96888 | 31.13366 | .98633 | 72.14583 | 13 |
| 14 | .95173 | 19.71737 | .96917 | 31.43671 | .98662 | 73.73586 | 14 |
| 15 | .95202 | 19.84283 | .96946 | 31.74554 | .98691 | 75.39655 | 15 |
| 16 | .95231 | 19.96982 | .96975 | 32.06030 | .98720 | 77.13274 | 16 |
| 17 | .95260 | 20.09838 | .97004 | 32.38118 | .98749 | 78.94968 | 17 |
| 18 | .95289 | 20.22852 | .97033 | 32.70835 | .98778 | 80.85315 | 18 |
| 19 | .95318 | 20.36027 | .97062 | 33.04199 | .98807 | 82.84947 | 19 |
| 20 | .95347 | 20.49368 | .97092 | 33.38233 | .98836 | 84.94561 | 20 |
| 21 | .95377 | 20.62876 | .97121 | 33.72952 | .98866 | 87.14924 | 21 |
| 22 | .95406 | 20.76555 | .97150 | 34.08380 | .98895 | 89.46886 | 22 |
| 23 | .95435 | 20.90403 | .97179 | 34.44539 | .98924 | 91.91387 | 23 |
| 24 | .95464 | 21.04440 | .97208 | 34.81452 | .98953 | 94.49471 | 24 |
| 25 | .95493 | 21.18653 | .97237 | 35.19141 | .98982 | 97.22303 | 25 |
| 26 | .95522 | 21.33050 | .97266 | 35.57633 | .99011 | 100.1119 | 26 |
| 27 | .95551 | 21.47635 | .97295 | 35.96953 | .99040 | 103.1757 | 27 |
| 28 | .95580 | 21.62413 | .97324 | 36.37127 | .99069 | 106.4311 | 28 |
| 29 | .95609 | 21.77386 | .97353 | 36.78185 | .99098 | 109.8966 | 29 |
| 30 | .95638 | 21.92559 | .97382 | 37.20155 | .99127 | 113.5980 | 30 |
| 31 | .95667 | 22.07935 | .97411 | 37.63068 | .99156 | 117.5444 | 31 |
| 32 | .95696 | 22.23520 | .97440 | 38.06957 | .99186 | 121.7780 | 32 |
| 33 | .95725 | 22.39316 | .97470 | 38.51855 | .99215 | 126.3253 | 33 |
| 34 | .95754 | 22.55329 | .97499 | 38.97797 | .99244 | 131.2223 | 34 |
| 35 | .95783 | 22.71563 | .97528 | 39.44820 | .99273 | 136.5111 | 35 |
| 36 | .95812 | 22.88022 | .97557 | 39.92963 | .99302 | 142.2406 | 36 |
| 37 | .95842 | 23.04712 | .97586 | 40.42266 | .99331 | 148.4684 | 37 |
| 38 | .95871 | 23.21637 | .97615 | 40.92772 | .99360 | 155.2623 | 38 |
| 39 | .95900 | 23.38802 | .97644 | 41.44525 | .99389 | 162.7083 | 39 |
| 40 | .95929 | 23.56212 | .97673 | 41.97571 | .99418 | 170.8883 | 40 |
| 41 | .95958 | 23.73873 | .97702 | 42.51961 | .99447 | 179.9350 | 41 |
| 42 | .95987 | 23.91790 | .97731 | 43.07746 | .99476 | 189.9868 | 42 |
| 43 | .96016 | 24.09969 | .97760 | 43.64980 | .99505 | 201.2212 | 43 |
| 44 | .96045 | 24.28414 | .97789 | 44.23720 | .99535 | 213.8600 | 44 |
| 45 | .96074 | 24.47134 | .97819 | 44.84026 | .99564 | 228.1339 | 45 |
| 46 | .96103 | 24.66132 | .97848 | 45.45963 | .99593 | 244.5540 | 46 |
| 47 | .96132 | 24.85417 | .97877 | 46.09596 | .99622 | 263.4427 | 47 |
| 48 | .96161 | 25.04994 | .97906 | 46.74997 | .99651 | 285.4795 | 48 |
| 49 | .96190 | 25.24869 | .97935 | 47.42241 | .99680 | 311.5230 | 49 |
| 50 | .96219 | 25.45051 | .97964 | 48.11406 | .99709 | 342.7752 | 50 |
| 51 | .96248 | 25.65546 | .97993 | 48.82576 | .99738 | 380.9723 | 51 |
| 52 | .96277 | 25.86360 | .98022 | 49.55840 | .99767 | 428.7187 | 52 |
| 53 | .96307 | 26.07503 | .98051 | 50.31290 | .99796 | 490.1070 | 53 |
| 54 | .96336 | 26.28981 | .98080 | 51.09027 | .99825 | 571.9681 | 54 |
| 55 | .96365 | 26.50804 | .98109 | 51.89156 | .99855 | 686.5496 | 55 |
| 56 | .96394 | 26.72978 | .98138 | 52.71790 | .99884 | 858.4369 | 56 |
| 57 | .96423 | 26.95513 | .98168 | 53.57046 | .99913 | 1144.916 | 57 |
| 58 | .96452 | 27.18417 | .98197 | 54.45053 | .99942 | 1717.874 | 58 |
| 59 | .96481 | 27.41700 | .98226 | 55.35946 | .99971 | 3436.747 | 59 |
| 60 | .96510 | 27.65371 | .98255 | 56.29869 | 1.00000 | Infinite | 60 |

## TABLE V.—SQUARES, CUBES, SQUARE ROOTS,

| No. | Squares. | Cubes. | Square Roots. | Cube Roots. | Reciprocals. |
|---|---|---|---|---|---|
| 1 | 1 | 1 | 1.0000000 | 1.0000000 | 1.000000000 |
| 2 | 4 | 8 | 1.4142136 | 1.2599210 | .500000000 |
| 3 | 9 | 27 | 1.7320508 | 1.4422496 | .333333333 |
| 4 | 16 | 64 | 2.0000000 | 1.5874011 | .250000000 |
| 5 | 25 | 125 | 2.2360680 | 1.7099759 | .200000000 |
| 6 | 36 | 216 | 2.4494897 | 1.8171206 | .166666667 |
| 7 | 49 | 343 | 2.6457513 | 1.9129312 | .142857143 |
| 8 | 64 | 512 | 2.8284271 | 2.0000000 | .125000000 |
| 9 | 81 | 729 | 3.0000000 | 2.0800837 | .111111111 |
| 10 | 100 | 1000 | 3.1622777 | 2.1544347 | .100000000 |
| 11 | 121 | 1331 | 3.3166248 | 2.2239801 | .090909091 |
| 12 | 144 | 1728 | 3.4641016 | 2.2894286 | .083333333 |
| 13 | 169 | 2197 | 3.6055513 | 2.3513347 | .076923077 |
| 14 | 196 | 2744 | 3.7416574 | 2.4101422 | .071428571 |
| 15 | 225 | 3375 | 3.8729833 | 2.4662121 | .066666667 |
| 16 | 256 | 4096 | 4.0000000 | 2.5198421 | .062500000 |
| 17 | 289 | 4913 | 4.1231056 | 2.5712816 | .058823529 |
| 18 | 324 | 5832 | 4.2426407 | 2.6207414 | .055555556 |
| 19 | 361 | 6859 | 4.3588989 | 2.6684016 | .052631579 |
| 20 | 400 | 8000 | 4.4721360 | 2.7144177 | .050000000 |
| 21 | 441 | 9261 | 4.5825757 | 2.7589243 | .047619048 |
| 22 | 484 | 10648 | 4.6904158 | 2.8020393 | .045454545 |
| 23 | 529 | 12167 | 4.7958315 | 2.8438670 | .043478261 |
| 24 | 576 | 13824 | 4.8989795 | 2.8844991 | .041666667 |
| 25 | 625 | 15625 | 5.0000000 | 2.9240177 | .040000000 |
| 26 | 676 | 17576 | 5.0990195 | 2.9624960 | .038461538 |
| 27 | 729 | 19683 | 5.1961524 | 3.0000000 | .037037037 |
| 28 | 784 | 21952 | 5.2915026 | 3.0365889 | .035714286 |
| 29 | 841 | 24389 | 5.3851648 | 3.0723168 | .034482759 |
| 30 | 900 | 27000 | 5.4772256 | 3.1072325 | .033333333 |
| 31 | 961 | 29791 | 5.5677644 | 3.1413806 | .032258065 |
| 32 | 1024 | 32768 | 5.6568542 | 3.1748021 | .031250000 |
| 33 | 1089 | 35937 | 5.7445626 | 3.2075343 | .030303030 |
| 34 | 1156 | 39304 | 5.8309519 | 3.2396118 | .029411765 |
| 35 | 1225 | 42875 | 5.9160798 | 3.2710663 | .028571429 |
| 36 | 1296 | 46656 | 6.0000000 | 3.3019272 | .027777778 |
| 37 | 1369 | 50653 | 6.0827625 | 3.3322218 | .027027027 |
| 38 | 1444 | 54872 | 6.1644140 | 3.3619754 | .026315789 |
| 39 | 1521 | 59319 | 6.2449980 | 3.3912114 | .025641026 |
| 40 | 1600 | 64000 | 6.3245553 | 3.4199519 | .025000000 |
| 41 | 1681 | 68921 | 6.4031242 | 3.4482172 | .024390244 |
| 42 | 1764 | 74088 | 6.4807407 | 3.4760266 | .023809524 |
| 43 | 1849 | 79507 | 6.5574385 | 3.5033981 | .023255814 |
| 44 | 1936 | 85184 | 6.6332496 | 3.5303483 | .022727273 |
| 45 | 2025 | 91125 | 6.7082039 | 3.5568933 | .022222222 |
| 46 | 2116 | 97336 | 6.7823300 | 3.5830479 | .021739130 |
| 47 | 2209 | 103823 | 6.8556546 | 3.6088261 | .021276600 |
| 48 | 2304 | 110592 | 6.9282032 | 3.6342411 | .020833333 |
| 49 | 2401 | 117649 | 7.0000000 | 3.6593057 | .020408163 |
| 50 | 2500 | 125000 | 7.0710678 | 3.6840314 | .020000000 |
| 51 | 2601 | 132651 | 7.1414284 | 3.7084298 | .019607843 |
| 52 | 2704 | 140608 | 7.2111026 | 3.7325111 | .019230769 |
| 53 | 2809 | 148877 | 7.2801099 | 3.7562858 | .018867925 |
| 54 | 2916 | 157464 | 7.3484692 | 3.7797631 | .018518519 |
| 55 | 3025 | 166375 | 7.4161985 | 3.8029525 | .018181818 |
| 56 | 3136 | 175616 | 7.4833148 | 3.8258624 | .017857143 |
| 57 | 3249 | 185193 | 7.5498344 | 3.8485011 | .017543860 |
| 58 | 3364 | 195112 | 7.6157731 | 3.8708766 | .017241379 |
| 59 | 3481 | 205379 | 7.6811457 | 3.8929965 | .016949153 |
| 60 | 3600 | 216000 | 7.7459667 | 3.9148676 | .016666667 |
| 61 | 3721 | 226981 | 7.8102497 | 3.9364972 | .016393443 |
| 62 | 3844 | 238328 | 7.8740079 | 3.9578915 | .016129032 |

| No. | Squares. | Cubes. | Square Roots. | Cube Roots. | Reciprocals. |
|---|---|---|---|---|---|
| 63 | 3969 | 250047 | 7.9372539 | 3.9790571 | .015873016 |
| 64 | 4096 | 262144 | 8.0000000 | 4.0000000 | .015625000 |
| 65 | 4225 | 274625 | 8.0622577 | 4.0207256 | .015384615 |
| 66 | 4356 | 287496 | 8.1240384 | 4.0412401 | .015151515 |
| 67 | 4489 | 300763 | 8.1853528 | 4.0615480 | .014925373 |
| 68 | 4624 | 314432 | 8.2462113 | 4.0816551 | .014705882 |
| 69 | 4761 | 328509 | 8.3066239 | 4.1015661 | .014492754 |
| 70 | 4900 | 343000 | 8.3666003 | 4.1212853 | .014285714 |
| 71 | 5041 | 357911 | 8.4261498 | 4.1408178 | .014084507 |
| 72 | 5184 | 373248 | 8.4852814 | 4.1601676 | .013888889 |
| 73 | 5329 | 389017 | 8.5440037 | 4.1793390 | .013698630 |
| 74 | 5476 | 405224 | 8.6023253 | 4.1983364 | .013513514 |
| 75 | 5625 | 421875 | 8.6602540 | 4.2171633 | .013333333 |
| 76 | 5776 | 438976 | 8.7177979 | 4.2358236 | .013157895 |
| 77 | 5929 | 456533 | 8.7749644 | 4.2543210 | .012987013 |
| 78 | 6084 | 474552 | 8.8317609 | 4.2726586 | .012820513 |
| 79 | 6241 | 493039 | 8.8881944 | 4.2908404 | .012658228 |
| 80 | 6400 | 512000 | 8.9442719 | 4.3088695 | .012500000 |
| 81 | 6561 | 531441 | 9.0000000 | 4.3267487 | .012345679 |
| 82 | 6724 | 551368 | 9.0553851 | 4.3444815 | .012195122 |
| 83 | 6889 | 571787 | 9.1104336 | 4.3620707 | .012048193 |
| 84 | 7056 | 592704 | 9.1651514 | 4.3795191 | .011904762 |
| 85 | 7225 | 614125 | 9.2195445 | 4.3968296 | .011764706 |
| 86 | 7396 | 636056 | 9.2736185 | 4.4140049 | .011627907 |
| 87 | 7569 | 658503 | 9.3273791 | 4.4310476 | .011494253 |
| 88 | 7744 | 681472 | 9.3808315 | 4.4479603 | .011363636 |
| 89 | 7921 | 704969 | 9.4339811 | 4.4647451 | .011235955 |
| 90 | 8100 | 729000 | 9.4868330 | 4.4814047 | .011111111 |
| 91 | 8281 | 753571 | 9.5393920 | 4.4979414 | .010989011 |
| 92 | 8464 | 778688 | 9.5916630 | 4.5143574 | .010869565 |
| 93 | 8649 | 804357 | 9.6436508 | 4.5306549 | .010752688 |
| 94 | 8836 | 830584 | 9.6953597 | 4.5468359 | .010638298 |
| 95 | 9025 | 857375 | 9.7467943 | 4.5629026 | .010526316 |
| 96 | 9216 | 884736 | 9.7979590 | 4.5788570 | .010416667 |
| 97 | 9409 | 912673 | 9.8488578 | 4.5947009 | .010309278 |
| 98 | 9604 | 941192 | 9.8994949 | 4.6104363 | .010204082 |
| 99 | 9801 | 970299 | 9.9498744 | 4.6260650 | .010101010 |
| 100 | 10000 | 1000000 | 10.0000000 | 4.6415888 | .010000000 |
| 101 | 10201 | 1030301 | 10.0498756 | 4.6570095 | .009900990 |
| 102 | 10404 | 1061208 | 10.0995049 | 4.6723287 | .009803922 |
| 103 | 10609 | 1092727 | 10.1488916 | 4.6875482 | .009708738 |
| 104 | 10816 | 1124864 | 10.1980390 | 4.7026694 | .009615385 |
| 105 | 11025 | 1157625 | 10.2469508 | 4.7176940 | .009523810 |
| 106 | 11236 | 1191016 | 10.2956301 | 4.7326235 | .009433962 |
| 107 | 11449 | 1225043 | 10.3440804 | 4.7474594 | .009345794 |
| 108 | 11664 | 1259712 | 10.3923048 | 4.7622032 | .009259259 |
| 109 | 11881 | 1295029 | 10.4403065 | 4.7768562 | .009174312 |
| 110 | 12100 | 1331000 | 10.4880885 | 4.7914199 | .009090909 |
| 111 | 12321 | 1367631 | 10.5356538 | 4.8058955 | .009009009 |
| 112 | 12544 | 1404928 | 10.5830052 | 4.8202845 | .008928571 |
| 113 | 12769 | 1442897 | 10.6301458 | 4.8345881 | .008849558 |
| 114 | 12996 | 1481544 | 10.6770783 | 4.8488076 | .008771930 |
| 115 | 13225 | 1520875 | 10.7238053 | 4.8629442 | .008695652 |
| 116 | 13456 | 1560896 | 10.7703296 | 4.8769990 | .008620690 |
| 117 | 13689 | 1601613 | 10.8166538 | 4.8909732 | .008547009 |
| 118 | 13924 | 1643032 | 10.8627805 | 4.9048681 | .008474576 |
| 119 | 14161 | 1685159 | 10.9087121 | 4.9186847 | .008403361 |
| 120 | 14400 | 1728000 | 10.9544512 | 4.9324242 | .008333333 |
| 121 | 14641 | 1771561 | 11.00 .0000 | 4.9460874 | .008264463 |
| 122 | 14884 | 1815848 | 11.0453610 | 4.9596757 | .008196721 |
| 123 | 15129 | 1860867 | 11.0905365 | 4.9731898 | .008130081 |
| 124 | 15376 | 1906624 | 11.1355287 | 4.9866310 | .008064516 |

## TABLE V.—SQUARES, CUBES, SQUARE ROOTS,

| No. | Squares. | Cubes. | Square Roots. | Cube Roots. | Reciprocals. |
|---|---|---|---|---|---|
| 125 | 15625 | 1953125 | 11.1803399 | 5.0000000 | .008000000 |
| 126 | 15876 | 2000376 | 11.2249722 | 5.0132979 | .007936508 |
| 127 | 16129 | 2048383 | 11.2694277 | 5.0265257 | .007874016 |
| 128 | 16384 | 2097152 | 11.3137085 | 5.0396842 | .007812500 |
| 129 | 16641 | 2146689 | 11.3578167 | 5.0527743 | .007751938 |
| 130 | 16900 | 2197000 | 11.4017543 | 5.0657970 | .007692308 |
| 131 | 17161 | 2248091 | 11.4455231 | 5.0787531 | .007633588 |
| 132 | 17424 | 2299968 | 11.4891253 | 5.0916434 | .007575758 |
| 133 | 17689 | 2352637 | 11.5325626 | 5.1044687 | .007518797 |
| 134 | 17956 | 2406104 | 11.5758369 | 5.1172299 | .007462687 |
| 135 | 18225 | 2460375 | 11.6189500 | 5.1299278 | .007407407 |
| 136 | 18496 | 2515456 | 11.6619038 | 5.1425632 | .007352941 |
| 137 | 18769 | 2571353 | 11.7046999 | 5.1551367 | .007299270 |
| 138 | 19044 | 2628072 | 11.7473401 | 5.1676493 | .007246377 |
| 139 | 19321 | 2685619 | 11.7898261 | 5.1801015 | .007194245 |
| 140 | 19600 | 2744000 | 11.8321596 | 5.1924941 | .007142857 |
| 141 | 19881 | 2803221 | 11.8743421 | 5.2048279 | .007092199 |
| 142 | 20164 | 2863288 | 11.9163753 | 5.2171034 | .007042254 |
| 143 | 20449 | 2924207 | 11.9582607 | 5.2293215 | .006993007 |
| 144 | 20736 | 2985984 | 12.0000000 | 5.2414828 | .006944444 |
| 145 | 21025 | 3048625 | 12.0415946 | 5.2535879 | .006896552 |
| 146 | 21316 | 3112136 | 12.0830460 | 5.2656374 | .006849315 |
| 147 | 21609 | 3176523 | 12.1243557 | 5.2776321 | .006802721 |
| 148 | 21904 | 3241792 | 12.1655251 | 5.2895725 | .006756757 |
| 149 | 22201 | 3307949 | 12.2065556 | 5.3014593 | .006711409 |
| 150 | 22500 | 3375000 | 12.2474487 | 5.3132928 | .006666667 |
| 151 | 22801 | 3442951 | 12.2882057 | 5.3250740 | .006622517 |
| 152 | 23104 | 3511808 | 12.3288280 | 5.3368033 | .006578947 |
| 153 | 23409 | 3581577 | 12.3693169 | 5.3484812 | .006535948 |
| 154 | 23716 | 3652264 | 12.4096736 | 5.3601084 | .006493506 |
| 155 | 24025 | 3723875 | 12.4498996 | 5.3716854 | .006451613 |
| 156 | 24336 | 3796416 | 12.4899960 | 5.3832126 | .006410256 |
| 157 | 24649 | 3869893 | 12.5299641 | 5.3946907 | .006369427 |
| 158 | 24964 | 3944312 | 12.5698051 | 5.4061203 | .006329114 |
| 159 | 25281 | 4019679 | 12.6095202 | 5.4175015 | .006289308 |
| 160 | 25600 | 4096000 | 12.6491106 | 5.4288352 | .006250000 |
| 161 | 25921 | 4173281 | 12.6885775 | 5.4401218 | .006211180 |
| 162 | 26244 | 4251528 | 12.7279221 | 5.4513618 | .006172840 |
| 163 | 26569 | 4330747 | 12.7671453 | 5.4625556 | .006134969 |
| 164 | 26896 | 4410944 | 12.8062485 | 5.4737037 | .006097561 |
| 165 | 27225 | 4492125 | 12.8452326 | 5.4848066 | .006060606 |
| 166 | 27556 | 4574296 | 12.8840987 | 5.4958647 | .006024096 |
| 167 | 27889 | 4657463 | 12.9228480 | 5.5068784 | .005988024 |
| 168 | 28224 | 4741632 | 12.9614814 | 5.5178484 | .005952381 |
| 169 | 28561 | 4826809 | 13.0000000 | 5.5287748 | .005917160 |
| 170 | 28900 | 4913000 | 13.0384048 | 5.5396583 | .005882353 |
| 171 | 29241 | 5000211 | 13.0766968 | 5.5504991 | .005847953 |
| 172 | 29584 | 5088448 | 13.1148770 | 5.5612978 | .005813953 |
| 173 | 29929 | 5177717 | 13.1529464 | 5.5720546 | .005780347 |
| 174 | 30276 | 5268024 | 13.1909060 | 5.5827702 | .005747126 |
| 175 | 30625 | 5359375 | 13.2287566 | 5.5934447 | .005714286 |
| 176 | 30976 | 5451776 | 13.2664992 | 5.6040787 | .005681818 |
| 177 | 31329 | 5545233 | 13.3041347 | 5.6146724 | .005649718 |
| 178 | 31684 | 5639752 | 13.3416641 | 5.6252263 | .005617978 |
| 179 | 32041 | 5735339 | 13.3790882 | 5.6357408 | .005586592 |
| 180 | 32400 | 5832000 | 13.4164079 | 5.6462162 | .005555556 |
| 181 | 32761 | 5929741 | 13.4536240 | 5.6566528 | .005524862 |
| 182 | 33124 | 6028568 | 13.4907376 | 5.6670511 | .005494505 |
| 183 | 33489 | 6128487 | 13.5277493 | 5.6774114 | .005464481 |
| 184 | 33856 | 6229504 | 13.5646600 | 5.6877340 | .005434783 |
| 185 | 34225 | 6331625 | 13.6014705 | 5.6980192 | .005405405 |
| 186 | 34596 | 6434856 | 13.6381817 | 5.7082675 | .005376344 |

| No. | Squares. | Cubes. | Square Roots. | Cube Roots. | Reciprocals. |
|---|---|---|---|---|---|
| 187 | 34969 | 6539203 | 13.6747943 | 5.7184791 | .005847594 |
| 188 | 35344 | 6644672 | 13.7113092 | 5.7286543 | .005319149 |
| 189 | 35721 | 6751269 | 13.7477271 | 5.7387936 | .005291005 |
| 190 | 36100 | 6859000 | 13.7840488 | 5.7488971 | .005263158 |
| 191 | 36481 | 6967871 | 13.8202750 | 5.7589652 | .005235602 |
| 192 | 36864 | 7077888 | 13.8564065 | 5.7689982 | .005208333 |
| 193 | 37249 | 7189057 | 13.8924440 | 5.7789966 | .005181347 |
| 194 | 37636 | 7301384 | 13.9283883 | 5.7889604 | .005154639 |
| 195 | 38025 | 7414875 | 13.9642400 | 5.7988900 | .005128205 |
| 196 | 38416 | 7529536 | 14.0000000 | 5.8087857 | .005102041 |
| 197 | 38809 | 7645373 | 14.0356688 | 5.8186479 | .005076142 |
| 198 | 39204 | 7762392 | 14.0712473 | 5.8284767 | .005050505 |
| 199 | 39601 | 7880599 | 14.1067360 | 5.8382725 | .005025126 |
| 200 | 40000 | 8000000 | 14.1421356 | 5.8480355 | .005000000 |
| 201 | 40401 | 8120601 | 14.1774469 | 5.8577660 | .004975124 |
| 202 | 40804 | 8242408 | 14.2126704 | 5.8674643 | .004950495 |
| 203 | 41209 | 8365427 | 14.2478068 | 5.8771307 | .004926108 |
| 204 | 41616 | 8489664 | 14.2828569 | 5.8867653 | .004901961 |
| 205 | 42025 | 8615125 | 14.3178211 | 5.8963685 | .004878049 |
| 206 | 42436 | 8741816 | 14.3527001 | 5.9059406 | .004854369 |
| 207 | 42849 | 8869743 | 14.3874946 | 5.9154817 | .004830918 |
| 208 | 43264 | 8998912 | 14.4222051 | 5.9249921 | .004807692 |
| 209 | 43681 | 9129329 | 14.4568323 | 5.9344721 | .004784689 |
| 210 | 44100 | 9261000 | 14.4913767 | 5.9439220 | .004761905 |
| 211 | 44521 | 9393931 | 14.5258390 | 5.9533418 | .004739336 |
| 212 | 44944 | 9528128 | 14.5602198 | 5.9627320 | .004716981 |
| 213 | 45369 | 9663597 | 14.5945195 | 5.9720926 | .004694836 |
| 214 | 45796 | 9800344 | 14.6287388 | 5.9814240 | .004672897 |
| 215 | 46225 | 9938375 | 14.6628783 | 5.9907264 | .004651163 |
| 216 | 46656 | 10077696 | 14.6969385 | 6.0000000 | .004629630 |
| 217 | 47089 | 10218313 | 14.7309199 | 6.0092450 | .004608295 |
| 218 | 47524 | 10360232 | 14.7648231 | 6.0184617 | .004587156 |
| 219 | 47961 | 10503459 | 14.7986486 | 6.0276502 | .004566210 |
| 220 | 48400 | 10648000 | 14.8323970 | 6.0368107 | .004545455 |
| 221 | 48841 | 10793861 | 14.8660687 | 6.0459435 | .004524887 |
| 222 | 49284 | 10941048 | 14.8996644 | 6.0550489 | .004504505 |
| 223 | 49729 | 11089567 | 14.9331845 | 6.0641270 | .004484305 |
| 224 | 50176 | 11239424 | 14.9666295 | 6.0731779 | .004464286 |
| 225 | 50625 | 11390625 | 15.0000000 | 6.0822020 | .004444444 |
| 226 | 51076 | 11543176 | 15.0332964 | 6.0911994 | .004424779 |
| 227 | 51529 | 11697083 | 15.0665192 | 6.1001702 | .004405286 |
| 228 | 51984 | 11852352 | 15.0996689 | 6.1091147 | .004385965 |
| 229 | 52441 | 12008989 | 15.1327460 | 6.1180332 | .004366812 |
| 230 | 52900 | 12167000 | 15.1657509 | 6.1269257 | .004347826 |
| 231 | 53361 | 12326391 | 15.1986842 | 6.1357924 | .004329004 |
| 232 | 53824 | 12487168 | 15.2315462 | 6.1446337 | .004310345 |
| 233 | 54289 | 12649337 | 15.2643375 | 6.1534495 | .004291845 |
| 234 | 54756 | 12812904 | 15.2970585 | 6.1622401 | .004273504 |
| 235 | 55225 | 12977875 | 15.3297097 | 6.1710058 | .004255319 |
| 236 | 55696 | 13144256 | 15.3622915 | 6.1797466 | .004237288 |
| 237 | 56169 | 13312053 | 15.3948043 | 6.1884628 | .004219409 |
| 238 | 56644 | 13481272 | 15.4272486 | 6.1971544 | .004201681 |
| 239 | 57121 | 13651919 | 15.4596248 | 6.2058218 | .004184100 |
| 240 | 57600 | 13824000 | 15.4919334 | 6.2144650 | .004166667 |
| 241 | 58081 | 13997521 | 15.5241747 | 6.2230843 | .004149378 |
| 242 | 58564 | 14172488 | 15.5563492 | 6.2316797 | .004132231 |
| 243 | 59049 | 14348907 | 15.5884573 | 6.2402515 | .004115226 |
| 244 | 59536 | 14526784 | 15.6204994 | 6.2487998 | .004098361 |
| 245 | 60025 | 14706125 | 15.6524758 | 6.2573248 | .004081633 |
| 246 | 60516 | 14886936 | 15.6843871 | 6.2658266 | .004065041 |
| 247 | 61009 | 15069223 | 15.7162336 | 6.2743054 | .004048583 |
| 248 | 61504 | 15252992 | 15.7480157 | 6.2827618 | .004032258 |

## TABLE V.—SQUARES, CUBES, SQUARE ROOTS,

| No. | Squares. | Cubes. | Square Roots. | Cube Roots. | Reciprocals. |
|---|---|---|---|---|---|
| 249 | 62001 | 15438249 | 15.7797338 | 6.2911946 | .004016064 |
| 250 | 62500 | 15625000 | 15.8113883 | 6.2996053 | .004000000 |
| 251 | 63001 | 15813251 | 15.8429795 | 6.3079935 | .003984064 |
| 252 | 63504 | 16003008 | 15.8745079 | 6.3163596 | .003968254 |
| 253 | 64009 | 16194277 | 15.9059737 | 6.3247035 | .003952569 |
| 254 | 64516 | 16387064 | 15.9373775 | 6.3330256 | .003937008 |
| 255 | 65025 | 16581375 | 15.9687194 | 6.3413257 | .003921569 |
| 256 | 65536 | 16777216 | 16.0000000 | 6.3496042 | .003906250 |
| 257 | 66049 | 16974593 | 16.0312195 | 6.3578611 | .003891051 |
| 258 | 66564 | 17173512 | 16.0623784 | 6.3660968 | .003875969 |
| 259 | 67081 | 17373979 | 16.0934769 | 6.3743111 | .003861004 |
| 260 | 67600 | 17576000 | 16.1245155 | 6.3825043 | .003846154 |
| 261 | 68121 | 17779581 | 16.1554944 | 6.3906765 | .003831418 |
| 262 | 68644 | 17984728 | 16.1864141 | 6.3988279 | .003816794 |
| 263 | 69169 | 18191447 | 16.2172747 | 6.4069585 | .003802281 |
| 264 | 69696 | 18399744 | 16.2480768 | 6.4150687 | .003787879 |
| 265 | 70225 | 18609625 | 16.2788206 | 6.4231583 | .003773585 |
| 266 | 70756 | 18821096 | 16.3095064 | 6.4312276 | .003759398 |
| 267 | 71289 | 19034163 | 16.3401346 | 6.4392767 | .003745318 |
| 268 | 71824 | 19248832 | 16.3707055 | 6.4473057 | .003731343 |
| 269 | 72361 | 19465109 | 16.4012195 | 6.4553148 | .003717472 |
| 270 | 72900 | 19683000 | 16.4316767 | 6.4633041 | .003703704 |
| 271 | 73441 | 19902511 | 16.4620776 | 6.4712736 | .003690037 |
| 272 | 73984 | 20123648 | 16.4924225 | 6.4792236 | .003676471 |
| 273 | 74529 | 20346417 | 16.5227116 | 6.4871541 | .003663004 |
| 274 | 75076 | 20570824 | 16.5529454 | 6.4950653 | .003649635 |
| 275 | 75625 | 20796875 | 16.5831240 | 6.5029572 | .003636364 |
| 276 | 76176 | 21024576 | 16.6132477 | 6.5108300 | .003623188 |
| 277 | 76729 | 21253933 | 16.6433170 | 6.5186839 | .003610108 |
| 278 | 77284 | 21484952 | 16.6733320 | 6.5265189 | .003597122 |
| 279 | 77841 | 21717639 | 16.7032931 | 6.5343351 | .003584229 |
| 280 | 78400 | 21952000 | 16.7332005 | 6.5421326 | .003571429 |
| 281 | 78961 | 22188041 | 16.7630546 | 6.5499116 | .003558719 |
| 282 | 79524 | 22425768 | 16.7928556 | 6.5576722 | .003546099 |
| 283 | 80089 | 22665187 | 16.8226038 | 6.5654144 | .003533569 |
| 284 | 80656 | 22906304 | 16.8522995 | 6.5731385 | .003521127 |
| 285 | 81225 | 23149125 | 16.8819430 | 6.5808443 | .003508772 |
| 286 | 81796 | 23393656 | 16.9115345 | 6.5885323 | .003496503 |
| 287 | 82369 | 23639903 | 16.9410743 | 6.5962023 | .003484321 |
| 288 | 82944 | 23887872 | 16.9705627 | 6.6038545 | .003472222 |
| 289 | 83521 | 24137569 | 17.0000000 | 6.6114890 | .003460208 |
| 290 | 84100 | 24389000 | 17.0293864 | 6.6191060 | .003448276 |
| 291 | 84681 | 24642171 | 17.0587221 | 6.6267054 | .003436426 |
| 292 | 85264 | 24897088 | 17.0880075 | 6.6342874 | .003424658 |
| 293 | 85849 | 25153757 | 17.1172428 | 6.6418522 | .003412969 |
| 294 | 86436 | 25412184 | 17.1464282 | 6.6493998 | .003401361 |
| 295 | 87025 | 25672375 | 17.1755640 | 6.6569302 | .003389831 |
| 296 | 87616 | 25934336 | 17.2046505 | 6.6644437 | .003378378 |
| 297 | 88209 | 26198073 | 17.2336879 | 6.6719403 | .003367003 |
| 298 | 88804 | 26463592 | 17.2626765 | 6.6794200 | .003355705 |
| 299 | 89401 | 26730899 | 17.2916165 | 6.6868831 | .003344482 |
| 300 | 90000 | 27000000 | 17.3205081 | 6.6943295 | .003333333 |
| 301 | 90601 | 27270901 | 17.3493516 | 6.7017593 | .003322259 |
| 302 | 91204 | 27543608 | 17.3781472 | 6.7091729 | .003311258 |
| 303 | 91809 | 27818127 | 17.4068952 | 6.7165700 | .003300330 |
| 304 | 92416 | 28094464 | 17.4355958 | 6.7239508 | .003289474 |
| 305 | 93025 | 28372625 | 17.4642492 | 6.7313155 | .003278689 |
| 306 | 93636 | 28652616 | 17.4928557 | 6.7386641 | .003267974 |
| 307 | 94249 | 28934443 | 17.5214155 | 6.7459967 | .003257329 |
| 308 | 94864 | 29218112 | 17.5499288 | 6.7533134 | .003246753 |
| 309 | 95481 | 29503629 | 17.5783958 | 6.7606143 | .003236246 |
| 310 | 96100 | 29791000 | 17.6068169 | 6.7678995 | .003225806 |

| No. | Squares. | Cubes. | Square Roots. | Cube Roots. | Reciprocals. |
|---|---|---|---|---|---|
| 311 | 96721 | 30080231 | 17.6351921 | 6.7751690 | .003215434 |
| 312 | 97344 | 30371328 | 17.6635217 | 6.7824229 | .003205128 |
| 313 | 97969 | 30664297 | 17.6918060 | 6.7896613 | .003194888 |
| 314 | 98596 | 30959144 | 17.7200451 | 6.7968844 | .003184713 |
| 315 | 99225 | 31255875 | 17.7482393 | 6.8040921 | .003174603 |
| 316 | 99856 | 31554496 | 17.7763888 | 6.8112847 | .003164557 |
| 317 | 100489 | 31855013 | 17.8044938 | 6.8184620 | .003154574 |
| 318 | 101124 | 32157432 | 17.8325545 | 6.8256242 | .003144654 |
| 319 | 101761 | 32461759 | 17.8605711 | 6.8327714 | .003134796 |
| 320 | 102400 | 32768000 | 17.8885438 | 6.8399037 | .003125000 |
| 321 | 103041 | 33076161 | 17.9164729 | 6.8470213 | .003115265 |
| 322 | 103684 | 33386248 | 17.9443584 | 6.8541240 | .003105590 |
| 323 | 104329 | 33698267 | 17.9722008 | 6.8612120 | .003095975 |
| 324 | 104976 | 34012224 | 18.0000000 | 6.8682855 | .003086420 |
| 325 | 105625 | 34328125 | 18.0277564 | 6.8753443 | .003076923 |
| 326 | 106276 | 34645976 | 18.0554701 | 6.8823888 | .003067485 |
| 327 | 106929 | 34965783 | 18.0831413 | 6.8894188 | .003058104 |
| 328 | 107584 | 35287552 | 18.1107703 | 6.8964345 | .003048780 |
| 329 | 108241 | 35611289 | 18.1383571 | 6.9034359 | .003039514 |
| 330 | 108900 | 35937000 | 18.1659021 | 6.9104232 | .003030303 |
| 331 | 109561 | 36264691 | 18.1934054 | 6.9173964 | .003021148 |
| 332 | 110224 | 36594368 | 18.2208672 | 6.9243556 | .003012048 |
| 333 | 110889 | 36926037 | 18.2482876 | 6.9313008 | .003003003 |
| 334 | 111556 | 37259704 | 18.2756669 | 6.9382321 | .002994012 |
| 335 | 112225 | 37595375 | 18.3030052 | 6.9451496 | .002985075 |
| 336 | 112896 | 37933056 | 18.3303028 | 6.9520533 | .002976190 |
| 337 | 113569 | 38272753 | 18.3575598 | 6.9589434 | .002967359 |
| 338 | 114244 | 38614472 | 18.3847763 | 6.9658198 | .002958580 |
| 339 | 114921 | 38958219 | 18.4119526 | 6.9726826 | .002949853 |
| 340 | 115600 | 39304000 | 18.4390889 | 6.9795321 | .002941176 |
| 341 | 116281 | 39651821 | 18.4661853 | 6.9863681 | .002932551 |
| 342 | 116964 | 40001688 | 18.4932420 | 6.9931906 | .002923977 |
| 343 | 117649 | 40353607 | 18.5202592 | 7.0000000 | .002915452 |
| 344 | 118336 | 40707584 | 18.5472370 | 7.0067962 | .002906977 |
| 345 | 119025 | 41063625 | 18.5741756 | 7.0135791 | .002898551 |
| 346 | 119716 | 41421736 | 18.6010752 | 7.0203490 | .002890173 |
| 347 | 120409 | 41781923 | 18.6279360 | 7.0271058 | .002881844 |
| 348 | 121104 | 42144192 | 18.6547581 | 7.0338497 | .002873563 |
| 349 | 121801 | 42508549 | 18.6815417 | 7.0405806 | .002865330 |
| 350 | 122500 | 42875000 | 18.7082869 | 7.0472987 | .002857143 |
| 351 | 123201 | 43243551 | 18.7349940 | 7.0540041 | .002849003 |
| 352 | 123904 | 43614208 | 18.7616630 | 7.0606967 | .002840909 |
| 353 | 124609 | 43986977 | 18.7882942 | 7.0673767 | .002832861 |
| 354 | 125316 | 44361864 | 18.8148877 | 7.0740440 | .002824859 |
| 355 | 126025 | 44738875 | 18.8414437 | 7.0806988 | .002816901 |
| 356 | 126736 | 45118016 | 18.8679623 | 7.0873411 | .002808989 |
| 357 | 127449 | 45499293 | 18.8944436 | 7.0939709 | .002801120 |
| 358 | 128164 | 45882712 | 18.9208879 | 7.1005885 | .002793296 |
| 359 | 128881 | 46268279 | 18.9472953 | 7.1071937 | .002785515 |
| 360 | 129600 | 46656000 | 18.9736660 | 7.1137866 | .002777778 |
| 361 | 130321 | 47045881 | 19.0000000 | 7.1203674 | .002770083 |
| 362 | 131044 | 47437928 | 19.0262976 | 7.1269360 | .002762431 |
| 363 | 131769 | 47832147 | 19.0525589 | 7.1334925 | .002754821 |
| 364 | 132496 | 48228544 | 19.0787840 | 7.1400370 | .002747253 |
| 365 | 133225 | 48627125 | 19.1049732 | 7.1465695 | .002739726 |
| 366 | 133956 | 49027896 | 19.1311265 | 7.1530901 | .002732240 |
| 367 | 134689 | 49430863 | 19.1572441 | 7.1595988 | .002724796 |
| 368 | 135424 | 49836032 | 19.1833261 | 7.1660957 | .002717391 |
| 369 | 136161 | 50243409 | 19.2093727 | 7.1725809 | .002710027 |
| 370 | 136900 | 50653000 | 19.2353841 | 7.1790544 | .002702708 |
| 371 | 137641 | 51064811 | 19.2613603 | 7.1855162 | .002695418 |
| 372 | 138384 | 51478848 | 19.2873015 | 7.1919663 | .002688172 |

## TABLE V.—SQUARES, CUBES, SQUARE ROOTS,

| No. | Squares. | Cubes. | Square Roots. | Cube Roots. | Reciprocals. |
|---|---|---|---|---|---|
| 373 | 139129 | 51895117 | 19.3132079 | 7.1984050 | .002680965 |
| 374 | 139876 | 52313624 | 19.3390796 | 7.2048322 | .002673797 |
| 375 | 140625 | 52734375 | 19.3649167 | 7.2112479 | .002666667 |
| 376 | 141376 | 53157376 | 19.3907194 | 7.2176522 | .002659574 |
| 377 | 142129 | 53582633 | 19.4164878 | 7.2240450 | .002652520 |
| 378 | 142884 | 54010152 | 19.4422221 | 7.2304268 | .002645503 |
| 379 | 143641 | 54439939 | 19.4679223 | 7.2367972 | .002638522 |
| 380 | 144400 | 54872000 | 19.4935887 | 7.2431565 | .002631579 |
| 381 | 145161 | 55306341 | 19.5192213 | 7.2495045 | .002624672 |
| 382 | 145924 | 55742968 | 19.5448203 | 7.2558415 | .002617801 |
| 383 | 146689 | 56181887 | 19.5703858 | 7.2621675 | .002610966 |
| 384 | 147456 | 56623104 | 19.5959179 | 7.2684824 | .002604167 |
| 385 | 148225 | 57066625 | 19.6214169 | 7.2747864 | .002597403 |
| 386 | 148996 | 57512456 | 19.6468827 | 7.2810794 | .002590674 |
| 387 | 149769 | 57960603 | 19.6723156 | 7.2873617 | .002583979 |
| 388 | 150544 | 58411072 | 19.6977156 | 7.2936330 | .002577320 |
| 389 | 151321 | 58863869 | 19.7230829 | 7.2998936 | .002570694 |
| 390 | 152100 | 59319000 | 19.7484177 | 7.3061436 | .002564103 |
| 391 | 152881 | 59776471 | 19.7737199 | 7.3123828 | .002557545 |
| 392 | 153664 | 60236288 | 19.7989899 | 7.3186114 | .002551020 |
| 393 | 154449 | 60698457 | 19.8242276 | 7.3248295 | .002544529 |
| 394 | 155236 | 61162984 | 19.8494332 | 7.3310369 | .002538071 |
| 395 | 156025 | 61629875 | 19.8746069 | 7.3372339 | .002531646 |
| 396 | 156816 | 62099136 | 19.8997487 | 7.3434205 | .002525253 |
| 397 | 157609 | 62570773 | 19.9248588 | 7.3495966 | .002518892 |
| 398 | 158404 | 63044792 | 19.9499373 | 7.3557624 | .002512563 |
| 399 | 159201 | 63521199 | 19.9749844 | 7.3619178 | .002506266 |
| 400 | 160000 | 64000000 | 20.0000000 | 7.3680630 | .002500000 |
| 401 | 160801 | 64481201 | 20.0249844 | 7.3741979 | .002493766 |
| 402 | 161604 | 64964808 | 20.0499377 | 7.3803227 | .002487562 |
| 403 | 162409 | 65450827 | 20.0748599 | 7.3864373 | .002481390 |
| 404 | 163216 | 65939264 | 20.0997512 | 7.3925418 | .002475248 |
| 405 | 164025 | 66430125 | 20.1246118 | 7.3986363 | .002469136 |
| 406 | 164836 | 66923416 | 20.1494417 | 7.4047206 | .002463054 |
| 407 | 165649 | 67419143 | 20.1742410 | 7.4107950 | .002457002 |
| 408 | 166464 | 67917312 | 20.1990099 | 7.4168595 | .002450980 |
| 409 | 167281 | 68417929 | 20.2237484 | 7.4229142 | .002444988 |
| 410 | 168100 | 68921000 | 20.2484567 | 7.4289589 | .002439024 |
| 411 | 168921 | 69426531 | 20.2731349 | 7.4349938 | .002433090 |
| 412 | 169744 | 69934528 | 20.2977831 | 7.4410189 | .002427184 |
| 413 | 170569 | 70444997 | 20.3224014 | 7.4470342 | .002421308 |
| 414 | 171396 | 70957944 | 20.3469899 | 7.4530399 | .002415459 |
| 415 | 172225 | 71473375 | 20.3715488 | 7.4590359 | .002409639 |
| 416 | 173056 | 71991296 | 20.3960781 | 7.4650223 | .002403846 |
| 417 | 173889 | 72511713 | 20.4205779 | 7.4709991 | .002398082 |
| 418 | 174724 | 73034632 | 20.4450483 | 7.4769664 | .002392344 |
| 419 | 175561 | 73560059 | 20.4694895 | 7.4829242 | .002386635 |
| 420 | 176400 | 74088000 | 20.4939015 | 7.4888724 | .002380952 |
| 421 | 177241 | 74618461 | 20.5182845 | 7.4948113 | .002375297 |
| 422 | 178084 | 75151448 | 20.5426386 | 7.5007406 | .002369668 |
| 423 | 178929 | 75686967 | 20.5669638 | 7.5066607 | .002364066 |
| 424 | 179776 | 76225024 | 20 5912603 | 7.5125715 | .002358491 |
| 425 | 180625 | 76765625 | 20.6155281 | 7.5184730 | .002352941 |
| 426 | 181476 | 77308776 | 20.6397674 | 7.5243652 | .002347418 |
| 427 | 182329 | 77854483 | 20.6639783 | 7.5302482 | .002341920 |
| 428 | 183184 | 78402752 | 20.6881609 | 7.5361221 | .002336449 |
| 429 | 184041 | 78953589 | 20.7123152 | 7.5419867 | .002331002 |
| 430 | 184900 | 79507000 | 20.7364414 | 7.5478423 | .002325581 |
| 431 | 185761 | 80062991 | 20.7605395 | 7.5536888 | .002320186 |
| 432 | 186624 | 80621568 | 20.7846097 | 7.5595263 | .002314815 |
| 433 | 187489 | 81182737 | 20.8086520 | 7.5653548 | .002309469 |
| 434 | 188356 | 81746504 | 20.8326667 | 7.5711743 | .002304147 |

| No. | Squares. | Cubes. | Square Roots. | Cube Roots. | Reciprocals. |
|---|---|---|---|---|---|
| 435 | 189225 | 82312875 | 20.8566536 | 7.5769849 | .002298851 |
| 436 | 190096 | 82881856 | 20.8806130 | 7.5827865 | .002293578 |
| 437 | 190969 | 83453453 | 20.9045450 | 7.5885793 | .002288330 |
| 438 | 191844 | 84027672 | 20.9284495 | 7.5943633 | .002283105 |
| 439 | 192721 | 84604519 | 20.9523268 | 7.6001385 | .002277904 |
| 440 | 193600 | 85184000 | 20.9761770 | 7.6059049 | .002272727 |
| 441 | 194481 | 85766121 | 21.0000000 | 7.6116626 | .002267574 |
| 442 | 195364 | 86350888 | 21.0237960 | 7.6174116 | .002262443 |
| 443 | 196249 | 86938307 | 21.0475652 | 7.6231519 | .002257336 |
| 444 | 197136 | 87528384 | 21.0713075 | 7.6288837 | .002252252 |
| 445 | 198025 | 88121125 | 21.0950231 | 7.6346067 | .002247191 |
| 446 | 198916 | 88716536 | 21.1187121 | 7.6403213 | .002242152 |
| 447 | 199809 | 89314623 | 21.1423745 | 7.6460272 | .002237136 |
| 448 | 200704 | 89915392 | 21.1660105 | 7.6517247 | .002232143 |
| 449 | 201601 | 90518849 | 21.1896201 | 7.6574138 | .002227171 |
| 450 | 202500 | 91125000 | 21.2132034 | 7.6630943 | .002222222 |
| 451 | 203401 | 91733851 | 21.2367606 | 7.6687665 | .002217295 |
| 452 | 204304 | 92345408 | 21.2602916 | 7.6744303 | .002212389 |
| 453 | 205209 | 92959677 | 21.2837967 | 7.6800857 | .002207506 |
| 454 | 206116 | 93576664 | 21.3072758 | 7.6857328 | .002202643 |
| 455 | 207025 | 94196375 | 21.3307290 | 7.6913717 | .002197802 |
| 456 | 207936 | 94818816 | 21.3541565 | 7.6970023 | .002192982 |
| 457 | 208849 | 95443993 | 21.3775583 | 7.7026246 | .002188184 |
| 458 | 209764 | 96071912 | 21.4009346 | 7.7082388 | .002183406 |
| 459 | 210681 | 96702579 | 21.4242853 | 7.7138448 | .002178649 |
| 460 | 211600 | 97336000 | 21.4476106 | 7.7194426 | .002173913 |
| 461 | 212521 | 97972181 | 21.4709106 | 7.7250325 | .002169197 |
| 462 | 213444 | 98611128 | 21.4941853 | 7.7306141 | .002164502 |
| 463 | 214369 | 99252847 | 21.5174348 | 7.7361877 | .002159827 |
| 464 | 215296 | 99897344 | 21.5406592 | 7.7417532 | .002155172 |
| 465 | 216225 | 100544625 | 21.5638587 | 7.7473109 | .002150538 |
| 466 | 217156 | 101194696 | 21.5870331 | 7.7528606 | .002145923 |
| 467 | 218089 | 101847563 | 21.6101828 | 7.7584023 | .002141328 |
| 468 | 219024 | 102503232 | 21.6333077 | 7.7639361 | .002136752 |
| 469 | 219961 | 103161709 | 21.6564078 | 7.7694620 | .002132196 |
| 470 | 220900 | 103823000 | 21.6794834 | 7.7749801 | .002127660 |
| 471 | 221841 | 104487111 | 21.7025344 | 7.7804904 | .002123142 |
| 472 | 222784 | 105154048 | 21.7255610 | 7.7859928 | .002118644 |
| 473 | 223729 | 105823817 | 21.7485632 | 7.7914875 | .002114165 |
| 474 | 224676 | 106496424 | 21.7715411 | 7.7969745 | .002109705 |
| 475 | 225625 | 107171875 | 21.7944947 | 7.8024538 | .002105263 |
| 476 | 226576 | 107850176 | 21.8174242 | 7.8079254 | .002100840 |
| 477 | 227529 | 108531333 | 21.8403297 | 7.8133892 | .002096436 |
| 478 | 228484 | 109215352 | 21.8632111 | 7.8188456 | .002092050 |
| 479 | 229441 | 109902239 | 21.8860686 | 7.8242942 | .002087683 |
| 480 | 230400 | 110592000 | 21.9089023 | 7.8297353 | .002083333 |
| 481 | 231361 | 111284641 | 21.9317122 | 7.8351688 | .002079002 |
| 482 | 232324 | 111980168 | 21.9544984 | 7.8405949 | .002074689 |
| 483 | 233289 | 112678587 | 21.9772610 | 7.8460134 | .002070393 |
| 484 | 234256 | 113379904 | 22.0000000 | 7.8514244 | .002066116 |
| 485 | 235225 | 114084125 | 22.0227155 | 7.8568281 | .002061856 |
| 486 | 236196 | 114791256 | 22.0454077 | 7.8622242 | .002057613 |
| 487 | 237169 | 115501303 | 22.0680765 | 7.8676130 | .002053388 |
| 488 | 238144 | 116214272 | 22.0907220 | 7.8729944 | .002049180 |
| 489 | 239121 | 116930169 | 22.1133444 | 7 8783684 | .002044990 |
| 490 | 240100 | 117649000 | 22.1359436 | 7.8837352 | .002040816 |
| 491 | 241081 | 118370771 | 22.1585198 | 7.8890946 | .002036660 |
| 492 | 242064 | 119095488 | 22.1810730 | 7.8944468 | .002032520 |
| 493 | 243049 | 119823157 | 22.2036033 | 7.8997917 | .002028398 |
| 494 | 244036 | 120553784 | 22.2261108 | 7.9051294 | .002024291 |
| 495 | 245025 | 121287375 | 22.2485955 | 7.9104599 | .002020202 |
| 496 | 246016 | 122023936 | 22.2710575 | 7.9157832 | .002016129 |

## Table V.—Squares, Cubes, Square Roots,

| No. | Squares. | Cubes. | Square Roots. | Cube Roots. | Reciprocals. |
|---|---|---|---|---|---|
| 497 | 247009 | 122763473 | 22.2934968 | 7.9210994 | .002012072 |
| 498 | 248004 | 123505992 | 22.3159136 | 7.9264085 | .002008032 |
| 499 | 249001 | 124251499 | 22.3383079 | 7.9317104 | .002004008 |
| 500 | 250000 | 125000000 | 22.3606798 | 7.9370053 | .002000000 |
| 501 | 251001 | 125751501 | 22.3830293 | 7.9422881 | .001996008 |
| 502 | 252004 | 126506008 | 22.4053565 | 7.9475739 | .001992032 |
| 503 | 253009 | 127263527 | 22.4276615 | 7.9528477 | .001988072 |
| 504 | 254016 | 128024064 | 22.4499443 | 7.9581144 | .001984127 |
| 505 | 255025 | 128787625 | 22.4722051 | 7.9633743 | .001980198 |
| 506 | 256036 | 129554216 | 22.4944438 | 7.9686271 | .001976285 |
| 507 | 257049 | 130323843 | 22.5166605 | 7.9738731 | .001972387 |
| 508 | 258064 | 131096512 | 22.5388553 | 7.9791122 | .001968504 |
| 509 | 259081 | 131872229 | 22.5610283 | 7.9843444 | .001964637 |
| 510 | 260100 | 132651000 | 22.5831796 | 7.9895697 | .001960784 |
| 511 | 261121 | 133432831 | 22.6053091 | 7.9947883 | .001956947 |
| 512 | 262144 | 134217728 | 22.6274170 | 8.0000000 | .001953125 |
| 513 | 263169 | 135005697 | 22.6495033 | 8.0052049 | .001949318 |
| 514 | 264196 | 135796744 | 22.6715681 | 8.0104032 | .001945525 |
| 515 | 265225 | 136590875 | 22.6936114 | 8.0155946 | .001941748 |
| 516 | 266256 | 137388096 | 22.7156334 | 8.0207794 | .001937984 |
| 517 | 267289 | 138188413 | 22.7376340 | 8.0259574 | .001934236 |
| 518 | 268324 | 138991832 | 22.7596134 | 8.0311287 | .001930502 |
| 519 | 269361 | 139798359 | 22.7815715 | 8.0362935 | .001926782 |
| 520 | 270400 | 140608000 | 22.8035085 | 8.0414515 | .001923077 |
| 521 | 271441 | 141420761 | 22.8254244 | 8.0466030 | .001919386 |
| 522 | 272484 | 142236648 | 22.8473193 | 8.0517479 | .001915709 |
| 523 | 273529 | 143055667 | 22.8691933 | 8.0568862 | .001912046 |
| 524 | 274576 | 143877824 | 22.8910463 | 8.0620180 | .001908397 |
| 525 | 275625 | 144703125 | 22.9128785 | 8.0671432 | .001904762 |
| 526 | 276676 | 145531576 | 22.9346899 | 8.0722620 | .001901141 |
| 527 | 277729 | 146363183 | 22.9564806 | 8.0773743 | .001897533 |
| 528 | 278784 | 147197952 | 22.9782506 | 8.0824800 | .001893939 |
| 529 | 279841 | 148035889 | 23.0000000 | 8.0875794 | .001890359 |
| 530 | 280900 | 148877000 | 23.0217289 | 8.0926723 | .001886792 |
| 531 | 281961 | 149721291 | 23.0434372 | 8.0977589 | .001883239 |
| 532 | 283024 | 150568768 | 23.0651252 | 8.1028390 | .001879699 |
| 533 | 284089 | 151419437 | 23.0867928 | 8.1079128 | .001876173 |
| 534 | 285156 | 152273304 | 23.1084400 | 8.1129803 | .001872659 |
| 535 | 286225 | 153130375 | 23.1300670 | 8.1180414 | .001869159 |
| 536 | 287296 | 153990656 | 23.1516738 | 8.1230962 | .001865672 |
| 537 | 288369 | 154854153 | 23.1732605 | 8.1281447 | .001862197 |
| 538 | 289444 | 155720872 | 23.1948270 | 8.1331870 | .001858736 |
| 539 | 290521 | 156590819 | 23.2163735 | 8.1382230 | .001855288 |
| 540 | 291600 | 157464000 | 23.2379001 | 8.1432529 | .001851852 |
| 541 | 292681 | 158340421 | 23.2594067 | 8.1482765 | .001848429 |
| 542 | 293764 | 159220088 | 23.2808935 | 8.1532939 | .001845018 |
| 543 | 294849 | 160103007 | 23.3023604 | 8.1583051 | .001841621 |
| 544 | 295936 | 160989184 | 23.3238076 | 8.1633102 | .001838235 |
| 545 | 297025 | 161878625 | 23.3452351 | 8.1683092 | .001834862 |
| 546 | 298116 | 162771336 | 23.3666429 | 8.1733020 | .001831502 |
| 547 | 299209 | 163667323 | 23.3880311 | 8.1782888 | .001828154 |
| 548 | 300304 | 164566592 | 23.4093998 | 8.1832695 | .001824818 |
| 549 | 301401 | 165469149 | 23.4307490 | 8.1882441 | .001821494 |
| 550 | 302500 | 166375000 | 23.4520788 | 8.1932127 | .001818182 |
| 551 | 303601 | 167284151 | 23.4733892 | 8.1981752 | .001814882 |
| 552 | 304704 | 168196608 | 23.4946802 | 8.2031319 | .001811594 |
| 553 | 305809 | 169112377 | 23.5159520 | 8.2080825 | .001808318 |
| 554 | 306916 | 170031464 | 23.5372046 | 8.2130271 | .001805054 |
| 555 | 308025 | 170953875 | 23.5584380 | 8.2179657 | .001801802 |
| 556 | 309136 | 171879616 | 23.5796522 | 8.2228985 | .001798561 |
| 557 | 310249 | 172808693 | 23.6008474 | 8.2278254 | .001795332 |
| 558 | 311364 | 173741112 | 23.6220236 | 8.2327463 | .001792115 |

| No. | Squares. | Cubes. | Square Roots. | Cube Roots. | Reciprocals. |
|---|---|---|---|---|---|
| 559 | 312481 | 174676879 | 23.6431808 | 8.2376614 | .001788909 |
| 560 | 313600 | 175616000 | 23.6643191 | 8.2425706 | .001785714 |
| 561 | 314721 | 176558481 | 23.6854386 | 8.2474740 | .001782531 |
| 562 | 315844 | 177504328 | 23.7065392 | 8.2523715 | .001779359 |
| 563 | 316969 | 178453547 | 23.7276210 | 8.2572633 | .001776199 |
| 564 | 318096 | 179406144 | 23.7486842 | 8.2621492 | .001773050 |
| 565 | 319225 | 180362125 | 23.7697286 | 8.2670294 | .001769912 |
| 566 | 320356 | 181321496 | 23.7907545 | 8.2719039 | .001766784 |
| 567 | 321489 | 182284263 | 23.8117618 | 8.2767726 | .001763668 |
| 568 | 322624 | 183250432 | 23.8327506 | 8.2816355 | .001760563 |
| 569 | 323761 | 184220000 | 23.8537209 | 8.2864928 | .001757469 |
| 570 | 324900 | 185193000 | 23.8746728 | 8.2913444 | .001754386 |
| 571 | 326041 | 186169411 | 23.8956063 | 8.2961903 | .001751313 |
| 572 | 327184 | 187149248 | 23.9165215 | 8.3010304 | .001748252 |
| 573 | 328329 | 188132517 | 23.9374184 | 8.3058651 | .001745201 |
| 574 | 329476 | 189119224 | 23.9582971 | 8.3106941 | .001742160 |
| 575 | 330625 | 190109375 | 23.9791576 | 8.3155175 | .001739130 |
| 576 | 331776 | 191102976 | 24.0000000 | 8.3203353 | .001736111 |
| 577 | 332929 | 192100033 | 24.0208243 | 8.3251475 | .001733102 |
| 578 | 334084 | 193100552 | 24.0416306 | 8.3299542 | .001730104 |
| 579 | 335241 | 194104539 | 24.0624188 | 8.3347553 | .001727116 |
| 580 | 336400 | 195112000 | 24.0831891 | 8.3395509 | .001724138 |
| 581 | 337561 | 196122941 | 24.1039416 | 8.3443410 | .001721170 |
| 582 | 338724 | 197137368 | 24.1246762 | 8.3491256 | .001718213 |
| 583 | 339889 | 198155287 | 24.1453929 | 8.3539047 | .001715266 |
| 584 | 341056 | 199176704 | 24.1660919 | 8.3586784 | .001712329 |
| 585 | 342225 | 200201625 | 24.1867732 | 8.3634466 | .001709402 |
| 586 | 343396 | 201230056 | 24.2074369 | 8.3682095 | .001706485 |
| 587 | 344569 | 202262003 | 24.2280829 | 8.3729668 | .001703578 |
| 588 | 345744 | 203297472 | 24.2487113 | 8.3777188 | .001700680 |
| 589 | 346921 | 204336469 | 24.2693222 | 8.3824653 | .001697793 |
| 590 | 348100 | 205379000 | 24.2899156 | 8.3872065 | .001694915 |
| 591 | 349281 | 206425071 | 24.3104916 | 8.3919423 | .001692047 |
| 592 | 350464 | 207474688 | 24.3310501 | 8.3966729 | .001689189 |
| 593 | 351649 | 208527857 | 24.3515913 | 8.4013981 | .001686341 |
| 594 | 352836 | 209584584 | 24.3721152 | 8.4061180 | .001683502 |
| 595 | 354025 | 210644875 | 24.3926218 | 8.4108326 | .001680672 |
| 596 | 355216 | 211708736 | 24.4131112 | 8.4155419 | .001677852 |
| 597 | 356409 | 212776173 | 24.4335834 | 8.4202460 | .001675042 |
| 598 | 357604 | 213847192 | 24.4540385 | 8.4249448 | .001672241 |
| 599 | 358801 | 214921799 | 24.4744765 | 8.4296383 | .001669449 |
| 600 | 360000 | 216000000 | 24.4948974 | 8.4343267 | .001666667 |
| 601 | 361201 | 217081801 | 24.5153013 | 8.4390098 | .001663894 |
| 602 | 362404 | 218167208 | 24.5356883 | 8.4436877 | .001661130 |
| 603 | 363609 | 219256227 | 24.5560583 | 8.4483605 | .001658375 |
| 604 | 364816 | 220348864 | 24.5764115 | 8.4530281 | .001655629 |
| 605 | 366025 | 221445125 | 24.5967478 | 8.4576906 | .001652893 |
| 606 | 367236 | 222545016 | 24.6170673 | 8.4623479 | .001650165 |
| 607 | 368449 | 223648543 | 24.6373,00 | 8.4670001 | .001647446 |
| 608 | 369664 | 224755712 | 24.6576560 | 8.4716471 | .001644737 |
| 609 | 370881 | 225866529 | 24.6779254 | 8.4762892 | .001642036 |
| 610 | 372100 | 226981000 | 24.6981781 | 8.4809261 | .001639344 |
| 611 | 373321 | 228099131 | 24.7184142 | 8.4855579 | .001636661 |
| 612 | 374544 | 229220928 | 24.7386338 | 8.4901848 | .001633987 |
| 613 | 375769 | 230346397 | 24.7588368 | 8.4948065 | .001631321 |
| 614 | 376996 | 231475544 | 24.7790234 | 8.4994233 | .001628664 |
| 615 | 378225 | 232608375 | 24.7991935 | 8.5040350 | .001626016 |
| 616 | 379456 | 233744896 | 24.8193473 | 8.5086417 | .001623377 |
| 617 | 380689 | 234885113 | 24.8394847 | 8.5132435 | .001620746 |
| 618 | 381924 | 236029032 | 24.8596058 | 8.5178403 | .001618123 |
| 619 | 383161 | 237176659 | 24.8797106 | 8.5224321 | .001615509 |
| 620 | 384400 | 238328000 | 24.8997992 | 8.5270189 | .001612908 |

| No. | Squares. | Cubes. | Square Roots. | Cube Roots. | Reciprocals. |
|---|---|---|---|---|---|
| 621 | 385641 | 239483061 | 24.9198716 | 8.5316009 | .001610306 |
| 622 | 386884 | 240641848 | 24.9239278 | 8.5361780 | .001607717 |
| 623 | 388129 | 241804867 | 24.9599679 | 8.5407501 | .001605136 |
| 624 | 389376 | 242970624 | 24.9799920 | 8.5453173 | .001602564 |
| 625 | 390625 | 244140625 | 25.0000000 | 8.5498797 | .001600000 |
| 626 | 391876 | 245314376 | 25.0199920 | 8. .544372 | .001597444 |
| 627 | 393129 | 246491883 | 25.0399681 | 8.5589899 | .001594896 |
| 628 | 394384 | 247673152 | 25.0599282 | 8.5635377 | .001592357 |
| 629 | 395641 | 248858189 | 25.0798724 | 8.5680807 | .001589825 |
| 630 | 396900 | 250047000 | 25.0998008 | 8.5726189 | .001587302 |
| 631 | 398161 | 251239591 | 25.1197134 | 8.5771523 | .001584786 |
| 632 | 399424 | 252435968 | 25.1396102 | 8.5816809 | .001582278 |
| 633 | 400689 | 253636137 | 25.1594913 | 8.5862047 | .001579779 |
| 634 | 401956 | 254840104 | 25.1793566 | 8.5907238 | .001577287 |
| 635 | 403225 | 256047875 | 25.1992063 | 8.5952380 | .001574803 |
| 636 | 404496 | 257259456 | 25.2190404 | 8.5997476 | .001572327 |
| 637 | 405769 | 258474853 | 25.2388589 | 8.6042525 | .001569859 |
| 638 | 407044 | 259694072 | 25.2586619 | 8.6087526 | .001567398 |
| 639 | 408321 | 260917119 | 25.2784493 | 8.6132480 | .001564945 |
| 640 | 409600 | 262144000 | 25.2982213 | 8.6177388 | .001562500 |
| 641 | 410881 | 263374721 | 25.3179778 | 8.6222248 | .001560062 |
| 642 | 412164 | 264609288 | 25.3377189 | 8.6267063 | .001557632 |
| 643 | 413449 | 265847707 | 25.3574447 | 8.6311830 | .001555210 |
| 644 | 414736 | 267089984 | 25.3771551 | 8.6356551 | .001552795 |
| 645 | 416025 | 268336125 | 25.3968502 | 8.6401226 | .001550388 |
| 646 | 417316 | 269586136 | 25.4165301 | 8.6445855 | .001547988 |
| 647 | 418609 | 270840023 | 25.4361947 | 8.6490437 | .001545595 |
| 648 | 419904 | 272097792 | 25.4558441 | 8.6534974 | .001543210 |
| 649 | 421201 | 273359449 | 25.4754784 | 8.6579465 | .001540832 |
| 650 | 422500 | 274625000 | 25.4950976 | 8.6623911 | .001538462 |
| 651 | 423801 | 275894451 | 25.5147016 | 8.6668310 | .001536098 |
| 652 | 425104 | 277167808 | 25.5342907 | 8.6712665 | .001533742 |
| 653 | 426409 | 278445077 | 25.5538647 | 8.6756974 | .001531394 |
| 654 | 427716 | 279726264 | 25.5734237 | 8.6801237 | .001529052 |
| 655 | 429025 | 281011375 | 25.5929678 | 8.6845456 | .001526718 |
| 656 | 430336 | 282300416 | 25.6124969 | 8.6889630 | .001524390 |
| 657 | 431649 | 283593393 | 25.6320112 | 8.6933759 | .001522070 |
| 658 | 432964 | 284890312 | 25.6515107 | 8.6977843 | .001519757 |
| 659 | 434281 | 286191179 | 25.6709953 | 8.7021882 | .001517451 |
| 660 | 435600 | 287496000 | 25.6904652 | 8.7065877 | .001515152 |
| 661 | 436921 | 288804781 | 25.7099203 | 8.7109827 | .001512859 |
| 662 | 438244 | 290117528 | 25.7293607 | 8.7153734 | .001510574 |
| 663 | 439569 | 291434247 | 25.7487864 | 8.7197596 | .001508296 |
| 664 | 440896 | 292754944 | 25.7681975 | 8.7241414 | .001506024 |
| 665 | 442225 | 294079625 | 25.7875939 | 8.7285187 | .001503759 |
| 666 | 443556 | 295408296 | 25.8069758 | 8.7328918 | .001501502 |
| 667 | 444889 | 296740963 | 25.8263431 | 8.7372604 | .001499250 |
| 668 | 446224 | 298077632 | 25.8456960 | 8.7416246 | .001497006 |
| 669 | 447561 | 299418309 | 25.8650343 | 8.7459846 | .001494768 |
| 670 | 448900 | 300763000 | 25.8843582 | 8.7503401 | .001492537 |
| 671 | 450241 | 302111711 | 25.9036677 | 8.7546918 | .001490313 |
| 672 | 451584 | 303464448 | 25.9229628 | 8.7590383 | .001488095 |
| 673 | 452929 | 304821217 | 25.9422435 | 8.7633809 | .001485884 |
| 674 | 454276 | 306182024 | 25.9615100 | 8.7677192 | .001483680 |
| 675 | 455625 | 307546875 | 25.9807621 | 8.7720532 | .001481481 |
| 676 | 456976 | 308915776 | 26.0000000 | 8.7763830 | .001479290 |
| 677 | 458329 | 310288733 | 26.0192237 | 8.7807084 | .001477105 |
| 678 | 459684 | 311665752 | 26.0384331 | 8.7850296 | .001474926 |
| 679 | 461041 | 313046839 | 26.0576284 | 8.7893466 | .001472754 |
| 680 | 462400 | 314432000 | 26.0768096 | 8.7936593 | .001470588 |
| 681 | 463761 | 315821241 | 26.0959767 | 8.7979679 | .001468429 |
| 682 | 465124 | 317214568 | 26.1151297 | 8.8022721 | .001466276 |

| No. | Squares. | Cubes. | Square Roots. | Cube Roots. | Reciprocals. |
|---|---|---|---|---|---|
| 683 | 466489 | 318611987 | 26.1342657 | 8.8065722 | .001464129 |
| 684 | 467856 | 320013504 | 26.1388937 | 8.8106681 | .001461988 |
| 685 | 469225 | 321419125 | 26.1725047 | 8.8151598 | .001459854 |
| 686 | 470596 | 322828856 | 26.1916017 | 8.8194474 | .001457726 |
| 687 | 471969 | 324242703 | 26.2106848 | 8.8237307 | .001455604 |
| 688 | 473344 | 325660672 | 26.2297541 | 8.8280099 | .001453488 |
| 689 | 474721 | 327082769 | 26.2488095 | 8.8322850 | .001451379 |
| 690 | 476100 | 328509000 | 26.2678511 | 8.8365559 | .001449275 |
| 691 | 477481 | 329939371 | 26.2868789 | 8.8408227 | .001447178 |
| 692 | 478864 | 331373888 | 26.3058929 | 8.8450854 | .001445087 |
| 693 | 480249 | 332812557 | 26.3248932 | 8.8493440 | .001443001 |
| 694 | 481636 | 334255384 | 26.3438797 | 8.8535985 | .001440922 |
| 695 | 483025 | 335702375 | 26.3628527 | 8.8578489 | .001438849 |
| 696 | 484416 | 337153536 | 26.3818119 | 8.8620952 | .001436782 |
| 697 | 485809 | 338608873 | 26.4007576 | 8.8663375 | .001434720 |
| 698 | 487204 | 340068392 | 26.4196896 | 8.8705757 | .001432665 |
| 699 | 488601 | 341532099 | 26.4386081 | 8.8748099 | .001430615 |
| 700 | 490000 | 343000000 | 26.4575131 | 8.8790400 | .001428571 |
| 701 | 491401 | 344472101 | 26.4764046 | 8.8832661 | .001426534 |
| 702 | 492804 | 345948408 | 26.4952826 | 8.8874882 | .001424501 |
| 703 | 494209 | 347428927 | 26.5141472 | 8.8917063 | .001422475 |
| 704 | 495616 | 348913664 | 26.5329983 | 8.8959204 | .001420455 |
| 705 | 497025 | 350402625 | 26.5518361 | 8.9001304 | .001418440 |
| 706 | 498436 | 351895816 | 26.5706605 | 8.9043366 | .001416431 |
| 707 | 499849 | 353393243 | 26.5894716 | 8.9085387 | .001414427 |
| 708 | 501264 | 354894912 | 26.6082694 | 8.9127369 | .001412429 |
| 709 | 502681 | 356400829 | 26 6270539 | 8.9169311 | .001410437 |
| 710 | 504100 | 357911000 | 26.6458252 | 8.9211214 | .001408451 |
| 711 | 505521 | 359425431 | 26.6645833 | 8.9253078 | .001406470 |
| 712 | 506944 | 360944128 | 26.6833281 | 8.9294902 | .001404494 |
| 713 | 508369 | 362467097 | 26.7020598 | 8.9336687 | .001402525 |
| 714 | 509796 | 363994344 | 26.7207784 | 8.9378433 | .001400560 |
| 715 | 511225 | 365525875 | 26.7394839 | 8.9420140 | .001398601 |
| 716 | 512656 | 367061696 | 26.7581763 | 8.9461809 | .001396648 |
| 717 | 514089 | 368601813 | 26.7768557 | 8.9503438 | .001394700 |
| 718 | 515524 | 370146232 | 26.7955220 | 8.9545029 | .001392758 |
| 719 | 516961 | 371694959 | 26.8141754 | 8.9586581 | .001390821 |
| 720 | 518400 | 373248000 | 26.8328157 | 8.9628095 | .001388889 |
| 721 | 519841 | 374805361 | 26.8514432 | 8.9669570 | .001386963 |
| 722 | 521284 | 376367048 | 26.8700577 | 8.9711007 | .001385042 |
| 723 | 522729 | 377933067 | 26.8886593 | 8.9752406 | .001383126 |
| 724 | 524176 | 379503424 | 26.9072481 | 8.9793766 | .001381215 |
| 725 | 525625 | 381078125 | 26.9258240 | 8.9835089 | .001379310 |
| 726 | 527076 | 382657176 | 26.9443872 | 8.9876373 | .001377410 |
| 727 | 528529 | 384240583 | 26.9629375 | 8.9917620 | .001375516 |
| 728 | 529984 | 385828352 | 26.9814751 | 8.9958829 | .001373626 |
| 729 | 531441 | 387420489 | 27.0000000 | 9.0000000 | .001371742 |
| 730 | 532900 | 389017000 | 27.0185122 | 9.0041134 | .001369863 |
| 731 | 534361 | 390617891 | 27.0370117 | 9.0082229 | .001367989 |
| 732 | 535824 | 392223168 | 27.0554985 | 9.0123288 | .001366120 |
| 733 | 537289 | 393832837 | 27.0739727 | 9.0164309 | .001364256 |
| 734 | 538756 | 395446904 | 27.0924344 | 9.0205293 | .001362398 |
| 735 | 540225 | 397065375 | 27.1108834 | 9.0246239 | .001360544 |
| 736 | 541696 | 398688256 | 27.1293199 | 9.0287149 | .001358696 |
| 737 | 543169 | 400315553 | 27.1477439 | 9.0328021 | .001356852 |
| 738 | 544644 | 401947272 | 27.1661554 | 9.0368857 | .001355014 |
| 739 | 546121 | 403583419 | 27.1845544 | 9.0409655 | .001353180 |
| 740 | 547600 | 405224000 | 27.2029410 | 9.0450419 | .001351351 |
| 741 | 549081 | 406869021 | 27.2213152 | 9.0491142 | .001349528 |
| 742 | 550564 | 408518488 | 27.2396769 | 9.0531831 | .001347709 |
| 743 | 552049 | 410172407 | 27.2580263 | 9.0572482 | .001345895 |
| 744 | 553536 | 411830784 | 27.2763634 | 9.0613098 | .001344086 |

| No. | Squares. | Cubes. | Square Roots. | Cube Roots. | Reciprocals. |
|---|---|---|---|---|---|
| 745 | 555025 | 413493625 | 27.2946881 | 9.0653677 | .001342282 |
| 746 | 556516 | 415160936 | 27.3180006 | 9.0694220 | .001340483 |
| 747 | 558009 | 416832723 | 27.3813007 | 9.0734726 | .001338688 |
| 748 | 559504 | 418508992 | 27.3495887 | 9.0775197 | .001336898 |
| 749 | 561001 | 420189749 | 27.3678644 | 9.0815631 | .001335113 |
| 750 | 562500 | 421875000 | 27.3861279 | 9.0856030 | .001333333 |
| 751 | 564001 | 423564751 | 27.4043792 | 9.0896392 | .001331558 |
| 752 | 565504 | 425259008 | 27.4226184 | 9.0936719 | .001329787 |
| 753 | 567009 | 426957777 | 27.4408455 | 9.0977010 | .001328021 |
| 754 | 568516 | 428661064 | 27.4590604 | 9.1017265 | .001326260 |
| 755 | 570025 | 430368875 | 27.4772632 | 9.1057485 | .001324503 |
| 756 | 571536 | 432081216 | 27.4954542 | 9.1097669 | .001322751 |
| 757 | 573049 | 433798093 | 27.5136330 | 9.1137818 | .001321004 |
| 758 | 574564 | 435519512 | 27.5317998 | 9.1177931 | .001319261 |
| 759 | 576081 | 437245479 | 27.5499546 | 9.1218010 | .001317523 |
| 760 | 577600 | 438976000 | 27.5680975 | 9.1258053 | .001315789 |
| 761 | 579121 | 440711081 | 27.5862284 | 9.1298061 | .001314060 |
| 762 | 580644 | 442450728 | 27.6043475 | 9.1338034 | .001312336 |
| 763 | 582169 | 444194947 | 27.62.4546 | 9.1377971 | .001310616 |
| 764 | 583696 | 445943744 | 27.6405499 | 9.1417874 | .001308901 |
| 765 | 585225 | 447697125 | 27.6586334 | 9.1457742 | .001307190 |
| 766 | 586756 | 449455096 | 27.6767050 | 9.1497576 | .001305483 |
| 767 | 588289 | 451217663 | 27.6947648 | 9.1537375 | .001303781 |
| 768 | 589824 | 452984832 | 27.7128129 | 9.1577139 | .001302083 |
| 769 | 591361 | 454756609 | 27.7308492 | 9.1616869 | .001300390 |
| 770 | 592900 | 456533000 | 27.7488739 | 9.1656565 | .001298701 |
| 771 | 594441 | 458314011 | 27.7668868 | 9.1696225 | .001297017 |
| 772 | 595984 | 460099648 | 27.7848880 | 9.1735852 | .001295337 |
| 773 | 597529 | 461889917 | 27.8028775 | 9.1775445 | .001293661 |
| 774 | 599076 | 463684824 | 27.8208555 | 9.1815003 | .001291990 |
| 775 | 600625 | 465484375 | 27.8388218 | 9.1854527 | .001290322 |
| 776 | 602176 | 467288576 | 27.8567766 | 9.1894018 | .001288660 |
| 777 | 603729 | 469097433 | 27.8747197 | 9.1933474 | .001287001 |
| 778 | 605284 | 470910952 | 27.8926514 | 9.1972897 | .001285347 |
| 779 | 606841 | 472729139 | 27.9105715 | 9.2012286 | .001283697 |
| 780 | 608400 | 474552000 | 27.9284801 | 9.2051641 | .001282051 |
| 781 | 609961 | 476379541 | 27.9463772 | 9.2090962 | .001280410 |
| 782 | 611524 | 478211768 | 27.9642629 | 9.2130250 | .001278772 |
| 783 | 613089 | 480048687 | 27.9821372 | 9.2169505 | .001277139 |
| 784 | 614656 | 481890304 | 28.0000000 | 9.2208726 | .001275510 |
| 785 | 616225 | 483736625 | 28.0178515 | 9.2247914 | .001273885 |
| 786 | 617796 | 485587656 | 28.0356915 | 9.2287068 | .001272265 |
| 787 | 619369 | 487443403 | 28.0535203 | 9.2326189 | .001270648 |
| 788 | 620944 | 489303872 | 28.0713377 | 9.2365277 | .001269036 |
| 789 | 622521 | 491169069 | 28.0891438 | 9.2404333 | .001267427 |
| 790 | 624100 | 493039000 | 28.1069386 | 9.2443355 | .001265823 |
| 791 | 625681 | 494913671 | 28.1247222 | 9.2482344 | .001264223 |
| 792 | 627264 | 496793088 | 28.1424946 | 9.2521300 | .001262626 |
| 793 | 628849 | 498677257 | 28.1602557 | 9.2560224 | .001261034 |
| 794 | 630436 | 500566184 | 28.1780056 | 9.2599114 | .001259446 |
| 795 | 632025 | 502459875 | 28.1957444 | 9.2637973 | .001257862 |
| 796 | 633616 | 504358336 | 28.2134720 | 9.2676798 | .001256281 |
| 797 | 635209 | 506261573 | 28.2311884 | 9.2715592 | .001254705 |
| 798 | 636804 | 508169592 | 28.2488938 | 9.2754352 | .001253133 |
| 799 | 638401 | 510082399 | 28.2665881 | 9.2793081 | .001251564 |
| 800 | 640000 | 512000000 | 28.2842712 | 9.2831777 | .001250000 |
| 801 | 641601 | 513922401 | 28.3019434 | 9.2870440 | .001248439 |
| 802 | 643204 | 515849608 | 28.3196045 | 9.2909072 | .001246883 |
| 803 | 644809 | 517781627 | 28.3372546 | 9.2947671 | .001245330 |
| 804 | 646416 | 519718464 | 28.3548938 | 9.2986239 | .001243781 |
| 805 | 648025 | 521660125 | 28.3725219 | 9.3024775 | .001242236 |
| 806 | 649636 | 523606616 | 28.3901391 | 9.3063278 | .001240695 |

| No. | Squares. | Cubes. | Square Roots. | Cube Roots. | Reciprocals. |
|---|---|---|---|---|---|
| 807 | 651249 | 525557943 | 28.4077454 | 9.3101750 | .001239157 |
| 808 | 652864 | 527514112 | 28.4253408 | 9.3140190 | .001237624 |
| 809 | 654481 | 529475129 | 28.4429253 | 9.3178599 | .001236094 |
| 810 | 656100 | 531441000 | 28.4604989 | 9.3216975 | .001234568 |
| 811 | 657721 | 533411731 | 28.4780617 | 9.3255320 | .001233046 |
| 812 | 659344 | 535387328 | 28.4956137 | 9.3293634 | .001231527 |
| 813 | 660969 | 537367797 | 28.5131549 | 9.3331916 | .001230012 |
| 814 | 662596 | 539353144 | 28.5306852 | 9.3370167 | .001228501 |
| 815 | 664225 | 541343375 | 28.5482048 | 9.3408386 | .001226994 |
| 816 | 665856 | 543338496 | 28.5657137 | 9.3446575 | .001225490 |
| 817 | 667489 | 545338513 | 28.5832119 | 9.3484731 | .001223990 |
| 818 | 669124 | 547343432 | 28.6006993 | 9.3522857 | .001222494 |
| 819 | 670761 | 549353259 | 28.6181760 | 9.3560952 | .001221001 |
| 820 | 672400 | 551368000 | 28.6356421 | 9.3599016 | .001219512 |
| 821 | 674041 | 553387661 | 28.6530976 | 9.3637049 | .001218027 |
| 822 | 675684 | 555412248 | 28.6705424 | 9.3675051 | .001216545 |
| 823 | 677329 | 557441767 | 28.6879766 | 9.3713022 | .001215067 |
| 824 | 678976 | 559476224 | 28.7054002 | 9.3750963 | .001213592 |
| 825 | 680625 | 561515625 | 28.7228132 | 9.3788873 | .001212121 |
| 826 | 682276 | 563559976 | 28.7402157 | 9.3826752 | .001210654 |
| 827 | 683929 | 565609283 | 28.7576077 | 9.3864600 | .001209190 |
| 828 | 685584 | 567663552 | 28.7749891 | 9.3902419 | .001207729 |
| 829 | 687241 | 569722789 | 28.7923601 | 9.3940206 | .001206273 |
| 830 | 688900 | 571787000 | 28.8097206 | 9.3977964 | .001204819 |
| 831 | 690561 | 573856191 | 28.8270706 | 9.4015691 | .001203369 |
| 832 | 692224 | 575930368 | 28.8444102 | 9.4053387 | .001201923 |
| 833 | 693889 | 578009537 | 28.8617394 | 9.4091054 | .001200480 |
| 834 | 695556 | 580093704 | 28.8790582 | 9.4128690 | .001199041 |
| 835 | 697225 | 582182875 | 28.8963666 | 9.4166297 | .001197605 |
| 836 | 698896 | 584277056 | 28.9136646 | 9.4203873 | .001196172 |
| 837 | 700569 | 586376253 | 28.9309523 | 9.4241420 | .001194743 |
| 838 | 702244 | 588480472 | 28.9482297 | 9.4278936 | .001193317 |
| 839 | 703921 | 590589719 | 28.9654967 | 9.4316423 | .001191895 |
| 840 | 705600 | 592704000 | 28.9827535 | 9.4353880 | .001190476 |
| 841 | 707281 | 594823321 | 29.0000000 | 9.4391307 | .001189061 |
| 842 | 708964 | 596947688 | 29.0172363 | 9.4428704 | .001187648 |
| 843 | 710649 | 599077107 | 29.0344623 | 9.4466072 | .001186240 |
| 844 | 712336 | 601211584 | 29.0516781 | 9.4503410 | .001184834 |
| 845 | 714025 | 603351125 | 29.0688837 | 9.4540719 | .001183432 |
| 846 | 715716 | 605495736 | 29.0860791 | 9.4577999 | .001182033 |
| 847 | 717409 | 607645423 | 29.1032644 | 9.4615249 | .001180638 |
| 848 | 719104 | 609800192 | 29.1204396 | 9.4652470 | .001179245 |
| 849 | 720801 | 611960049 | 29.1376046 | 9.4689661 | .001177856 |
| 850 | 722500 | 614125000 | 29.1547595 | 9.4726824 | .001176471 |
| 851 | 724201 | 616295051 | 29.1719043 | 9.4763957 | .001175088 |
| 852 | 725904 | 618470208 | 29.1890390 | 9.4801061 | .001173709 |
| 853 | 727609 | 620650477 | 29.2061637 | 9.4838136 | .001172333 |
| 854 | 729316 | 622835864 | 29.2232784 | 9.4875182 | .001170960 |
| 855 | 731025 | 625026375 | 29.2403830 | 9.4912200 | .001169591 |
| 856 | 732736 | 627222016 | 29.2574777 | 9.4949188 | .001168224 |
| 857 | 734449 | 629422793 | 29.2745623 | 9.4986147 | .001166861 |
| 858 | 736164 | 631628712 | 29.2916370 | 9.5023078 | .001165501 |
| 859 | 737881 | 633839779 | 29.3087018 | 9.5059980 | .001164144 |
| 860 | 739600 | 636056000 | 29.3257566 | 9.5096854 | .001162791 |
| 861 | 741321 | 638277381 | 29.3428015 | 9.5133699 | .001161440 |
| 862 | 743044 | 640503928 | 29.3598365 | 9.5170515 | .001160093 |
| 863 | 744769 | 642735647 | 29.3768616 | 9.5207303 | .001158749 |
| 864 | 746496 | 644972544 | 29.3938769 | 9.5244063 | .001157407 |
| 865 | 748225 | 647214625 | 29.4108823 | 9.5280794 | .001156069 |
| 866 | 749956 | 649461896 | 29.4278779 | 9.5317497 | .001154734 |
| 867 | 751689 | 651714363 | 29.4448637 | 9.5354172 | .001153403 |
| 868 | 753424 | 653972032 | 29.4618397 | 9.5390818 | .001152074 |

| No. | Squares. | Cubes. | Square Roots. | Cube Roots. | Reciprocals. |
|---|---|---|---|---|---|
| 869 | 755161 | 656234909 | 29.4788059 | 9.5427437 | .001150748 |
| 870 | 756900 | 658503000 | 29.4957624 | 9.5464027 | .001149425 |
| 871 | 758641 | 660776311 | 29.5127091 | 9.5500589 | .001148106 |
| 872 | 760384 | 663054848 | 29.5296461 | 9.5537123 | .001146789 |
| 873 | 762129 | 665338617 | 29.5465734 | 9.5573630 | .001145475 |
| 874 | 763876 | 667627624 | 29.5634910 | 9.5610108 | .001144165 |
| 875 | 765625 | 669921875 | 29.5803989 | 9.5646559 | .001142857 |
| 876 | 767376 | 672221376 | 29.5972972 | 9.5682982 | .001141553 |
| 877 | 769129 | 674526133 | 29.6141858 | 9.5719377 | .001140251 |
| 878 | 770884 | 676836152 | 29.6310648 | 9.5755745 | .001138952 |
| 879 | 772641 | 679151439 | 29.6479342 | 9.5792085 | .001137656 |
| 880 | 774400 | 681472000 | 29.6647939 | 9.5828397 | .001136364 |
| 881 | 776161 | 683797841 | 29.6816442 | 9.5864682 | .001135074 |
| 882 | 777924 | 686128968 | 29.6984848 | 9.5900939 | .001133787 |
| 883 | 779689 | 688465387 | 29.7153159 | 9.5937169 | .001132503 |
| 884 | 781456 | 690807104 | 29.7321375 | 9.5973373 | .001131222 |
| 885 | 783225 | 693154125 | 29.7489496 | 9.6009548 | .001129944 |
| 886 | 784996 | 695506456 | 29.7657521 | 9.6045696 | .001128668 |
| 887 | 786769 | 697864103 | 29.7825452 | 9.6081817 | .001127396 |
| 888 | 788544 | 700227072 | 29.7993289 | 9.6117911 | .001126126 |
| 889 | 790321 | 702595369 | 29.8161030 | 9.6153977 | .001124859 |
| 890 | 792100 | 704969000 | 29.8328678 | 9.6190017 | .001123596 |
| 891 | 793881 | 707347971 | 29.8496231 | 9.6226030 | .001122334 |
| 892 | 795664 | 709732288 | 29.8663690 | 9.6262016 | .001121076 |
| 893 | 797449 | 712121957 | 29.8831056 | 9.6297975 | .001119821 |
| 894 | 799236 | 714516984 | 29.8998328 | 9.6333907 | .001118568 |
| 895 | 801025 | 716917375 | 29.9165506 | 9.6369812 | .001117318 |
| 896 | 802816 | 719323136 | 29.9332591 | 9.6405690 | .001116071 |
| 897 | 804609 | 721734273 | 29.9499583 | 9.6441542 | .001114827 |
| 898 | 806404 | 724150792 | 29.9666481 | 9.6477367 | .001113586 |
| 899 | 808201 | 726572699 | 29.9833287 | 9.6513166 | .001112347 |
| 900 | 810000 | 729000000 | 30.0000000 | 9.6548938 | .001111111 |
| 901 | 811801 | 731432701 | 30.0166620 | 9.6584684 | .001109878 |
| 902 | 813604 | 733870808 | 30.0333148 | 9.6620403 | .001108647 |
| 903 | 815409 | 736314327 | 30.0499584 | 9.6656096 | .001107420 |
| 904 | 817216 | 738763264 | 30.0665928 | 9.6691762 | .001106195 |
| 905 | 819025 | 741217625 | 30.0832179 | 9.6727403 | .001104972 |
| 906 | 820836 | 743677416 | 30.0998339 | 9.6763017 | .001103753 |
| 907 | 822649 | 746142643 | 30.1164407 | 9.6798604 | .001102536 |
| 908 | 824464 | 748613312 | 30.1330383 | 9.6834166 | .001101322 |
| 909 | 826281 | 751089429 | 30.1496269 | 9.6869701 | .001100110 |
| 910 | 828100 | 753571000 | 30.1662063 | 9.6905211 | .001098901 |
| 911 | 829921 | 756058031 | 30.1827765 | 9.6940694 | .001097695 |
| 912 | 831744 | 758550528 | 30.1993377 | 9.6976151 | .001096491 |
| 913 | 833569 | 761048497 | 30.2158899 | 9.7011583 | .001095290 |
| 914 | 835396 | 763551944 | 30.2324329 | 9.7046989 | .001094092 |
| 915 | 837225 | 766060875 | 30.2489669 | 9.7082369 | .001092896 |
| 916 | 839056 | 768575296 | 30.2654919 | 9.7117723 | .001091703 |
| 917 | 840889 | 771095213 | 30.2820079 | 9.7153051 | .001090513 |
| 918 | 842724 | 773620632 | 30.2985148 | 9.7188354 | .001089325 |
| 919 | 844561 | 776151559 | 30.3150128 | 9.7223631 | .001088139 |
| 920 | 846400 | 778688000 | 30.3315018 | 9.7258883 | .001086957 |
| 921 | 848241 | 781229961 | 30.3479818 | 9.7294109 | .001085776 |
| 922 | 850084 | 783777448 | 30.3644529 | 9.7329309 | .001084599 |
| 923 | 851929 | 786330467 | 30.3809151 | 9.7364484 | .001083423 |
| 924 | 853776 | 788889024 | 30.3973683 | 9.7399634 | .001082251 |
| 925 | 855625 | 791453125 | 30.4138127 | 9.7434758 | .001081081 |
| 926 | 857476 | 794022776 | 30.4302481 | 9.7469857 | .001079914 |
| 927 | 859329 | 796597983 | 30.4466747 | 9.7504930 | .001078749 |
| 928 | 861184 | 799178752 | 30.4630924 | 9.7539979 | .001077586 |
| 929 | 863041 | 801765089 | 30.4795013 | 9.7575002 | .001076426 |
| 930 | 864900 | 804357000 | 30.4959014 | 9.7610001 | .001075269 |

| No. | Squares. | Cubes. | Square Roots. | Cube Roots. | Reciprocals. |
|---|---|---|---|---|---|
| 931 | 866761 | 806954491 | 30.5122926 | 9.7644974 | .001074114 |
| 932 | 868624 | 809557568 | 30.5286750 | 9.7679922 | .001072961 |
| 933 | 870489 | 812166237 | 30.5450487 | 9.7714845 | .001071811 |
| 934 | 872356 | 814780504 | 30.5614136 | 9.7749743 | .001070664 |
| 935 | 874225 | 817400375 | 30.5777697 | 9.7784616 | .001069519 |
| 936 | 876096 | 820025856 | 30.5941171 | 9.7819466 | .001068376 |
| 937 | 877969 | 822656953 | 30.6104557 | 9.7854288 | .001067236 |
| 938 | 879844 | 825293672 | 30.6267857 | 9.7889087 | .001066098 |
| 939 | 881721 | 827936019 | 30.6431069 | 9.7923861 | .001064963 |
| 940 | 883600 | 830584000 | 30.6594194 | 9.7958611 | .001063830 |
| 941 | 885481 | 833237621 | 30.6757233 | 9.7993336 | .001062699 |
| 942 | 887364 | 835896888 | 30.6920185 | 9.8028036 | .001061571 |
| 943 | 889249 | 838561807 | 30.7083051 | 9.8062711 | .001060445 |
| 944 | 891136 | 841232384 | 30.7245830 | 9.8097362 | .001059322 |
| 945 | 893025 | 843908625 | 30.7408523 | 9.8131989 | .001058201 |
| 946 | 894916 | 846590536 | 30.7571130 | 9.8166591 | .001057082 |
| 947 | 896809 | 849278123 | 30.7733651 | 9.8201169 | .001055966 |
| 948 | 898704 | 851971392 | 30.7896086 | 9.8235723 | .001054852 |
| 949 | 900601 | 854670349 | 30.8058436 | 9.8270252 | .001053741 |
| 950 | 902500 | 857375000 | 30.8220700 | 9.8304757 | .001052632 |
| 951 | 904401 | 860085351 | 30.8382879 | 9.8339238 | .001051525 |
| 952 | 906304 | 862801408 | 30.8544972 | 9.8373695 | .001050420 |
| 953 | 908209 | 865523177 | 30.8706981 | 9.8408127 | .001049318 |
| 954 | 910116 | 868250664 | 30.8868904 | 9.8442536 | .001048218 |
| 955 | 912025 | 870983875 | 30.9030743 | 9.8476920 | .001047120 |
| 956 | 913936 | 873722816 | 30.9192497 | 9.8511280 | .001046025 |
| 957 | 915849 | 876467493 | 30.9354166 | 9.8545617 | .001044932 |
| 958 | 917764 | 879217912 | 30.9515751 | 9.8579929 | .001043841 |
| 959 | 919681 | 881974079 | 30.9677251 | 9.8614218 | .001042753 |
| 960 | 921600 | 884736000 | 30.9838668 | 9.8648483 | .001041667 |
| 961 | 923521 | 887503681 | 31.0000000 | 9.8682724 | .001040583 |
| 962 | 925444 | 890277128 | 31.0161248 | 9.8716941 | .001039501 |
| 963 | 927369 | 893056347 | 31.0322413 | 9.8751135 | .001038422 |
| 964 | 929296 | 895841344 | 31.0483494 | 9.8785305 | .001037344 |
| 965 | 931225 | 898632125 | 31.0644491 | 9.8819451 | .001036269 |
| 966 | 933156 | 901428696 | 31.0805405 | 9.8853574 | .001035197 |
| 967 | 935089 | 904231063 | 31.0966236 | 9.8887673 | .001034126 |
| 968 | 937024 | 907039232 | 31.1126984 | 9.8921749 | .001033058 |
| 969 | 938961 | 909853209 | 31.1287648 | 9.8955801 | .001031992 |
| 970 | 940900 | 912673000 | 31.1448230 | 9.8989830 | .001030928 |
| 971 | 942841 | 915498611 | 31.1608729 | 9.9023835 | .001029866 |
| 972 | 944784 | 918330048 | 31.1769145 | 9.9057817 | .001028807 |
| 973 | 946729 | 921167317 | 31.1929479 | 9.9091776 | .001027749 |
| 974 | 948676 | 924010424 | 31.2089731 | 9.9125712 | .001026694 |
| 975 | 950625 | 926859375 | 31.2249900 | 9.9159624 | .001025641 |
| 976 | 952576 | 929714176 | 31.2409987 | 9.9193513 | .001024590 |
| 977 | 954529 | 932574833 | 31.2569992 | 9.9227379 | .001023541 |
| 978 | 956484 | 935441352 | 31.2729915 | 9.9261222 | .001022495 |
| 979 | 958441 | 938313739 | 31.2889757 | 9.9295042 | .001021450 |
| 980 | 960400 | 941192000 | 31.3049517 | 9.9328839 | .001020408 |
| 981 | 962361 | 944076141 | 31.3209195 | 9.9362613 | .001019368 |
| 982 | 964324 | 946966168 | 31.3368792 | 9.9396363 | .001018330 |
| 983 | 966289 | 949862087 | 31.3528308 | 9.9430092 | .001017294 |
| 984 | 968256 | 952763904 | 31.3687743 | 9.9463797 | .001016260 |
| 985 | 970225 | 955671625 | 31.3847097 | 9.9497479 | .001015228 |
| 986 | 972196 | 958585256 | 31.4006369 | 9.9531138 | .001014199 |
| 987 | 974169 | 961504803 | 31.4165561 | 9.9564775 | .001013171 |
| 988 | 976144 | 964430272 | 31.4324673 | 9.9598389 | .001012146 |
| 989 | 978121 | 967361669 | 31.4483704 | 9.9631981 | .001011122 |
| 990 | 980100 | 970299000 | 31.4642654 | 9.9665549 | .001010101 |
| 991 | 982081 | 973242271 | 31.4801525 | 9.9699095 | .001009082 |
| 992 | 984064 | 976191488 | 31.4960315 | 9.9732619 | .001008065 |

## TABLE V.—SQUARES, CUBES, SQUARE ROOTS,

| No. | Squares. | Cubes. | Square Roots. | Cube Roots. | Reciprocals. |
|---|---|---|---|---|---|
| 993 | 986049 | 979146657 | 31.5119025 | 9.9766120 | .001007049 |
| 994 | 988036 | 982107784 | 31.5277655 | 9.9799599 | .001006036 |
| 995 | 990025 | 985074875 | 31.5436206 | 9.9833055 | .001005025 |
| 996 | 992016 | 988047936 | 31.5594677 | 9.9866488 | .001004016 |
| 997 | 994009 | 991026973 | 31.5753068 | 9.9899900 | .001003009 |
| 998 | 996004 | 994011992 | 31.5911380 | 9.9933289 | .001002004 |
| 999 | 998001 | 997002999 | 31.6069613 | 9.9966656 | .001001001 |
| 1000 | 1000000 | 1000000000 | 31.6227766 | 10.0000000 | .001000000 |
| 1001 | 1002001 | 1003003001 | 31.6385840 | 10.0033322 | .0009990010 |
| 1002 | 1004004 | 1006012008 | 31.6543836 | 10.0066622 | .0009980040 |
| 1003 | 1006009 | 1009027027 | 31.6701752 | 10.0099899 | .0009970090 |
| 1004 | 1008016 | 1012048064 | 31.6859590 | 10.0133155 | .0009960159 |
| 1005 | 1010025 | 1015075125 | 31.7017349 | 10.0166389 | .0009950249 |
| 1006 | 1012036 | 1018108216 | 31.7175030 | 10.0199601 | .0009940358 |
| 1007 | 1014049 | 1021147343 | 31.7332633 | 10.0232791 | .0009930487 |
| 1008 | 1016064 | 1024192512 | 31.7490157 | 10.0265958 | .0009920635 |
| 1009 | 1018081 | 1027243729 | 31.7647603 | 10.0299104 | .0009910803 |
| 1010 | 1020100 | 1030301000 | 31.7804972 | 10.0332228 | .0009900990 |
| 1011 | 1022121 | 1033364331 | 31.7962262 | 10.0365330 | .0009891197 |
| 1012 | 1024144 | 1036433728 | 31.8119474 | 10.0398410 | .0009881423 |
| 1013 | 1026169 | 1039509197 | 31.8276609 | 10.0431469 | .0009871668 |
| 1014 | 1028196 | 1042590744 | 31.8433666 | 10.0464506 | .0009861933 |
| 1015 | 1030225 | 1045678375 | 31.8590646 | 10.0497521 | .0009852217 |
| 1016 | 1032256 | 1048772096 | 31.8747549 | 10.0530514 | .0009842520 |
| 1017 | 1034289 | 1051871913 | 31.8904374 | 10.0563485 | .0009832842 |
| 1018 | 1036324 | 1054977832 | 31.9061123 | 10.0596435 | .0009823183 |
| 1019 | 1038361 | 1058089859 | 31.9217794 | 10.0629364 | .0009813543 |
| 1020 | 1040400 | 1061208000 | 31.9374388 | 10.0662271 | .0009803922 |
| 1021 | 1042441 | 1064332261 | 31.9530906 | 10.0695156 | .0009794319 |
| 1022 | 1044484 | 1067462648 | 31.9687347 | 10.0728020 | .0009784736. |
| 1023 | 1046529 | 1070599167 | 31.9843712 | 10.0760863 | .0009775171 |
| 1024 | 1048576 | 1073741824 | 32.0000000 | 10.0793684 | .0009765625 |
| 1025 | 1050625 | 1076890625 | 32.0156212 | 10.0826484 | .0009756098 |
| 1026 | 1052676 | 1080045576 | 32.0312348 | 10.0859263 | .0009746589 |
| 1027 | 1054729 | 1083206683 | 32.0468407 | 10.0892019 | .0009737098 |
| 1028 | 1056784 | 1086373952 | 32.0624391 | 10.0924755 | .0009727626 |
| 1029 | 1058841 | 1089547389 | 32.0780298 | 10.0957469 | .0009718173 |
| 1030 | 1060900 | 1092727000 | 32.0936131 | 10.0990163 | .0009708738 |
| 1031 | 1062961 | 1095912791 | 32.1091887 | 10.1022835 | .0009699321 |
| 1032 | 1065024 | 1099104768 | 32.1247568 | 10.1055487 | .00 9689922 |
| 1033 | 1067089 | 1102302937 | 32.1403173 | 10.1088117 | .0009680542 |
| 1034 | 1069156 | 1105507304 | 32.1558704 | 10.1120726 | .0009671180 |
| 1035 | 1071225 | 1108717875 | 32.1714159 | 10.1153314 | .0009661836 |
| 1036 | 1073296 | 1111934656 | 32.1869539 | 10.1185882 | .0009652510 |
| 1037 | 1075369 | 1115157653 | 32.2024844 | 10.1218428 | .0009643202 |
| 1038 | 1077444 | 1118386872 | 32.2180074 | 10.1250953 | .0009633911 |
| 1039 | 1079521 | 1121622319 | 32.2335229 | 10.1283457 | .0009624639 |
| 1040 | 1081600 | 1124864000 | 32.2490310 | 10.1315941 | .0009615385 |
| 1041 | 1083681 | 1128111921 | 32.2645316 | 10.1348403 | .0009606148 |
| 1042 | 1085764 | 1131366088 | 32.2800248 | 10.1380845 | .0009596929 |
| 1043 | 1087849 | 1134626507 | 32.2955105 | 10.1413266 | .0009587738 |
| 1044 | 1089936 | 1137893184 | 32.3109888 | 10.1445667 | .0009578544 |
| 1045 | 1092025 | 1141166125 | 32.3264598 | 10.1478047 | .0009569378 |
| 1046 | 1094116 | 1144445336 | 32.3419233 | 10 1510406 | .0009560229 |
| 1047 | 1096209 | 1147730823 | 32.3573794 | 10 1542744 | .0009551098 |
| 1048 | 1098304 | 1151022592 | 32.3728281 | 10.1575062 | .0009541985 |
| 1049 | 1100401 | 1154320649 | 32.3882695 | 10.1607359 | .0009532888 |
| 1050 | 1102500 | 1157625000 | 32.4037035 | 10.1639636 | .0009523810 |
| 1051 | 1104601 | 1160935651 | 32.4191301 | 10.1671893 | .0009514748 |
| 1052 | 1106704 | 1164252608 | 32.4345495 | 10.1704129 | .0009505708 |
| 1053 | 1108809 | 1167575877 | 32.4499615 | 10.1736344 | .0009496676 |
| 1054 | 1110916 | 1170905464 | 32.4653662 | 10.1768539 | .0009487666 |

## TABLE VI.—DEFLECTION ANGLES FOR A SPIRAL WHEN THE CHORD = $R_1 \div 60$.

| Sta. 0 | Sta. 1 | Sta. 2 | Sta. 3 | Sta. 4 | Sta. 5 | Sta. 6 | Sta. 7 |
|---|---|---|---|---|---|---|---|
| | $R = 60C$ | $R = 30C$ | $R = 20C$ | $R = 15C$ | $R = 12C$ | $R = 10C$ | $R = 8.6C$ |
| Ins. at 0 | 0° 29′ | 1° 12′ | 2° 14′ | 3° 35′ | 5° 15′ | 7° 14′ | 9° 32′ |
| 0° 00′ | Ins. at 1 | 1 26 | 2 38 | 4 08 | 5 58 | 8 07 | 10 35 |
| 0 00 | 0 48 | Ins. at 2 | 3 06 | 4 46 | 6 46 | 9 04 | 11 42 |
| 0 00 | 0 53 | 2 01 | Ins. at 3 | 5 25 | 7 34 | 10 02 | 12 49 |
| 0 00 | 1 02 | 2 28 | 4 04 | Ins. at 4 | 8 21 | 10 59 | 13 55 |
| 0 00 | 1 12 | 2 42 | 4 32 | 6 41 | Ins. at 5 | 11 56 | 15 02 |
| 0 00 | 1 21 | 3 01 | 5 01 | 7 20 | 9 57 | Ins. at 6 | 16 09 |
| 0 00 | 1 31 | 3 21 | 5 30 | 7 58 | 10 45 | 13 51 | Ins. at 7 |

## TABLE VII.—RADII, FROG ANGLES, AND FROG DISTANCES FOR GIVEN NUMBER OF FROG.

| No. of Frog. | Radius. | Frog Angle. | Frog Distance. | No. of Frog. | Radius. | Frog Angle. | Frog Distance. |
|---|---|---|---|---|---|---|---|
| | Feet. | | Feet. | | Feet. | | Feet. |
| 2 | 37.66 | 28° 04′ | 18.83 | 5 | 235.41 | 11° 25′ | 47.08 |
| 2½ | 58.85 | 22 37 | 23.54 | 5½ | 284.83 | 10 23 | 51.79 |
| 3 | 84.75 | 18 55 | 28.25 | 6 | 338.99 | 9 32 | 56.50 |
| 3½ | 115.35 | 16 16 | 32.96 | 6½ | 397.83 | 8 48 | 61.20 |
| 4 | 150.67 | 14 15 | 37.66 | 7 | 461.39 | 8 10 | 65.91 |
| 4½ | 190.67 | 12 41 | 42.37 | 7½ | 529.65 | 7 38 | 70.62 |
| 5 | 235.41 | 11 25 | 47.08 | 8 | 602.63 | 7 09 | 75.33 |

## TABLE VIII.—FROG ANGLES AND FROG DISTANCES FOR GIVEN RADIUS.

| Radius. | Frog Angle. | Frog Distance. | Radius. | Frog Angle. | Frog Dis. tance. | Radius. | Frog Angle. | Frog Distance. |
|---|---|---|---|---|---|---|---|---|
| Feet. | | Feet. | Feet. | | Feet. | Feet. | | Feet. |
| 35 | 29° 04′ | 18.15 | 60 | 22° 24′ | 23.77 | 85 | 18° 53′ | 28.29 |
| 36 | 28 41 | 18.41 | 61 | 22 18 | 23.96 | 86 | 18 47 | 28.46 |
| 37 | 28 19 | 18.67 | 62 | 22 08 | 24.16 | 87 | 18 40 | 28.62 |
| 38 | 27 57 | 18.92 | 63 | 21 52 | 24.36 | 88 | 18 34 | 28.79 |
| 39 | 27 36 | 19.16 | 64 | 21 42 | 24.55 | 89 | 18 28 | 28.95 |
| 40 | 27 16 | 19.40 | 65 | 21 33 | 24.74 | 90 | 18 22 | 29.11 |
| 41 | 26 57 | 19.65 | 66 | 21 23 | 24.93 | 91 | 18 16 | 29.27 |
| 42 | 26 38 | 19.89 | 67 | 21 13 | 25.12 | 92 | 18 10 | 29.43 |
| 43 | 26 20 | 20.12 | 68 | 21 04 | 25.30 | 93 | 18 04 | 29.59 |
| 44 | 26 03 | 20.36 | 69 | 20 55 | 25.49 | 94 | 17 59 | 29.75 |
| 45 | 25 45 | 20.59 | 70 | 20 47 | 25.67 | 95 | 17 53 | 29 91 |
| 46 | 25 29 | 20.81 | 71 | 20 38 | 25.86 | 96 | 17 47 | 30.07 |
| 47 | 25 13 | 21.04 | 72 | 20 30 | 26.04 | 97 | 17 42 | 30.22 |
| 48 | 24 57 | 21.26 | 73 | 20 21 | 26.22 | 98 | 17 37 | 30.38 |
| 49 | 24 42 | 21.48 | 74 | 20 13 | 26.40 | 99 | 17 32 | 30.53 |
| 50 | 24 28 | 21.70 | 75 | 20 05 | 26.57 | 100 | 17 26 | 30.69 |
| 51 | 24 14 | 21.92 | 76 | 19 57 | 26.75 | 105 | 17 01 | 31.44 |
| 52 | 24 01 | 22.13 | 77 | 19 50 | 26.93 | 110 | 16 38 | 32.18 |
| 53 | 23 48 | 22.34 | 78 | 19 42 | 27.10 | 115 | 16 17 | 32 91 |
| 54 | 23 35 | 22.55 | 79 | 19 35 | 27.27 | 120 | 15 56 | 33.62 |
| 55 | 23 22 | 22.76 | 80 | 19 28 | 27.44 | 125 | 15 37 | 34.31 |
| 56 | 23 10 | 22.96 | 81 | 19 21 | 27.62 | 130 | 15 19 | 34.99 |
| 57 | 22 58 | 23.17 | 82 | 19 14 | 27.79 | 135 | 15 02 | 35.66 |
| 58 | 22 46 | 23.37 | 83 | 19 07 | 27.96 | 140 | 14 46 | 36.31 |
| 59 | 22 35 | 23.57 | 84 | 19 00 | 28.12 | 145 | 14 31 | 36.95 |
| 60 | 22 24 | 23.77 | 85 | 18 53 | 28.29 | 150 | 14 17 | 37.58 |

## TABLE IX.—USEFUL NUMBERS AND FORMULÆ.

| Title. | Symbol. | Number. | Logarithm. |
|---|---|---|---|
| Ratio of circumference to diameter..... | $\pi$ | 3.1415927 | 0.4971499 |
| Reciprocal of same ..... | $\frac{1}{\pi}$ | 0.3183099 | 9.5028501 |
| Degrees in arc of length equal to radius.... | $\frac{180^\circ}{\pi}$ | 57.295780 | 1.7581226 |
| Minutes " " " " " .... | $\frac{10800'}{\pi}$ | 3437.7468 | 3.5362739 |
| Seconds " " " " " .... | $\frac{648000''}{\pi}$ | 206264.81 | 5.3144251 |
| Length of 1° arc, radius unity..... | $\frac{\pi}{180^\circ}$ | .01745329 | 8.2418774 |
| Length of 1′ arc, " " ..... | $\frac{\pi}{10800}$ | .00029089 | 6.4637261 |
| Length of 1″ arc, " " ..... | $\frac{\pi}{648000}$ | .000004848 | 4.6855749 |
| Radius by which 1 foot of arc = 1 degree. | | 57.295780 | 1.7581226 |
| Radius " " $\frac{1}{60}$ " " = 1 minute. | | 343.77468 | 2.5362739 |
| Radius " " $\frac{1}{3600}$ " " = 10 seconds | | 206.26481 | 2.3144251 |
| Factors for dividing a line into extreme and mean ratio..... | | 0.6180340<br>0.3819660 | 9.7910124<br>9.5820248 |
| Base of hyperbolic logarithms..... | $\varepsilon$ | 2.7182818 | 0.4342945 |
| Modulus of common system of logs. = log $\varepsilon$ | $M$ | 0.4342945 | 9.6377843 |
| Reciprocal of same = hyp. log. 10..... | $\frac{1}{M}$ | 2.3025851 | 0.3622157 |
| Length of seconds pendulum at New York in inches ..... | | 39.11256 | 1.5923162 |
| Length of seconds pendulum at New York in feet..... | | 3.25938 | 0.5131350 |
| Acceleration due to gravity at New York... | $g$ | 32.1688 | 1.5074347 |
| Square root of same ..... | $\sqrt{g}$ | 5.67175 | 0.7537173 |
| Yards in 1 metre..... | | 1.093623 | 0.0388676 |
| Feet in 1 " ..... | | 3.280869 | 0.5159869 |
| Inches in 1 " ..... | | 39.37043 | 1.5951701 |
| Metres in 1 foot..... | | 0.304797 | 9.4840111 |
| Metres in 1 yard..... | | 0.914392 | 9.9611324 |
| Metres in 1 mile..... | | 1609.330 | 3.2066450 |

## TABLE IX.—USEFUL NUMBERS AND FORMULÆ

| Title. | Symbol. | Number. | Logarithm. |
|---|---|---|---|
| Cubic inches in 1 U. S. gallon | | 231. | 2.3636120 |
| " " " 1 Imperial gallon | | 277.274 | 2.4429092 |
| " " " 1 U. S. bushel | | 2150.42 | 3.3325233 |
| Cubic feet in 1 U. S. gallon | | 0.133681 | 9.1260683 |
| " " " 1 Imperial gallon | | 0.160459 | 9.2053655 |
| " " " 1 U. S. bushel | | 1.244456 | 0.0949796 |
| Weight of 1 cub. foot of water, barom. 30 in. | | | |
| ther. 39°.83 Fah.; pounds | | 62.379 | 1.7950384 |
| " 62° " " | | 62.321 | 1.7946349 |
| Weight in grains, 1 cubic inch, at 62° Fah. | | 252.458 | 2.4021892 |
| No. of grains in 1 pound avoir. | | 7000. | 3.8450980 |
| " " " 1 ounce " | | 437.5 | 2.6409781 |

$r$ = radius of circular arc;
$l$ = length of arc;
$a°$ = degrees in same arc.

$$a° = \frac{l}{r} \cdot \frac{180°}{\pi}$$
$$r = \frac{l}{a°} \cdot \frac{180°}{\pi}$$
$$l = a° r \cdot \frac{\pi}{180°}$$

Radius by which the length of chord $c$ in feet $= \frac{a'}{10}$ in minutes;

$$r = \frac{\frac{1}{2}a'}{10 \sin \frac{1}{2}a'}$$

Hyp. log $x$ = com. log $x \times \frac{1}{M}$, or

com. log (hyp. log $x$) = com. log (com. log $x$) + 0.3622157

Com. log $x = M \times$ hyp. log $x$; or

com. log (com. log $x$) = 9.6377843 + com. log (hyp. log $x$)

Circumference of circle (radius $= r$) ..... $2\pi r$

Area of circle ..... $\pi r^2$

Area of sector (length of arc $= l$) ..... $\frac{1}{2}lr$

Area of sector (angle of arc $= a°$) ..... $\frac{a}{360}\pi r^2$

Approximate area of segment (chord $= c$, mid. ord. $= m$) ..... $\frac{2}{3}cm$